Anderson

MECHANICS AND HEAT

PHYSICS FOR TECHNICAL STUDENTS

MECHANICS

AND

HEAT

BY

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PREFACE

The present volume is the outgrowth of mimeograph notes which the author has used in connection with a course of lectures given during the past six years. Since the author has also conducted the recitations for several sections during this time, the successive revisions of the notes have been made by one viewing the work from two angles, that of class instructor, as well as that of lecturer. It is believed that in this way a keener realization of the student's difficulties, and a better appreciation of what parts should be revised, have been obtained than would have been possible without this two-fold contact.

We now have a large and rapidly increasing number of students who are interested primarily in the practical side of education. With the needs of these students in mind, the practical side of the subject has been emphasized throughout the book. This method, it is believed, will sustain interest in the subject by showing its application to everyday affairs, and will, it is hoped, be appreciated by both students and instructors in Agriculture and Engineering. In this connection, attention is directed to sections 18, 19, 20, 29, 30, 39, 44, 54, 56, 60, 62, 63, 76, 80, 83, 108, 109, 111, 134, 138, 170, 185, 189, 190, 195, 200, 204, 205, 206, 218 and Chapters VII, XII, XVII, and XVIII.

More space than usual has been devoted to the treatment of Force, Torque, Translatory Motion, and Rotary Motion. It is felt that the great importance of these topics, which underlie so much of the subsequent work of the student, warrants such treatment. Probably everyone who has taught the theory of electrical measuring instruments, for example, has realized that the student's greatest handicap is the lack of a thorough grasp of the fundamental principles of mechanics. The student who has thoroughly mastered elementary mechanics has done much toward preparing himself for effective work in technical lines.

The sketches, which are more numerous than is usual in such a text, are chosen with special reference to the help they will be in enabling the student to readily grasp important or difficult principles. Wherever possible, every principle involved in the

text is brought up again in a problem; so that in working all of the problems a review of practically the entire book is obtained. For a complete course, the text should be accompanied by lectures and laboratory work.

In the treatment of many of the subjects, the author is indebted to various authors of works in Physics, among whom may be mentioned Professors Spinney, Duff, Watson, and Crew. The order in which the different subjects are treated is that which seems most logical and most teachable, and was given much thought.

Thanks are due Professor G. M. Wilcox, of the Department of Physics, Armour Institute, and Professor W. Weniger, of the Department of Physics at Oregon Agricultural College, for their careful reading of the original mimeograph notes and for the numerous suggestions which they offered. I wish also to thank my colleagues, Professor H. J. Plagge and Professor W. Kunerth, for reading of the manuscript and proofs, and for valuable suggestions. Thanks are also due to Professor W. R. Raymond of the English Department of this College for reading much of the manuscript during revision, and to Professor J. C. Bowman of the same department, for reading practically all of the manuscript just before it went to press.

IOWA STATE COLLEGE,
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W. B. A.

CONTENTS

PART II

PROPERTIES OF MATTER

CHAPTER VIII

CHAPTER IX

CHAPTER X

CHAPTER XI

CHAPTER XII

PART III

HEAT

CHAPTER XIII

CHAPTER XIV

CHAPTER XV

PART I

MECHANICS

MECHANICS AND HEAT

CHAPTER I

MEASUREMENT

1. The Three Fundamental Quantities.—The measurement of physical quantities is absolutely essential to an exact and scientific study of almost any physical phenomenon. For this reason, *Measurement* is usually the topic first discussed in a course in Physics. The popular expressions, "quite a distance," a "large quantity," etc., are too indefinite to satisfy the scientific mind. A physical quantity may be defined as anything that can be measured. The measurement of length, mass, and time are of special importance and will therefore be considered first. Indeed, almost all physical quantities may be expressed in terms of one or more of these three quantities, for which reason they are called the *fundamental quantities*. In the case of some physical quantities this is at once apparent. Thus, to measure the area of a piece of land, it is, as a rule, only necessary to measure the distance across it north and south (say L_1) and then east and west (L_2). The product of these two dimensions, L_1L_2, is then an area. If it is required to find how many "yards" of earth have been removed in digging a cellar, not only the length and width must be known, but also the depth (L_3). The result evidently involves a *length* (*i.e.*, distance) only, since *volume* = $L_1L_2L_3$. Coal, grain, etc., are measured in terms of *mass*. If the quantity involved is the time between two dates it is, of course, measured in terms of *time*. If a train goes from one city to another in a known time T, its average velocity is the distance between the two points (*i.e.*, a *length*) divided by the *time* required to traverse that distance, or

$$Velocity = \frac{L}{T}$$

A force may be measured in terms of the rate at which it changes the *velocity* of a body of known *mass* upon which it acts. Velocity, as we have just seen, is a quantity involving both

length and time; hence, force must be a quantity involving all three fundamental quantities. In like manner it may be shown that other physical quantities, *e.g.*, power, work, electric charge, electric current, etc., are expressible in terms of one or more of the three fundamental quantities—*length*, *mass*, and *time*.

2. Units and Numerics.—In order to measure and record the value of any quantity, it is necessary to have a unit of that same quantity in which to express the result. Thus if we measure the length of a board with a foot rule and find that we must apply it ten times, and that the remainder is then half the length of the rule, we say that the length of the board is 10½ ft. If this same board is measured with a yard stick, 3½ yds. is the result; while, if the inch is the unit, 126 inches is the result. Here the foot, the yard, or the inch is the *Unit*, and the 10½, 3½, or 126 is the *Numeric*. Evidently the larger the unit, the smaller the numeric, and *vice versa*. Thus, in expressing a weight of 2 tons as 4000 lbs., the numeric becomes 2000 times as large because the unit chosen is 1/2000 as large as before.

3. Fundamental Units.—In the *British System* of measurement, which is used in practical work in the United States and Great Britain, the units of length, mass, and time are respectively the *foot*, the *pound*, and the *second*. It is often termed the *foot-pound-second* system, or briefly the "F.P.S." system. Since, as has been pointed out, nearly all physical quantities may be expressed in terms of one or more of the above *quantities*, the above *units* are called *Fundamental Units*. (The fundamental units of the metric system are given in Sec. 5.)

4. Standards of Length, Mass, and Time.—If measurements made now are to be properly interpreted several hundred years later, it is evident that the units involved must not be subject to change. To this end the British Government has had made, and keeps at London, a bronze bar having near each end a fine transverse scratch on a gold plug. The distance between these two scratches, when the temperature of the bar is 62° Fahrenheit, is the *standard yard*. At the same place is kept a piece of platinum of 1 lb. mass. This bar and this piece of platinum are termed the *Standards* of length and mass respectively. The standard for time measurement is the *mean solar day*, and the second is then fixed as the $1/60 \times 1/60 \times 1/24$, or 1/86400 part of a mean solar day.

The Day—Sidereal, Solar, and Mean Solar.—Very few things so commonplace as the day, are so little understood. The time that elapses between two successive passages of a star (a true star, not a planet) across the meridian (a north and south line), in other words the time interval from "star noon" to "star noon," is a *Sidereal Day*. From "sun noon" to "sun noon" is a *Solar Day*. The longest solar day is nearly a minute longer than the shortest. The average of the 365 solar days is the *Mean Solar Day*. The mean solar day is the day commonly used. It is exactly 24 hours. The sidereal day, which is the exact time required for the earth to make one revolution on its axis, is nearly four minutes shorter than the mean solar day.

The cause for the four minutes difference between the sidereal day and the solar day may be indicated by two or three homely illustrations. If a silver dollar is rolled around another dollar, without slipping, it will be found that the moving dollar makes *two* rotations about its axis, while making *one* revolution about the stationary dollar. The moon always keeps the same side toward the earth, and for this very reason rotates once upon its axis for each revolution about the earth. Compare constantly facing a chair while you walk once around it. You will find that you have turned around (on an axis) once for each revolution about the chair. If, now, you turn around in the *same direction* as before, *three* times per revolution, you will find that you face the chair but *twice* per revolution. For exactly the same reason the earth musr rotate 366 times on its axis during one revolution about the sun, in ordet to "face" the sun 365 times. Consequently the sidereal day is, using round numbers, 365/366 as long as the mean solar day, or about four minutes shorter.

Variation in the Solar Day.—If the orbit of the earth around the sun were an exact circle, and if, further, the axis of rotation of the earth were at right angles to the plane of its orbit (plane of the ecliptic), then all solar days would be of equal length. The orbit, however, is slightly elliptical, the earth being nearer to the sun in winter and farther from it in summer than at other seasons; and the axis of the earth lacks 23°.5 of being at right angles to the plane of the ecliptic.

Let S, (Fig. 1) represent the sun, E, the earth on a certain day, and E', the earth a sidereal day later (distance EE' is exaggerated). Let the curved arrow indicate the rotary motion of the earth and the straight arrow, the motion in its orbit. When the earth is at E, it is noon at point A; *i.e.*, AS is vertical; while at E', the earth having made exactly one revolution, the vertical at A is AB, and it will not be noon until the vertical (hence the earth) rotates through the angle θ. This requires about four minutes (θ being much smaller than drawn), causing the solar day to be about four minutes longer than the sidereal day. The stars are so distant that if AS points toward a star, then AB, which is parallel to it, points at the same star so far as the eye can detect.

Hence the sidereal day gives, as above stated, the exact time of one revolution of the earth.

When the earth is nearest to the sun (in December) it travels fastest; *i.e.*, when AS is shortest, EE' is longest. Obviously both of these changes increase θ and hence make the solar day longer. The effect of the above 23°.5 angle, in other words, the effect due to the obliquity of the earth's axis, is best explained by use of a model. We may simply state, however, that due to this cause the solar day in December is still further lengthened. As a result it is nearly a minute longer than the shortest solar day, which is in September.

When the solar days are longer than the mean solar day (24 hour day) the sun crosses the meridian, *i.e.*, "transit" occurs, later and later each day; while when they are shorter, the transit occurs earlier each day. In February, transit occurs at about 12:15 mean solar time (*i.e.*, clock time), at which date the almanac records sun "slow" 15 minutes. In early November the sun is about 15 minutes "fast." These are the two extremes.

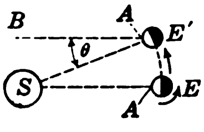

FIG. 1.

5. The Metric System.—This system is in common use in most civilized countries except the United States and Great Britain, while its scientific use is universal. The fundamental units of the *Metric System* of measurement are the *centimeter*, the *gram*, and the *second*. It is accordingly called the *centimeter-gram-second* system, or briefly the "C.G.S." system. This system far surpasses the British system in simplicity and facility in computation, because its different units for the measurement of the same quantity are related by a ratio of 10, or 10 to some integral power, as 100, 1000, etc. The *centimeter* (cm.) is the 1/100 part of the length of a certain platinum bar when at the temperature of melting ice. This bar, whose length is 1 meter (m.), is kept at Paris by the French Government. The *gram* is the 1/1000 part of the mass of a certain piece of platinum (the standard kilogram) kept at the same place. The milligram is 1/1000 gm., and the millimeter (mm.) is 1/1000 meter. The *second* is the same as in the British system. The above meter bar and kilogram mass are respectively the *Standards* of length and mass in the Metric System.

6. Conversion of Units.—In this course both systems of units will be used, because both are frequently met in general reading. Some practice will also be given in converting results expressed in terms of the units of one system into units of the

other (see problems at the close of this chapter). To do this it is only necessary to know that 1 inch = 2.54 cm. and 1 kilogram (= 1000 gm.) = 2.2046 lbs., or approximately 2.2 lbs. These two ratios should be memorized, and perhaps also the fact that the meter = 39.37 in. From the first ratio it will be seen that the numeric is made 30.48 (or 12×2.54) times as large whenever a certain length is expressed in centimeters instead of in feet. The relation between all other units in the two systems can readily be obtained if the above *two* ratios are known.

7. Measurement of Length.—The method employed in measuring the length of any object or the distance between any two points, will depend upon the magnitude of the distance to be measured, and the accuracy with which the result must be determined. For many purposes, either the meter stick or the foot rule answers very well; while for other purposes, such as the measurement of the thickness of a sheet of paper, both are obviously useless. For more accurate measurements, several instruments are in use, prominent among which are the vernier caliper, the micrometer caliper, and the micrometer microscope.

8. The Vernier Caliper.—In Fig. 2 is shown a simplified form of the vernier caliper from which the important principle of the vernier may be readily comprehended. This vernier cali-

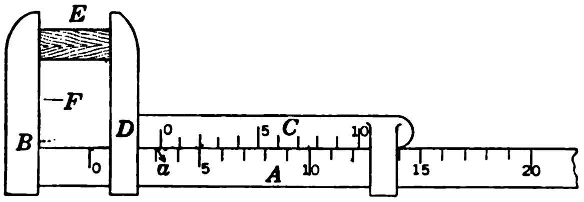

Fig. 2.

per consists of a bar A, having marked near one edge a scale in millimeter divisions Rigidly attached to A is the jaw B, whose face F is accurately perpendicular to A, and parallel to the face of jaw D, attached to bar C. C may be slid along A until D strikes B, if there is nothing between the jaws. While in this position, a scale of equal divisions is ruled upon C having its zero line in coincidence with the zero line of A, and its tenth line in coincidence with the *ninth* line on A. The scale on C is called the *vernier* scale and that on A, the *main* scale. Obviously, the vernier divisions are 1/10 mm. shorter than the main

scale divisions; *i.e.*, they are 9/10 as long, since 10 vernier divisions just equal 9 scale divisions.

To measure the length of the block E, place it between the jaws D and B, as shown. Since the two zero lines coincide when the jaws are together, the length of the block must be equal to the distance between the two zeros, or 3 mm., plus the small distance a. But if line 2 on the vernier coincides with a line on the main scale, as shown, then a is simply the difference in length between 2 *vernier* divisions and 2 *main scale* divisions, or 0.2 mm. The length of E is then 3.2 mm.

If C were slid to the right 1/10 mm., line 3 on the vernier would coincide with a main scale line, and a would then equal 0.3 mm.; so that the distance between the jaws would be 3.3 mm. Evidently, the above 1/10 mm. is the *least* motion of C that can be *directly* measured by the vernier. This distance (1/10 mm.) is called the *sensitiveness* of this vernier. If the divisions on A had been made 1/20 inch, and 25 vernier divisions had been made equal to 24 main scale divisions, then the sensitiveness or difference between the length of a main scale division and a vernier division would be 1/500 inch. For the vernier divisions, being 1/25 *division* shorter than the main scale divisions (*i.e.*, 24/25 as long), are $1/25 \times 1/20$ or 1/500 inch shorter.

This arrangement of two scales of slightly different spacing, free to slide past each other, is an application of the *Vernier Principle*. This principle is much employed in making measuring instruments. Instead of having 10 vernier spaces equal to 9 spaces on the main scale, the ratio may be 25 to 24 as mentioned, or 50 to 49, 16 to 15, etc., according to the use that is to be made of the instrument. In the case of circular verniers and scales on surveying instruments, the above-mentioned ratio is usually 30 to 29 or else 60 to 59, because they are to be read in degrees, minutes, and seconds of arc. If the vernier principle is thoroughly understood, there should be no difficulty in reading any vernier, whether straight or circular, in which a convenient ratio is employed.

9. The Micrometer Caliper.—The micrometer caliper (Fig. 3) consists of a metal yoke A, a stop S, a screw B whose threads fit accurately the threads cut in the hole through A, and a sleeve C rigidly connected to B. When B and S are in contact, the edge E of C is at the zero of scale D; consequently the distance from S to B, in other words the thickness of the block F

as sketched, is equal to the distance from this zero to E. If the figure represents the very common form of micrometer caliper in which the "pitch" of B (*i.e.*, the distance B advances for each revolution) is 1/2 mm., D is a scale of millimeter divisions, and the circumference of C at E is divided into 50 equal divisions; then the thickness of F is 4.5 mm. plus the slight distance that B moves when E turns through 6 of its divisions, or 6/50 of a revolution. But $6/50 \times 1/2$ mm. = 0.06 mm.; so that the thickness of E is $4.5+0.06$ or 4.56 mm. It should be explained that if the instrument is properly adjusted, then, when B and S are in contact, the zero of E and the zero of D coincide. Accordingly if the zero of E were exactly in line with scale D, then 4.5 would be the result. As sketched, however, it is 6/50 of a revolution past the position of alignment with D, which

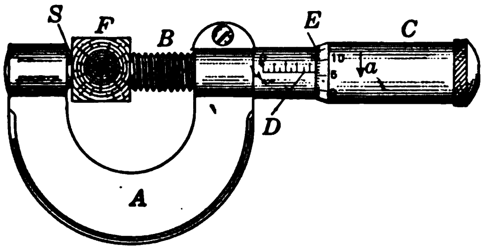

FIG. 3.

adds 0.06 mm. to the distance between B and S as already shown.

If C were turned in the direction of arrow a through 1/50 revolution, then line 7 of E, instead of line 6, would come in line with D, and B would have moved $1/50 \times 1/2$ mm., or 0.01 mm. farther from S. This, the least change in setting that can be read directly without estimating, is called the *Sensitiveness* of an instrument (see Sec. 8). Thus the sensitiveness of this micrometer caliper is 0.01 mm.

10. The Micrometer Microscope.—The micrometer microscope consists of an ordinary compound microscope, having movable crosshairs in the barrel of the instrument where the magnified image of the object to be measured is formed. These crosshairs may be moved by turning a micrometer screw similar to B in Fig. 3.

If it is known how many turns are required to cause the crosshairs to move over one space of a millimeter scale, placed on

the stage of the microscope, and also what part of a turn will cause them to move the width of a small object also placed on the stage, the diameter of the object can be at once calculated.

11. Measurement of Mass, Inertia.—Consider two large pieces of iron, provided with suitable handles for seizing them, each one resting upon a light and nearly frictionless truck on a level steel track, and hence capable of being moved in a horizontal direction witb great freedom. If a person is brought blindfolded and permitted to touch only the handles, he can very quickly tell by jerking tbem to and fro horizontally, which one contains the greater amount of iron. If one piece of iron is removed and replaced by a piece of wood of the same size as the remaining piece of iron, he would immediately detect that the piece of wood moved more easily and would perhaps think it to be a very small piece of iron. The difference which he detects is certainly not difference in *volume*, as he is not permitted either to see or to feel them; neither is it difference in *weight*, since he does not *lift* them. It is difference in *Mass* that he detects. Hence *Mass* may be defined as *that property of matter by virtue of which it resists being suddenly set into motion, or, if already in motion, resists being suddenly brought to rest.*

Inertia and *Mass* are synonymous; inertia being used in a general way only, while mass is used in a general, qualitative way and also in a quantitative way. Thus we speak of a large mass, great inertia, a 5-lb. mass, etc., but not of 5 lbs. inertia.

If it were possible, by the above method, for the person to make accurate determinations, and if he found that one piece had just twice as much mass as the other, then upon weighing them it would be found that one piece was exactly twice as heavy as the other. In other words, the *Weight of any body is proportional to its Mass.* The weight of a body is simply the attractive pull of the earth upon it; hence we see that the pull of the earth upon any body depends upon the mass of the body, and therefore affords a very convenient, and also very accurate means of comparing masses.

Thus the druggist, using a simple beam balance, "weighs out" a pound mass of any chemical by placing a standard pound mass in one pan and then pouring enough of the chemical into the other pan to exactly "balance" it. That is, the amount of chemical in one pan is varied until the pull of the earth on the chemical at one end of the beam is made exactly equal to the

pull of the earth on the standard pound mass at the other end. He then knows, since the pull of the earth on each is equal, that their weights, and consequently their masses, are equal. Weights, and hence masses, may be compared also by means of the steel-yard, the spring balance, and the platform scale. These devices will be discussed later in the course.

The *mass* of a body is absolutely *constant* wherever it is determined, while its *weight* becomes very *slightly less* as it is taken up a mountain or taken toward the equator. This is due partly to the fact that the body is slightly farther from the earth's center at those points, and partly to the rotary motion of the earth (see centrifugal force, Sec. 58). The polar diameter of the earth is about 27 miles less than its equatorial diameter. A given object weighed at St. Louis and then at St. Paul with the same *spring balance* should show an increase in weight at the latter place; whereas if weighed with the same *beam balance* at both places, there should be no difference in the weights read. The weight of the object actually does increase, but the weight of the counterbalancing standard masses used with the beam balance also increases in the same proportion.

12. Measurement of Time.—A modern instrument for measuring time must have these three essentials: (1) a device for measuring equal intervals of time, *i.e.*, for time "spacing," (2) a driving mechanism, (3) a recording mechanism. In the case of the clock, (1) is the pendulum, (2) is the mainspring or weights, train of wheels and escapement, and (3) is the train of wheels and the hands. In the watch, the balance wheel and hairspring take the place of the pendulum.

The necessity for the pendulum or its equivalent, and the recording mechanism, is obvious. Friction makes the driving mechanism necessary. The escapement clutch attached to the pendulum is shaped with such a slant that each time it releases a cog of the escapement wheel it receives from that wheel a slight thrust just sufficient to compensate for friction, which would otherwise soon bring the pendulum to rest. If the pendulum, as it vibrates, releases a cog each second, and if the escapement wheel has 20 cogs, the latter will, of course, make a revolution in 20 seconds. It is then an easy matter to design a connecting train of geared wheels and pinions between it and the post to which the minute hand is attached, so that the latter will make one revolution in an hour. In the same

way the hour hand is caused to make one revolution in twelve hours.

In the hourglass of olden times, and in the similar device, the clepsydra or water dropper of the Ancient Greeks, only the time "spacing" is automatic. The observer became the driving mechanism by inverting the hourglass at the proper moment; and by either remembering or recording how many times he had inverted it, he became also the recording mechanism.

Other time measurers, in which only time spacing is present, are the earth and the moon. The rotation of the earth about its axis determines our day, while its revolution about the sun determines our year. The revolution of the moon about the earth determines our lunar month, which is about 28 days.

PROBLEMS

1. What is the height in feet and inches of a man who is 1 m. 80 cm. tall? Reduce 5 ft. 4.5 in. to centimeters.

2. What does a 160-lb. man weigh in grams? In kilograms? Reduce 44 kilograms 240 grams to pounds.

3. Reduce 100 yds. to meters. What part of a mile is the kilometer?

4. A cubic centimeter of gold weighs 19.3 gm. Find the weight of 1 cu. ft. of gold in grams. In pounds.

5. One cm.^{3} of glycerine weighs 1.27 gms. How many pounds will 1 gallon (231 in.^{3}) weigh?

6. If a man can run 100 yds. in 10 sec., how long will he require for the 100 meter dash? Assume the same average velocity for both.

7. If, in Fig. 2, the main scale divisions were 1/16 inch, and 20 vernier divisions were equal to 19 divisions on the main scale, other conditions being as shown, what would be the length of E?

8. The pitch of a certain micrometer caliper is 1/20 inch and the screw head has 25 divisions. After setting upon a block and then removing it, 7 complete turns and 4 divisions are required to cause the screw to advance to the stop. What is the thickness of the block?

9. Between the jaws of a vernier caliper (Fig. 2) is placed a block of such length that line 5 of the vernier scale coincides with line 10 of the main scale, and consequently the zero of the vernier scale is a short distance to the right of line 5 of the main scale. If the main scale divisions are 1/2 mm., and 25 vernier divisions are equal in length to 24 main scale divisions, what is the length of the block?

10. What is the sensitiveness (see Sec. 8) of the vernier caliper in problem 7? In problem 9? What is the sensitiveness of the micrometer caliper in problem 8?

CHAPTER II

VECTORS

13. Scalars and Vectors Defined.—All physical quantities may be divided into two general classes, *Scalars* and *Vectors.* A scalar quantity is one that is fully specified if its magnitude only is given; while to specify a vector quantity completely, not only its *magnitude,* but also its *direction* must be given. Hence vectors might be called *directed quantities.*

Such quantities as volume, mass, work, energy, and quantity of heat or of electricity, do not have associated with them any idea of direction, and are therefore scalars. Force, pressure, and velocity, must have direction as well as magnitude given or they are not completely specified; therefore they are vectors. Thus, if the statement is made that a certain ship left port at a speed of 20 miles per hour, the motion of the ship is not fully known. The statement that the ship's *velocity* was 20 miles an hour due north, completely specifies the motion of the ship, and conveys the *full* meaning of velocity. This distinction between speed and velocity is not always observed in popular language, but it must be observed in technical work.

If two forces F_1 and F_2 act upon a body, say a boat in still water, they will produce no effect, if equal and *opposed; i.e.,* if the angle between the two forces is 180°. If this angle is zero, *i.e.,* if both forces act in the same direction, their *Resultant* F (Sec. 15), or the single *force* that would produce the *same effect* upon the boat as both F_1 and F_2, is simply their sum, or

$$F = F_1 + F_2 \tag{1}$$

If F_1 is greater than F_2, then when the angle between them is 180°, that is when F_1 and F_2 are oppositely directed, we have

$$F = F_1 - F_2 \tag{2}$$

The resultant F has in Eq. 1 its maximum value, and in Eq. 2 its minimum value. It may have any value varying between these limits, as the angle between F_1 and F_2 varies from zero to 180°.

In contrast with the above statements, observe that in scalar addition the result is always simply the arithmetical sum. Thus, 15 qts. and 10 qts. are 25 qts.; while the resultant of a 15-lb. pull and a 10-lb. pull may have any value between 5 lbs. and 25 lbs. and it may also have any direction, depending upon the directions of the two pulls.

Note that such *physical objects* as a stone or a train are *neither scalars* nor *vectors*. Several *physical quantities* relating to a stone are *scalars;* viz., its mass, volume, and density; while some are *vectors;* viz., its weight, and, if in motion, its velocity.

14. Representation of Vectors by Straight Lines.—A very simple and rapid method of calculating vectors, called the *Graphical Method,* depends upon the fact that a vector may be completely represented by a straight line having at one end an arrow head. Thus to represent the velocity of a southwest

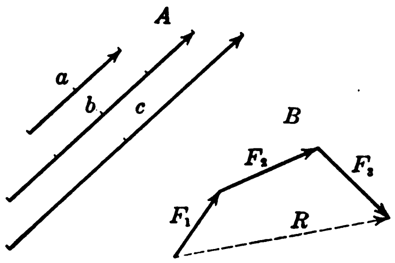

FIG. 4.

wind blowing at the rate of 12 miles an hour, a line (*a*) 2 cm. long, or (*b*) 4 cm. long, or (*c*) 2 inches long, may be used as shown at A, Fig. 4. In case (*a*), 1 cm. represents 6 miles an hour; while in case (*b*) it represents 3 mi. an hour. In case (*c*) the scale is chosen the same as in case (*a*), except that 1 in., instead of 1 cm., represents 6 miles an hour velocity. Any convenient scale may be chosen. In each case the length of the line represents the *magnitude* of the vector quantity; and the direction of the line represents the *direction* of the vector quantity.

15. Addition of Vectors, Resultant.—The vector sum or *Resultant* (see Sec. 13) of two or more forces or other vectors differs in general from either the arithmetical or the algebraic sum. By the *Graphical method,* it may be found as follows. Choose a suitable scale and represent the first force F_1 by a line

having an arrow head as shown at B, Fig. 4. Next, from the arrow point of this line, draw a second line representing the second force F_2, and from the arrow point of F_2 draw a line representing F_3, etc. Finally connect the beginning of the first line with the arrow point of the last by a straight line. The length of this line, say in inches, multiplied by the number of pounds which one inch represents in the scale chosen, gives the

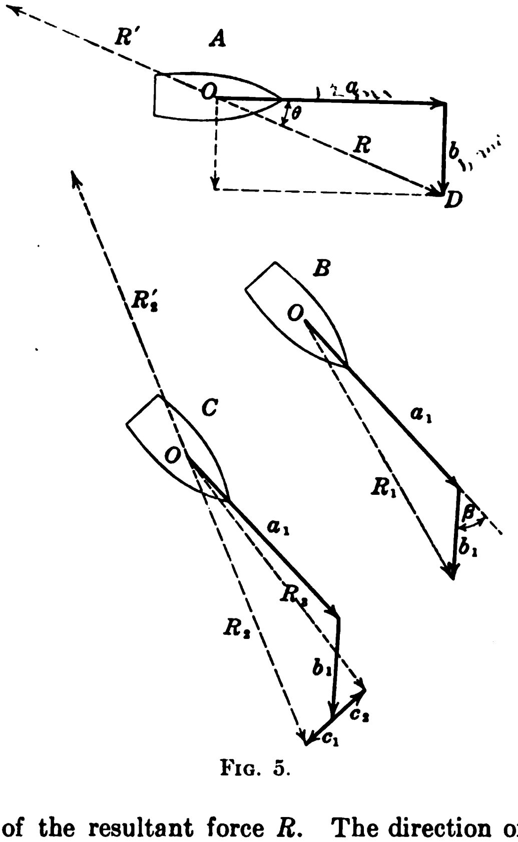

Fig. 5.

magnitude of the resultant force R. The direction of this line gives the direction of the resultant force. Obviously, the same scale must be used throughout. An example involving several *velocities* will further illustrate this method of adding vectors. Although in this course we shall apply the graphical method to only *force* and *velocity*, it should be borne in mind that it may be, and indeed is, applied to any vector quantity.

A steamboat, which travels 12 miles an hour in still water, is

headed due east across a stream which flows south at the rate of 5 miles an hour. Let us find the velocity of the steamboat. In an hour, the boat would move eastward a distance of 12 miles due to the action of the propeller, even if the river did not flow; while if the propeller should stop, the flow of the river alone would cause the boat to drift southward 5 miles in an hour. Consequently, if subjected to the action of both propeller and current for an hour, the steamboat would be both 12 miles farther east and 5 miles farther south, or at D (case A, Fig. 5). By choosing 1 cm. to represent 4 miles per hr., the "steam" velocity would be represented by a line a, 3 cm. in length; while the "drift" velocity of 5 miles an hour to this *same scale*, would be represented by a line b, 1.25 cm. in length. The length (3.25 cm.) of the line OD or R represents the *magnitude* of the steamboat's velocity, and the direction of this line gives the course of the boat, or the *direction* of its velocity. The velocity is then 4×3.25 or 13 miles an hour to the south of east by an angle θ as shown. This velocity R, of 13 miles per hour, is the *resultant* or vector sum of the two velocities a and b, and is evidently the actual velocity of the steamboat.

By the analytical method, the magnitude of the resultant velocity is given by the equation

$$R = \sqrt{(12)^2 + (5)^2}$$

and its direction is known from the equation

$$\tan \theta = 5/12 = 0.417$$

from which $$\theta = 22°.38.$$

If the steamboat is headed southeast, then a_1 and b_1 (case B, Fig. 5) represent the "steam" and "drift" velocities respectively, and the magnitude of the resultant velocity R_1, in miles per hr., will be found by multiplying the length of R_1 in centimeters by 4. If the analytical method is employed, we have from trigonometry,

$$R_1^2 = a_1^2 + b_1^2 + 2a_1b_1 \cos \beta$$

Suppose, further, that it is required to find the actual velocity of a man who is walking toward the right side of the steamboat at the rate of 2 miles an hour, when the boat is headed as shown in case B. Let a_1, b_1, and c_1 represent the "steam," "drift," and "walking" velocities respectively; then R_2 represents the

actual velocity of the man as shown in case C, Fig. 5. If the man walks toward the *left* side of the boat, his "walking" velocity is c_2 and his actual velocity is R_3. In these cases his velocity could also be found by the analytical method, but not so readily.

16. The Vector Polygon.—In cases A and B (Fig. 5), the vector triangle is used in determining the resultant; while in case C, the vector polygon, whose sides are a_1, b_1, c_1 and R_2, is so used. In general, however, many vectors are involved, the *closing* side of the polygon represents the resultant of all the other vectors.

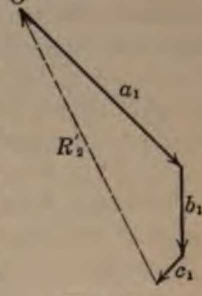

Fig. 5a.

If a man were to run toward the left and rear end of the steamboat in the direction R' at the speed of 13 miles per hour (case A), he would appear to an observer on shore to be standing still with respect to the shore. Hence his actual velocity is zero. Since R' is equal to R and oppositely directed, we see that the three vectors a, b, and R' would form exactly the same triangle as a, b, and R, but for the fact that the arrow head on R' points in the opposite direction to that on R. Thus vectors forming a *closed triangle* have a resultant equal to zero.

Again, suppose that the man while walking toward the right side of the ship, case C (Fig. 5), and therefore having an actual velocity R_2, should throw a ball with an equal velocity R'_2 in a direction exactly opposite to that of R_2 (*i.e.*, $R'_2 = -R_2$). It will be evident at once that the ball under these circumstances would simply stand still in the air as far as *horizontal* motion is concerned. It will be seen that there are four horizontal velocities imparted to the ball. First, the "steam" velocity a_1 (Fig. 5a), second, the "drift" velocity b_1, third, the "walking" velocity c_1, and fourth, the "throwing" velocity R'_2. These four velocities, however, form a closed polygon and the actual velocity of the ball is zero. Hence we may now make the general statement that when *any number of velocities* (or forces or any other vectors) *form a Closed Triangle or a Closed Polygon, their resultant is zero.* This fact is of great importance and convenience in the treatment of forces in equilibrium and will be made use of in some of the problems at the close of this chapter.

17. Vectors in Equilibrium.—The method of the preceding sections applies equally well if the vectors involved are any other

quantities; *e.g.*, forces, instead of velocities; and the constructions are made in the same way. This method has many important applications in connection with forces, among which is the calculation of the proper elevation of the outer rail on a curve (Sec. 62) in order that the weight, or better, the thrust of a train shall be equal upon both rails; and the calculation of the proper strength for the different parts of bridges and other structures.

In Sec. 16 it was shown that to find in what direction and with what speed the man must throw the ball in order to make its actual velocity zero, a line R'_2 must be drawn equal to R_2, but oppositely directed. R_2 is the resultant of the three veloci-

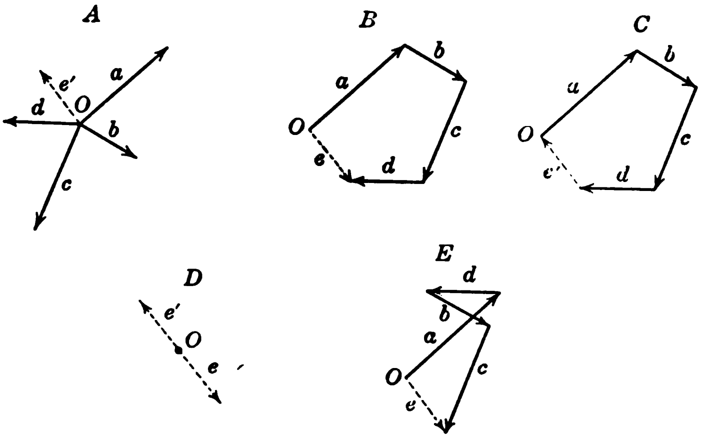

FIG. 6.

ties a_1, b_1, and c_1, while R'_2 is the *Antiresultant* (anti = opposed to) or *Equilibrant.*

Thus it will be seen that in the graphical method the antiresultant of any number of velocities is represented by a line drawn from the arrow point of the last velocity to the beginning of the first velocity. In other words, it is represented by the closing side of the vector polygon. Observe that in this case the arrow heads all point in the *same* way around the polygon; while, if the closing side is the resultant, its arrow head is directed oppositely to all the others.

The case of several forces in equilibrium, or so-called "balanced forces," is of special importance. The construction is the same as that shown in Fig. 5*a*. Suppose that a body floating in still water is acted upon by four horizontal forces, whose

values are represented both in magnitude and direction by the lines a, b, c, and d of A (Fig. 6). Let it be required to find the magnitude and direction of a fifth force e', which applied to the body will produce equilibrium, so that the body will have no tendency to move in any direction; in other words, let us find the antiresultant of a, b, c, and d. From B (Fig. 6) we find the *resultant* e, or that single force which would exactly *replace* a, b, c, and d; *i.e.*, which *alone* would move the body in the same direction, and with the same speed as would these four forces. The construction C shows how e' is found. Obviously, e' and e alone (D, Fig. 6) would produce equilibrium, and since e is exactly equivalent to a, b, c, and d, it follows that a, b, c, d, and e' produce equilibrium. From E (Fig. 6) it will be seen that the resultant is the same if the vectors a, b, c, and d are taken in a different order.

This is true for the reason that wherever, in the construction of the polygon, we choose to draw d, say, the pencil point will thereby be moved a definite distance to the left. Likewise drawing b moves the pencil a definite distance to the right and downward. Consequently the final position of the pencil after drawing lines a, b, c, and d, which position determines the resultant e, can in no wise depend upon the *order* of drawing these lines.

18. The Crane.—The crane, in its simplest form, is shown in Fig. 7. B is a rigid beam, pivoted at its lower end and fastened at its upper end by a cable C to a post A. D is the "block and tackle" for raising the object L whose weight is W. After the object is raised, the beam B may be swung around horizontally; and then, by means of the block and tackle, the object may be lowered and deposited where it is wanted. By shortening the cable C it is possible to raise the weight higher, but the "sweep" of the crane is of course shortened thereby.

The traveling crane, used in factories, is mounted on a "carriage" which may be run back and forth on a track sometimes extending the entire length of the building, so that a massive machine weighing several tons may readily be picked up and carried to any part of the building.

In choosing the size of the cable and the beam for a crane as sketched, it is necessary to know what pull will be exerted on C, and what end thrust on B when the maximum load is being lifted. These two forces, c and b, we shall now proceed to find.

In Sec. 17 it was shown that any number of forces or any other vectors in equilibrium are represented by a closed polygon. Three forces in equilibrium will accordingly form a closed triangle. The point O, at the upper end of the beam B, is obviously in equilibrium and is acted upon by the three forces W, c, and b; which forces, graphically represented, must therefore form a *closed triangle.* The directions of b and c are known but not their magnitudes. W, however, is fully specified both as to direction and magnitude. Hence the forces acting upon O

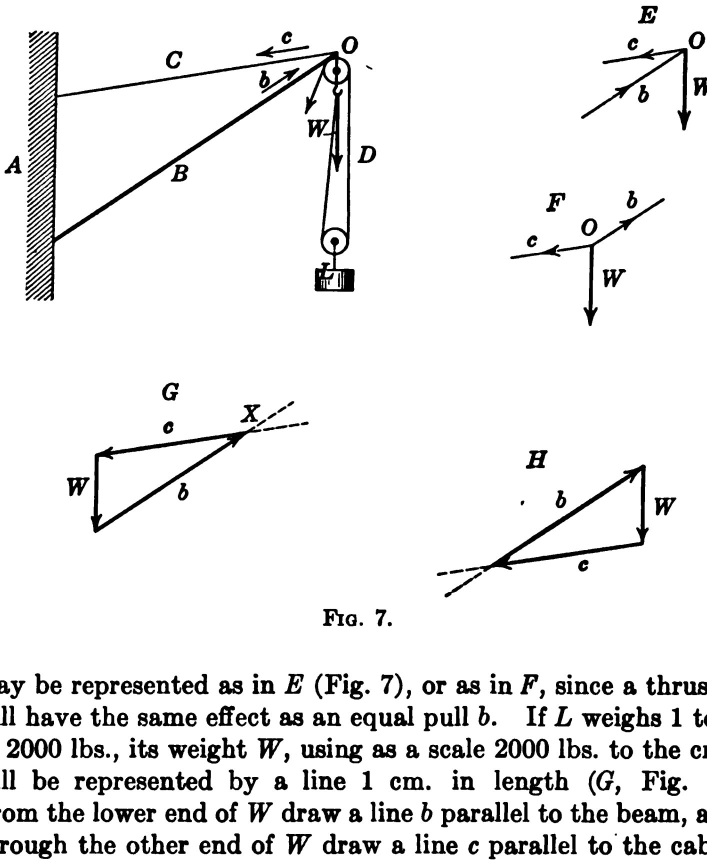

FIG. 7.

may be represented as in E (Fig. 7), or as in F, since a thrust b will have the same effect as an equal pull b. If L weighs 1 ton, or 2000 lbs., its weight W, using as a scale 2000 lbs. to the cm., will be represented by a line 1 cm. in length (G, Fig. 7). From the lower end of W draw a line b parallel to the beam, and through the other end of W draw a line c parallel to the cable. The intersection of these two lines at X determines the magnitude of both b and c. For the three forces have the required directions, and they also form a closed triangle, thus representing equilibrium. The length of b in centimeters times 2000 lbs.

gives the thrust on the beam. The value of c is found in the same way. The construction may also be made as shown in H.

The problem will be seen to be simply this: *Given one side W of a triangle, both in direction and length, and the directions only of the other two sides b and c; let it be required to construct the triangle.*

19. Resolution of Vectors into Components.—*The resolution of a vector V into two components, consists in finding the magnitude of two vectors, V_1 and V_2, whose directions are given, and whose vector sum shall be the given vector V.* It is thus seen to be the converse of vector addition. The method will be best understood from one or two applications. We shall here apply it to velocities and forces, but it applies equally well to any other vector quantity.

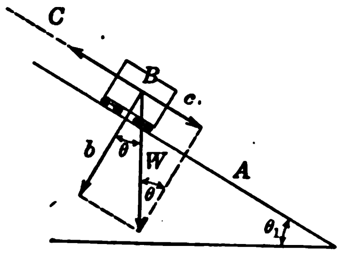

Fig. 8.

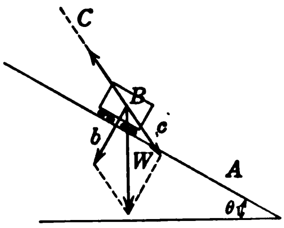

Fig. 9.

A ship is traveling with a uniform velocity of 20 mi. per hr. in a direction somewhat south of east. An hour later the ship is 18 mi. farther east and 8.7 mi. farther south than when first observed. Under such circumstances the velocity of the ship may be resolved into an eastward component of 18 mi. per hr. and a southward component of 8.7 mi. per hr. Had the ship been headed nearly south, the southward component would have been the larger. We shall next resolve a force into two components.

Consider a car B (Fig. 8) of weight W, held by a cable C from running down the inclined track A. Let it be required to find the pull c that the car exerts upon the cable, and also the force b that it exerts against the track. The latter is of course at right angles to the track, but it is not equal to the weight of the car, as might at first be supposed. In fact, the weight of the car W, or the force with which the earth pulls upon it, gives rise to the two forces, b and c. The directions of b and c are

known, but not their magnitudes. In order to find their magnitudes, first draw W to a suitable scale. Then, from the arrow point of W, draw two lines, one *parallel to b* and *intersecting c*, the other *parallel to c* and *intersecting b*. These *intersections* determine the magnitudes of both b and c, as shown. We may also determine b and c by the method used in the solution of the crane problem.

If the cable is attached to a higher point, the construction is as shown in Fig. 9. It will be noticed that under these conditions the c component has become larger, and the b component smaller, than in Fig. 8. If the cable is fastened directly above the car, the b component is zero; that is, the car is simply suspended by the cable.

In case a force is resolved into two components at right angles to each other, their values may be readily found by the analytical method. Thus in Fig. 8, $c = W \sin\theta$, and $b = W \cos\theta$. Note that $\theta_1 = \theta$.

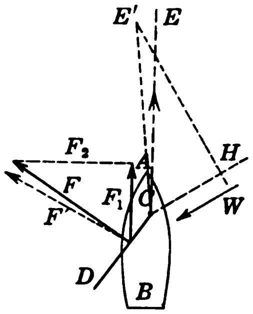

Fig. 10.

20. Sailing Against the Wind.—Although sailing "into the wind" by "tacking" has been practised by sea-faring people from time immemorial, it is still a puzzle to many. Let AB (Fig. 10) represent a sailing vessel, CD its sail, CE the direction in which it is headed, and W the direction of the wind. If the sail CD were frictionless and perfectly flat, the reaction of the air in striking it would give rise to a force F strictly at right angles to the sail. A push (force) against a frictionless surface, whether exerted by the wind or by any other means, must be normal to the surface; otherwise it would have a component parallel to the surface, which is impossible if there is no friction. This force F may be resolved into the two components F_1 and F_2 as shown. Although as sketched, F_2 is greater than the useful component F_1, nevertheless the *sidewise* drift of the ship is small compared with its forward motion, because of its greater resistance to motion in that direction. Making slight allowance for this leeward drift, we have CE' for the course of the ship. Obviously, in going from C to E', the ship goes the distance CH "into" the wind.

In case the boat is moving north at a high velocity, the wind, to

a person on the boat, will appear to come from a point much more nearly north than it would to a stationary observer. In other words, the angle between the plane of the sail and the *real* direction of the wind, is always greater than the angle between this plane and the *apparent* direction of the wind as observed by an occupant of the boat. It is, however, the apparent direction or, perhaps better, the relative velocity of the wind, that determines the reacting thrust upon the sail. Hence strictly, W (Fig. 10) should represent the apparent direction of the wind. It is a matter of common observation that, to a man driving rapidly north, an east wind appears to come from a point considerably north of east.

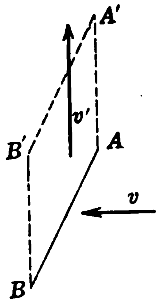

FIG. 11.

Because of the very slight friction of the wind on the sail, F' is more nearly the direction of the push on the sail. The useful component of F', which drives the ship, is obviously slightly less than F_1 as found above for the theoretical case of no friction.

21. Sailing Faster Than the Wind.—It is possible, strange though it may seem, to make an iceboat travel faster than the wind that drives it. Let AB (Fig. 11) represent the sail (only) of an iceboat which is traveling due north, and v the velocity of the wind. If the runner friction were zero, so that no power would be derived from the moving air, the air would move on *unchanged* in both *direction* and *speed*. Considering the air that strikes at A, this would evidently require the sail to travel the distance AA' while the wind traveled from A to B'. Hence the velocity v' of the boat would be AA'/AB' times that of the wind, or $v'/v = AA'/AB'$. The slight friction between the runners and the ice reduces this ratio somewhat; nevertheless, under favorable circumstances, an iceboat may travel twice as fast as the wind. Velocities as high as 85 mi. per hr. have been maintained for short distances.

PROBLEMS

1. A balloon is traveling at the rate of 20 miles an hour due southeast. Find its eastward and southward components of velocity by both the graphical and analytical methods.

2. Find the force required to draw a wagon, which with its load weighs 2500 lbs., up a grade rising 40 ft. in a distance of 200 ft. measured on the grade. Neglect friction, and use the graphical method.

3. Find R_1, case B (Fig. 5) if $\beta = 60°$. ($\cos 60° = 0.5$). Use the analytical method.

4. A boat which travels at the rate of 10 mi. an hr. in still water, is headed S.W. across a stream flowing south at the rate of 4 mi. an hr. A man on the deck runs at the rate of 7 mi. an hr. toward a point on the boat which is due east of him. Find the actual velocity of the man with respect to the earth, and also that of the boat. Use graphical method.

5. By the graphical method, find the resultant and antiresultant of the following four forces: 10 lbs. N., 12 lbs. N.E., 15 lbs. E., and 8 lbs. S.

6. If the beam B (Fig. 7) is 30 ft. in length and makes an angle of 45° with the horizontal, and the guy cable C is fastened 15 ft. above the lower end of B, what will be the thrust on B and the pull on C if the load L weighs 3000 lbs.? Use the graphical method.

7. After a man has traveled 4 miles east, and 4 miles N., how far must he travel N.W. before he will be due north of the starting place, and how far will he then be from the starting place? Solve by both the graphical method and the analytical method.

8. A certain gun, with a light charge of powder, gives its projectile an initial (muzzle) velocity of 300 ft. per sec. when stationary. If this gun is on a car whose velocity is 100 ft. per sec. north, what will be the muzzle velocity of the projectile if the gun is fired N.? If fired S.? If fired E.?

9. A south wind is blowing at the rate of 30 mi. per hr. Find, by the graphical method and also by the analytical method, the apparent velocity of the wind as observed by a man standing on a car which is traveling east at the rate of 40 mi. per hr.

10. The instruments on a ship going due north at the rate of 20 miles an hour record a wind velocity of 25 miles per hour from the N.E. What is the actual velocity of the wind? Use the graphical method.

11. A tight rope, tied to two posts A and B which are 20 ft. apart, is pulled sidewise at its middle point a distance of 1 ft. by a force of 100 lbs. By two graphical methods (Sec. 18 and 19) find the pull exerted on the posts. Solve also by the analytical method.

12. Neglecting friction, find the pull on the cable and the thrust on the track in drawing a 1000-lb. car up a 45° incline. The cable is parallel to the track.

13. Find the pull and the thrust (Prob. 12) if the cable is (a) horizontal; (b) inclined 30° above the horizontal.

CHAPTER III

TRANSLATORY MOTION

22. Kinds of Motion.—All motion may be classed as either *translatory* motion or *rotary* motion, or as a combination of these two. A body has motion of translation only, when *any* line (which means every line) in the body remains parallel to its original position throughout the motion. It may also be defined as a motion in which each particle of the body describes a path of the same form and length as that of every other particle, and at the same speed at any given instant; so that the motion of any *one particle* represents completely the motion of the entire body. Thus if A, B, C, and D represent the positions of a

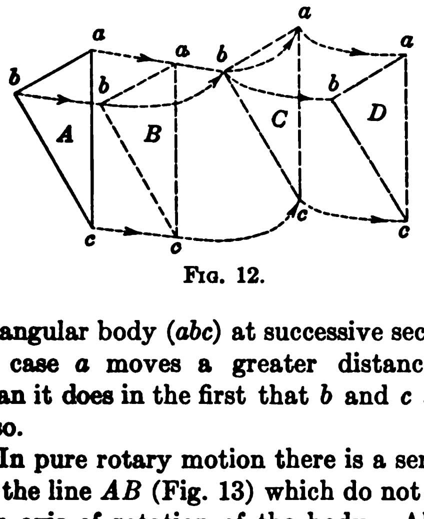

Fig. 12.

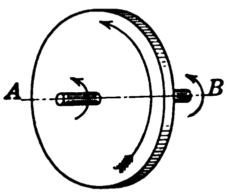

Fig. 13.

triangular body (abc) at successive seconds, it will be noted that in case a moves a greater distance in the second second than it does in the first that b and c and all other particles do also.

In pure rotary motion there is a series of particles, *e.g.*, those in the line AB (Fig. 13) which do not move. This line is called the axis of rotation of the body. All other particles move in circular paths about this axis as a center, those particles farthest from the axis having the highest velocity.

Having obtained a clear notion of rotary motion, we may consider a body to have pure translatory motion if it moves from one point to another by any path, however straight or crooked, without any motion of rotation. The rifle ball has what is

termed *Screw* motion. The motion of a steamship might seem to be pure translatory motion, and indeed it closely approximates such motion when the sea is calm. In a rough sea its motion is very complicated, consisting of a combination of translatory motion, with to-and-fro rotation about three axes. In the "rolling" of a ship, the axis is lengthwise of the ship or longitudinal. The "pitching" of a ship is a to-and-fro rotation about a transverse axis. As the ship swerves slightly from its course, it rotates about a vertical axis.

Both translatory and rotary motion may be either *uniform*, or accelerated; that is, the velocity may be either *constant* or *changing*. Accelerated motion is of two kinds, uniformly accelerated and nonuniformly accelerated. Thus there are three types each of both translatory and rotary motion. Before discussing these types of motion, it will be necessary to define and discuss velocity and acceleration.

23. Speed, Average Speed, Velocity, and Average Velocity.— As already mentioned (Sec. 13), speed is a scalar quantity and velocity is a vector quantity. Both designate rate of motion; but the former does not take into account the direction of the motion, whereas the latter does.

Average speed, which may be designated by $\bar{s}$ (read "barred s") is given by

$$\bar{s} = \frac{D}{t} \tag{3}$$

in which D is the total distance traversed by a body in a given time t. Average velocity $\bar{v}$ is given by the equation

$$\bar{v} = \frac{d}{t} \tag{4}$$

in which d is the distance from start to finish measured in a *straight line*, and t is the time required. Observe that d has, in addition to magnitude, a *definite* direction, and is therefore a *vector;* whereas D is simply the distance as measured along the path traversed, which may be quite tortuous, and is therefore a scalar. The *Speed* of a body at any given *instant* is the distance which the body would travel in unit time if it maintained that particular rate of motion; while the *Velocity* of the body at that same *instant* has the same numerical value as the speed, and is defined in the same way except that it must also state the

direction of the motion. An example will serve to further illustrate the significance of the above four quantities.

Suppose that a fox hunt, starting at a certain point, terminates 10 hrs. later at a point 20 miles farther north. Suppose further that during this time the dog travels 100 miles. Then d (Eq. 4) is 20 miles due north (a vector), D (Eq. 3) is 100 miles (scalar), $\bar{v}$ is 2 miles an hour north (vector), and $\bar{s}$ is 10 miles an hour (scalar). If the dog's *speed* s at a given instant is 15 miles an hour (often written 15 mi./hr. and called 15 mi. per hr.), then an hour later, if he continues to run at that same speed, he will be 15 miles from this point as measured *along the trail;* whereas if the dog's *velocity* at that same instant is 15 miles per hour east, then, an hour later, *if he maintains that same velocity,* he will be at a point 15 miles farther *east.*

If the hunter travels 40 miles, while a friend, traveling a straight road, travels only 20 miles in the ten hours, then the hunter's average speed is twice that of his friend and only two-fifths that of the dog; whereas the average velocity $\bar{v}$ is the same for all three, viz., 2 miles an hour. We thus see that the average velocity of a body is that velocity which, unchanged in either *magnitude* or *direction,* would cause the body to move from one point to the other in the same time that it *actually does* require.

24. Acceleration.—If a body moves at a uniform speed in a straight line it is said to have uniform velocity, and its velocity is the distance traversed divided by the time required. If its speed is not uniform its velocity changes (in magnitude), *and the rate at which its velocity changes is called the acceleration, a.* If the velocity of a body is not changing at a uniform rate, then the change in velocity that occurs in a given time, divided by that time, gives the *average* rate of change of the velocity of the body, or its *average* acceleration for that time. Since the second is the unit of time usually employed, we see that the average acceleration is the change (gain or loss) in velocity per second. The acceleration of such a body at any particular *instant* is numerically the change in velocity that *would occur* in 1 sec. if the acceleration were to have that same value for the second; *i.e.,* if the velocity were to continue to change at that same rate for the second.

If the velocity is increasing, the acceleration is positive; if decreasing, it is said to be negative. Thus the motion of a train

when approaching a station with brakes applied, is accelerated motion. As it starts from the station it also has accelerated motion, but in this case the acceleration is positive, since it is in the direction of the velocity; while in the former case, the acceleration is negative.

If the *acceleration* of a body is *constant*, for example if the body continues to move faster and faster, and the increase in velocity each succeeding second or other unit of time is the same, its motion is said to be *uniformly accelerated*. Thus if the velocity of a body expressed in feet per second, *e.g.*, the velocity of a street car, has the values 10, 12, 14, 16, 18, etc., for successive seconds; then the acceleration a for this interval is constant, and has the value 2 ft. per sec. per sec., or

$$a = 2 \text{ ft. per sec. per sec. (also written } 2\,\frac{\text{ft.}}{\text{sec.}^2})$$

If a certain train is observed to have the above velocities for successive *minutes*, then the motion of the train is uniformly accelerated, since its acceleration is constant; but it is less than above given for the street car, in fact, 1/60 as great, or 2 ft. per sec. per *min.*; that is,

$$a = 2 \text{ ft. per sec. per min. (also written } \frac{2\text{ ft.}}{\text{sec. min.}})$$

This means that the gain of velocity each minute is 2 ft. per sec. A freely falling body, or a car running down a grade due to its weight only, are examples of uniformly accelerated motion. In order that a body may have accelerated motion, it must be acted upon by an applied or external force differing from that required to overcome all friction effects upon the body.

25. Accelerating Force.—Force may be defined as that which produces or tends to produce change in the velocity of a body to which it is applied; *i.e.*, force tends to accelerate a body. A force may be applied to a body either as a *push* or a *pull*. It has been shown experimentally that it requires, for example, exactly twice as great a force to give twice as great an acceleration to a given mass which is perfectly free to move; and also that if the mass be doubled it requires twice as much force to produce the same acceleration. In other words, the force (F) is proportional to the resulting acceleration (a), and also proportional to the mass

(M) of the body accelerated. These facts are expressed by the equation

$$F = Ma \tag{5}$$

For, to increase a n-fold, F must be increased n-fold; in other words, the resulting acceleration of a body is *directly* proportional to the applied force, and is also *inversely* proportional to the mass of the body.

Eq. 5 is sometimes written $F = kMa$. If the units of force, mass, and acceleration are properly chosen (see below), k becomes unity and may be omitted.

Units of Force.—Imagine the masses now to be considered, to be perfectly free to move on a level frictionless surface, and let the accelerating force be horizontal. Then the unit force in the metric system, the **Dyne,** *is that force which will give unit mass* (1 gm.) *unit acceleration* (1 cm. per sec. per sec.); while in the British system, unit force, the **Poundal,** *is that force which will give unit mass* (1 lb.) *unit acceleration* (1 ft. per sec. per sec.). Thus, to cause the velocity of a 10-gm. mass to change by 4 cm. per sec. in 1 sec.; *i.e.*, to give it an acceleration of 4 cm. per sec. per sec., will require an accelerating force of 40 dynes, as may be seen by substituting in Eq. 5.

The relation between these units and the common gravitational units, the gram weight and the pound weight, will be explained under the study of gravitation (Sec. 32); but we may here simply state without explanation that 1 gram weight is equal to 980 dynes (approx.), and that 1 pound weight is equal to 32.2 poundals (approx.).

In general, only a part of the force applied to a body is used in accelerating it, the remainder being used to overcome friction or other resistance. The part that is used in producing acceleration is called the *Accelerating Force.* It should be emphasized that Eq. 5 holds only if F is the accelerating force. Thus if a stands for the acceleration in the motion of a train, and M for the mass of the train, then F is *not* the *total* pull exerted by the drawbar of the engine, but only the *excess* pull above that needed to overcome the friction of the car wheels on axle bearings and on the track, air friction, etc. If an 8000-lb. pull is just sufficient to maintain the speed of a certain train at 40 miles an hour on a level track, then a pull of 9000 lbs. would cause its speed to increase, and 7000 lbs., to decrease. The accelerating force, *i.e.*,

the F of Eq. 5, would be 1000 lbs., *i.e.*, 32,200 poundals, in each case.

In the case of a freely falling body, the accelerating force is of course the pull of the earth upon the body, or its weight; while in the case of a lone car running down a grade, it is the *component* of the car's weight parallel to the grade (see Fig. 8), minus the force required to overcome friction, that gives the accelerating force. We may now make the statement that when a body is in motion its velocity will not change if the force applied is just sufficient to overcome friction; while if the force is increased, the velocity will increase, and the acceleration will be positive and proportional to this increase or *excess* of force. If the applied force is decreased so as to become less than that needed to overcome friction, then, of course, the velocity decreases, and the acceleration is negative and proportional to the *deficiency* of the applied force.

26. Uniform Motion and Uniformly Accelerated Motion.—This subject will be best understood if discussed in connection with a specific example. Suppose that a train, traveling on a straight track and at a uniform speed from a town A to a town B 20 miles north of A, requires 30 minutes time. In this case its velocity

$$v = \frac{\text{distance traversed}}{\text{time required}} = \frac{d}{t} = \frac{20 \text{ miles}}{30 \text{ min.}}$$

or 2/3 of a mile per min. north. Since the velocity is constant, the train is said to have *Uniform Motion.* If the track is level, the pull on the drawbar of the engine must be just sufficient to overcome friction, since there is no acceleration and hence no accelerating force. Thus, uniform motion may be defined as the motion of a body which experiences no acceleration. This train would have to be a through train; for if it is a train that stops at A, its velocity just as it leaves A would be increasing; *i.e.*, there would be an acceleration. Consequently there would have to be an accelerating force; that is, the pull on the drawbar would have to be *greater* than the force required to overcome friction. In this case the motion would be *accelerated motion.*

In case the accelerating force is constant, for example, if the pull on the drawbar exceeds the force required to overcome friction by, say 4000 lbs. constantly for the first minute, then the acceleration (a) is constant or uniform, and the motion for this

first minute would be *Uniformly Accelerated Motion*. For, from $F = Ma$ (Eq. 5), we see that if the accelerating force F (here 4000 lbs.) is constant, a will also be constant; *i.e.*, the velocity of the train will increase at a uniform rate. As a rule, this excess pull is not constant, so that the acceleration varies, and the train has *nonuniformly accelerated* motion.

Let us further consider the motion of the above train if the accelerating force is constant, and its motion, consequently, uniformly accelerated. Suppose that its velocity as it passes a certain bridge is 20 ft. per sec. and that we represent it by v_o; while its velocity 10 seconds later (or t sec. later) is 34.6 ft. per sec., represented by v_t. Its total change of velocity in this time t is $v_t - v_o$, hence the acceleration

$$a = \frac{v_t - v_o}{t} = \frac{34.6 - 20}{10} = 1.46 \text{ ft. per sec. per sec.} \tag{6}$$

It is customary to represent the velocity *first considered* by v_o, and the velocity t seconds later by v_t, as we have here done. If we *first* consider the motion of the train just as it starts from A, *i.e.*, as it starts from rest, then v_o is zero, and v_t is its velocity t seconds after leaving A. If t is 60 sec., then v_t is the velocity of the train 60 seconds after leaving A.

Let us suppose that one minute after leaving A (from rest) the velocity of the train is 60 miles per hour. This is the same as 1 mile per min. or 88 ft. per sec. The total change in velocity in the first *minute* is then 60 miles per *hour*, and hence the acceleration is 60 miles per hour per *minute*, or

$$a = 60 \text{ miles per hr. per min.}$$

This same acceleration is 1 mile per minute per minute or

$$a = 1 \text{ mi. per min. per min.}$$

It is also 88 ft. per second per minute, or

$$a = 88 \text{ ft. per sec. per min.} = \frac{88}{60} \text{ ft. per sec. per sec.}$$
$$= 1.46 \text{ ft. per sec. per sec.}$$

This equation states that the change of velocity in one minute is 88 ft. per sec., while in one second it is of course 1/60 of this, or 1.46 ft. per sec. Ten seconds after the train leaves A,

its velocity is 10×1.46 or 14.6 ft. per sec. Observe that when v_0 is zero, Eq. 6 may be written

$$v_t = at \tag{7}$$

27. Universal Gravitation.—Any two masses of matter exert upon each other a force of attraction. This property of matter is called *Universal Gravitation.* Thus a book held in the hand experiences a very feeble upward pull due to the ceiling and other material above it; side pulls in every direction due to the walls, etc.; and finally, a very strong downward pull due to the earth. This downward pull or force is the only one that is large enough to be measured by any ordinary device, and is what is known as the *weight* of the body.

That there is a gravitational force of attraction exerted by every body upon every other body, was shown experimentally by Lord Cavendish. A light rod with a small metal ball at each end was suspended in a horizontal position by a vertical wire attached to its center. A large mass, say A, placed near one of these balls B and upon the same level with it, was found to exert upon the ball a slight pull which caused the rod to rotate and twist the suspending wire very slightly. Comparing this slight pull on B due to A, with the pull of the earth upon B, *i.e.*, with B's weight, Cavendish was able to compute the mass of the earth. In popular language, he *Weighed the Earth.*

From the mass of the earth and its volume Lord Cavendish determined the average density of the earth to be about 5.5 times that of water. The surface soil and surface rocks—sandstone, limestone, etc.—have an average density of but 2.5 times that of water. Hence the deeper strata of the earth are the *more dense,* and consequently as a body is lowered into a mine and approaches closer and closer to the more dense material, its weight might be expected to *increase.* The upward attraction upon the body exerted by the overlying mass of earth and rocks should cause its weight to *decrease.* The former more than offsets the latter, so that there is a slight *increase* in the weight of a body as it is carried down into a deep mine.

Newton's Law of Gravitation.—Sir Isaac Newton was the first to express clearly the law of universal gravitation by means of an equation. He made the very logical assumption that the attractive gravitational force (F) exerted between two masses M_1 and M_2, when placed a distance

d apart, would be proportional to the product of the masses, and inversely proportional to the square of the distance between them (Sec. 28), *i.e.*,

$$F = k\frac{M_1 M_2}{d^2}$$

If, in this equation, M_1 and M_2 are expressed in grams, the distance in centimeters, and F in dynes, then k, the *proportionality constant* or proportionality factor (Sec. 28) is shown by experiment to be 0.0000000666. If M_1, M_2, and d are all unity, then $F = k$. In other words, the gravitational attraction between two 1-gm. masses when 1 cm. apart is 0.0000000666 dynes. Since the dyne is a small force, this will be seen to be a very small force. Lord Cavendish used this equation in computing his results.

28. The Law of the Inverse Square of the Distance.—This law is one of the most important laws of physics and has many applications, a few of which we shall now consider. We are all familiar with the fact that as we recede from a source of light, for example a lamp, the intensity of the light decreases. That the intensity of illumination at a point varies inversely as the square of the distance from that point to the light source, has been repeatedly verified by experiment, and it may also be demonstrated by a simple line of reasoning as follows: Imagine a lamp which radiates light equally in all directions, to be placed first at the center of a hollow sphere of 1 ft. radius, and later at the center of a similar hollow sphere whose radius is 3 ft. In each case the hollow sphere would receive all of the light emitted by the lamp, but in the second case this light would be distributed over 9 times as much surface as in the first. Hence, the illumination would be 1/9 as intense, and we have therefore proved that the intensity of illumination varies inversely as the square of the distance from the lamp.

An exactly similar proof would show that the same law applies in the case of heat radiation, or indeed in the case of any effect which acts equally in all directions from the source. This law has been shown to hold rigidly in the case of the gravitational attraction between bodies, for example between the different members of the solar system.

Proportionality Factor.—In all cases in which one quantity is proportional to another, the fact may be stated by an equation if we introduce a proportionality factor (k). Thus the weight of a certain quantity of

water is proportional to its volume; *i.e.*, 3 times as great volume will have 3 times as great weight, and so on. We may then write

$$W \propto V, \text{ but not } W = V$$

We may, however, write

$$W = kV$$

in which k is called the *proportionality factor*. In this case k (in the English system) would be numerically the weight of a cubic foot of water, or 62.4 (1 cu. ft. weighs 62.4 lbs.), V being the number of cubic feet whose weight is sought.

We may add another illustration of the use of the proportionality factor. We have just seen that the illumination (I) at a point varies inversely as the square of the distance from the source. We also know that it should vary as the candle power ($C.P.$) of the source. Hence we may write

$$I \propto \frac{C.P.}{d^2}, \text{ or } I = k\frac{C.P.}{d^2}$$

A third illustration has already been given at the close of Sec. 27.

29. Planetary Motion.—The earth revolves about the sun once a year in a nearly circular orbit of approximately 93,000,000 miles radius. The other seven planets of the solar system have similar orbits. The planets farthest from the sun have, of course, correspondingly longer orbits, and they also travel more slowly; so that their "year" is very much longer than ours. Thus Neptune, the most distant planet, requires about 165 years to traverse its orbit, while Mercury, which is the closest planet to the sun, has an 88-day "year." The moon revolves about the earth once each lunar month in an orbit of approximately 240,000 miles radius. Several of the planets have moons revolving about them while they themselves revolve about the sun.

If a stone is whirled rapidly around in a circular path by means of an attached string, we readily observe that a considerable pull must be exerted by the string to cause the stone to follow its constantly curving path (Sec. 58). In the case of the earth and the other planets, it is the gravitational attraction between planet and sun that produces the required inward pull. Our moon is likewise held to its path by means of the gravitational attraction between the earth and the moon. The amount of pull required to keep the moon in its course has been computed, and found to be in close agreement with the computed gravitational pull that

the earth should exert upon a body at that distance. In computing the latter it was assumed that the inverse square law (Sec. 28) applied.

Since the moon is approximately 60 times as far from the center of the earth as we are, it follows that the pull of the earth upon a pound mass at the moon is $(1/60)^2$ or 1/3600 pound. By means of the formulas developed in Sec. 58, the student can easily show that this force would exactly suffice to cause the moon to follow its constantly curving path if it had only one *pound of mass*. Since the mass of the moon is vastly greater than one pound, it requires a correspondingly greater force or pull to keep it to its orbit, but its greater mass also causes the gravitational pull between it and the earth to be correspondingly greater so that this pull just suffices.

30. The Tides.—A complete discussion of the subject of tides is beyond the scope of this work, but a brief discussion of this important phenomenon may be of interest. Briefly stated, the

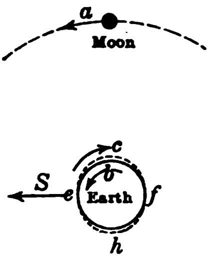

FIG. 14.

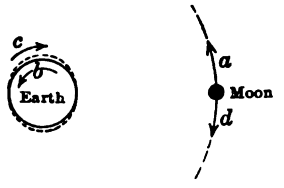

FIG. 15.

main cause of tides is the fact that the gravitational attraction of the moon upon unit mass is *greater* for the ocean upon the side of the earth toward it, than for the main body of the earth; while for the ocean lying upon the opposite side of the earth, it is *less*. This follows directly from a consideration of the law of inverse squares (Sec. 28).

This difference in lunar gravitational attraction *tends* to heap the water slightly upon the side of the earth *toward* it and also upon the *opposite* side; consequently if the earth always presented the same side to the moon, these two "heaps" would be permanent and stationary (Fig. 14). As the earth rotates from *west* to *east*, however, these two "heaps" or *tidal waves* travel from

east to *west* around the earth once each lunar day (about 24 hrs. 50 min.), *tending*, of course, to keep directly under the moon. Due to the inertia of the water, the tidal wave lags behind the moon; so that high tide does not occur when the moon is overhead (Fig. 14), but more nearly at the time it is setting, and also when it is rising (Fig. 15). Since the moon revolves about the earth from west to east in approximately 28 days, we see why the lunar day, moonrise to moonrise, or strictly speaking, "moon noon" to "moon noon," is slightly longer than the solar day (Sec. 4).

Every body of the solar system, so far as known, except Neptune's moon, revolves in a counterclockwise direction both about its axis and also in its orbit as *viewed* from the *North Star*. Hence the arrows a, b, c, and d respectively represent the motion of the moon, rotation of the earth, motion of the tides, and *apparent* motion of the moon with respect to the earth. Consequently, according to this convention, the moon rises at the left and sets at the right, which is at variance with the usual geographical convention.

Although the sun has a vastly greater mass than the moon, its much greater distance from the earth reduces its tidal effect to less than half that of the moon. During new moon, when the sun and moon are on the *same* side of the earth, or at full moon, when on *opposite* sides, their tidal effects are evidently additive, and therefore produce the maximum high tides known as *Spring tides*. During first quarter and last quarter their tidal effects are subtractive, giving the minimum high tides or *Neap tides*. For if the sun were in the direction S (Fig. 14) it would *tend* to produce high tide at e and f, and low tide at c and h.

On small islands in mid-ocean, the tidal rise is but a few feet; while in funnel-shaped bays facing eastward, such as the Bay of Fundy, for example, it is from 40 to 50 feet.

If the earth were completely surrounded by an ocean of uniform depth, the above simple theory would explain the behavior of the tides. Under such circumstances tides would always travel *westward*. The irregular form and varying depth of the ocean make the problem vastly more complex. Thus the tide comes to the British Isles from the southwest. (Ency. Brit.). This tide, which is simply a large long wave produced by the true tidal effect in a distant portion of the open ocean, first reaches the west coasts of Ireland and England, and then, passing through the English Channel, reaches London several hours later.

31. Acceleration of Gravity and Accelerating Force in Free Fall.—Since the earth exerts the same pull upon a body whether at *rest* or in *motion*, it will be evident that the *accelerating force* in the case of a falling body is simply its *weight* W, and hence we have from Eq. 5, Sec. 25.

$$W = Ma, \text{ or } W = Mg \tag{8}$$

in which M is the mass of the falling body, and g is its acceleration. It is customary to use g instead of a to designate the *acceleration* of *gravity*, *i.e.*, the acceleration of a freely falling body. From Eq. 8 we see that $g = W/M$, and since a mass n times as large has n times as great weight, g must be constant; *i.e.*, a 10-lb. mass should fall no faster than a 1-lb. mass, neglecting air friction. If it were not for air friction, a feather would fall just as fast as a stone. This has been demonstrated by placing a coin and a feather in a glass tube ("guinea and feather" experiment) and then exhausting the air from the tube by means of an air pump. Upon inverting the tube, it is found that the coin and the feather fall equally fast; hence they must both experience the same constant acceleration. From Eq. 8 it follows that g varies in value with change of altitude or latitude just as does the weight W of a body (Sec. 11).

Since the acceleration of gravity, g, represents the rate at which any falling body gains velocity, it is at once evident that it is a very important constant. Its value has been repeatedly determined with great care, and it has been found that

$$g = 980.6 \text{ cm. per sec. per sec.} \tag{9}$$

for points whose latitude is about 45°. For points farther north it is slightly greater than this (983.2 at pole); and for points farther south, slightly less (978 at equator.) The above equation states that in one second a falling body acquires an additional velocity of 980.6 cm. per sec. Since 980.6 cm. per sec. = 32.17 ft. per sec., we have

$$g = 32.17 \text{ ft. per sec. per sec.} \tag{9a}$$

We may *define* the *Acceleration* of *Gravity as the rate of change of velocity of a freely falling body;* hence it is *numerically* the additional velocity acquired by a body in each second of free fall. If it were not for air friction, a body would add this 32.17 ft. per sec. (980.6 cm. per sec.) to its velocity every second, how-

ever rapidly it might be falling. Though a close study of the effects of air friction upon the acceleration is beyond the scope of this course, we readily see that when a falling body has acquired such a velocity that the air friction resisting its fall is equal to *one-third* of its weight, then only *two-thirds* of its weight remains as the accelerating force. Its acceleration would then, of course, be only two-thirds g. When a falling body, for example a hailstone, has acquired such a velocity that the air friction encountered is just equal to its weight, then its *entire* weight is used in overcoming friction, the accelerating force acting upon it has become zero, and its acceleration is zero; *i.e.*, it makes no further gain in velocity.

32. Units of Weight and Units of Force Compared.—From Eq. 5 (Sec. 25) we see that the logical unit of force is that force which will give unit mass unit acceleration, or unit change of velocity in unit time. Hence, in the *metric system*, unit force, or the *Dyne* (See also "Units of Force," Sec. 25), is that force which *will give one gram mass an acceleration of* 1 *cm. per sec. per sec.*, *i.e.*, a change in velocity of 1 cm. per sec. in a second. In the case of a gram mass falling, the accelerating force is a gram weight, and the velocity imparted to it in one second is found by experiment (in latitude 45°) to be 980.6 cm. per sec. (Sec. 31); whence g equals 980.6 cm. per sec. per sec. It follows at once, then, that a gram weight equals 980.6 dynes, since it produces when applied to a gram mass 980.6 times as great an acceleration as the dyne does. Likewise in the *British system*, unit force (the *Poundal*) is that force which will give unit mass (the pound) unit velocity (1 ft. per sec.) in unit time (the second). But in the case of a pound mass freely falling, the accelerating force is one pound weight, and this force, as experiment shows, imparts to it a velocity of 32.17 ft. per sec. in one second. It follows at once that one pound force, or one pound weight, equals 32.17 poundals, since it produces 32.17 times as great acceleration with the same mass (see Eq. 5).

The poundal and the dyne are the absolute units of force. The pound, ton, gram, kilogram, etc., are some of the units of force in common use. Forces are measured by spring balances and other weighing devices.

In Eq. 8, the weight is expressed in absolute units; in which case $W = Mg$. If W is expressed in grams weight or pounds weight, then we have simply $W = M$ (numerically), *i.e.*, a 100-

gm. mass weighs 100 *grams*, or 98,060 *dynes*. Likewise a 10-lb. mass weighs 10 *lbs.*, or 321.7 *poundals* (latitude 45°).

The Engineer's Units of Force and Mass.—In engineering work the pound is used as the unit of force instead of the poundal. Transposing Eq. 8, Sec. 31, we have $M = W/g$. Now in physics, W is expressed in poundals, M being in pounds, while in engineering work W is expressed in pounds. Since the pound is 32.17 times as large a unit as the poundal, M must be expressed in the engineering system in a unit 32.17 times as large as the pound mass (close approximation). This 32.17-lb. mass is sometimes called the *Slug*.

As a summary, let us write the equation $F = Ma$, and the similar equation restricted to gravitational acceleration; namely, $W = Mg$, indicating the units for each symbol in all three systems—the Metric, the British, and the Engineering systems.

Metric System:

$F = Ma$ and $W = Mg$, *i.e.*, F or W (dynes)
$= M$(gm.) $\times a$ or g (cm. per sec. per sec.).

British System:

$F = Ma$ and $W = Mg$, *i.e.*, F or W (poundals)
$= M$(lbs.) $\times a$ or g (ft. per sec. per sec.).

Engineering System:

$F = \frac{W}{g}\, a = (Ma)$ and $W = Mg$, *i.e.*, F or W (pounds)
$= M$(slugs) $\times a$ or g (ft. per sec. per sec.).

Thus, *practically*, the engineering system differs from the British system in that the *units of mass, force, and weight are* 32.17 *times as large as the corresponding units in the British system.*

Some regret that the engineering system was ever introduced. It is now firmly established, however, and the labor involved in mastering this third system is very slight, indeed, if the British system is thoroughly understood. Furthermore, this system has in some cases certain advantages.

Observe that the word "pound" is used for the unit of *mass* and also for one of the units of *force*. Having defined the pound force as the weight of a pound mass, we *may* (and frequently do) use it (the pound force) as the unit in measuring forces which have absolutely nothing to do with either mass or weight. Thus in stretching a clothes line with a force of, say 50 lbs., it is clear that this 50-lb. force has nothing to do with the mass or weight of the clothes line, or post, or anything else. The pound force

is used almost exclusively as the unit of force in engineering work. Objection to its use as a unit is sometimes made because of the fact that the weight of a 1-lb. mass varies with g. Since g varies from 978 at the equator to 983.2 at the poles (Sec. 31), we see that the weight of a 1-lb. mass (or any other mass) is about 1/2 per cent. greater at the poles than at the equator. This slight variation in the value of the pound force may well be ignored in practically all engineering problems. If the standard pound force is defined as the weight of a 1-lb. mass in latitude 45° ($g=980.6$), it becomes as definite and accurate as any other unit of force.

33. Motion of Falling Bodies; Velocity—Initial, Final, and Average.—The initial velocity of a body is usually represented by v_o (Sec. 26), and the final velocity by v_t. An example will serve the double purpose of illustrating exactly what these terms mean as applied to falling bodies, and also of showing how their numerical values are found.

Suppose that a body has been falling for a short time before we observe it and that we wish to discuss its motion for the succeeding eight seconds of fall. Suppose that its initial velocity v_o, observed at the beginning of this eight-second interval, is 20 ft. per sec. Its final velocity v_t at the close of this eight-second interval would be found as follows. It will at once be granted that the *final velocity* v_t will be equal to the *initial velocity* plus the *acquired velocity*. But by definition (Sec. 31), g is numerically the velocity acquired or gained in one second of free fall. Hence in two seconds the acquired velocity would be $2g$, in 3 seconds $3g$, and in t seconds the velocity acquired would be gt. Accordingly

$$v_t=v_o+gt \tag{10}$$

In the present problem $v_t=20+32.17\times 8=277.36$ ft. per sec.

Average velocity is commonly represented by $\bar{v}$ (read "barred v"), and in the case of *falling bodies* it is equal to half the sum of the initial and final velocities. Hence

$$\bar{v}=\frac{v_o+v_t}{2}=\frac{v_o+(v_o+gt)}{2}=v_o+\tfrac{1}{2}gt \tag{11}$$

In general, the average velocity of a train would not be even approximately equal to half the sum of the initial and final velocities. We ought therefore to prove the validity of Eq. 11.

We readily see that the average value of all numbers from 40 to 100 is 140 ÷ 2 or 70. If the velocity of a train is 10 feet per sec., and each succeeding minute it gains 2 feet per second, then its velocities for the succeeding minutes are respectively 10, 12, 14, 16, 18, 20, 22 feet per second, and its correct average velocity would be, under these special circumstances, one-half the sum of the initial and final velocities. Adding all these numbers and dividing by 7 gives an average of 16, but one-half the sum of the first and last is also 16.

We may now make the general statement that one-half the sum of the first and last of a series of numbers gives a correct value for the average, provided the successive values of the numbers in the series differ by a constant amount. Now the velocity each successive second is g feet per second (approximately 32 feet per second) *greater* than for the preceding second; consequently, *in all cases of falling bodies, the average velocity is half the sum of the initial and final velocities*, as given in Eq. 11.

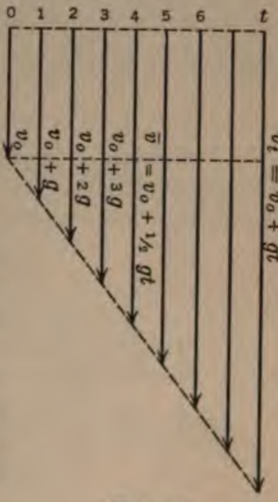

FIG. 16.

The above facts are shown graphically in Fig. 16, in which the successive lines 1, 2, 3, 4, 5, . . . t represent the velocities of a body after falling 1, 2, 3, 4, . . . t seconds respectively. Observe that the velocity at any time, *e.g.*, after 6 seconds, consists of two parts; that above the horizontal dotted line being the *initial* velocity v_o, and that below, the *acquired* velocity (or gt), at that instant. It will be evident, as the figure shows, that the average velocity will be attained when *half* of the time, viz., 4 seconds, has elapsed, and hence $\bar{v} = v_o + \frac{1}{2}gt$; whereas the final velocity v_t is attained after the *whole* time t has elapsed, and is therefore $v_o + gt$, as given above (Eq. 10).

In case the body falls from rest, v_o is zero, and the conditions would be represented by only the portion of Fig. 16 below the dotted line. In this case the entire velocity v_t at any *instant* would be merely gt or that acquired previous to that instant, and the average velocity $\bar{v}$ for a given time t would be $\frac{1}{2}gt$.

34. Distance Fallen in a Given Time.—In general, the distance d traversed by any body in a given time is its *average* velocity $\bar{v}$ times this time, or $d=\bar{v}t$. Introducing the value of $\bar{v}$ from Eq. 11 gives,

$$d=\bar{v}t=(v_o+\frac{gt}{2})t=v_ot+\tfrac{1}{2}gt^2 \tag{12}$$

If $v_o=0$, *i.e.*, if the body falls from rest, and the distance it falls in seconds is wanted, then, from Eq. 12,

$$d=\tfrac{1}{2}gt^2 \tag{13}$$

If $v_o=0$, Eq. 10 may be written $t=\frac{v_t}{g}$. Substituting this value of t in Eq. 13, we obtain

$$v_t=\sqrt{2gd}=\sqrt{2gh} \tag{14}$$

In this equation, v_t is the velocity acquired by a body in falling from rest through a distance d (or h).

It will be observed that v_ot of Eq. 12 is the distance which the body with initial velocity v_o would travel in t seconds if there were no *acceleration;* while $\frac{1}{2}gt^2$ is the distance it would travel in this *same time* if there had been no *initial velocity, i.e.*, had it fallen from rest. The distance it actually *does* travel, since there are both initial velocity and acceleration, is simply the sum (vector sum) of these two. If a person throws a stone vertically upward with a velocity v_o, then the distance from that person's hand to the stone after t seconds will be $v_ot-\frac{1}{2}gt^2$. For evidently the distances the stone would go, due to its initial velocity alone, and due to falling alone, are directly opposite as indicated by the minus sign. Finally, if a person on a high cliff throws a stone at an angle of 45° (upward) from the horizontal with a velocity of 20 ft. per second, let us find the distance from his hand to the stone 3 seconds later. Due to its initial velocity alone, it would be 60 ft. distant, represented by line a (Fig. 17), while due to

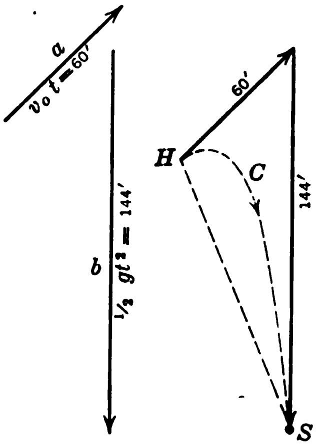

FIG. 17.

falling alone it would be approximately 144 ft. distant, represented by line *b*. Hence due to both, we have, by vector construction, *HS* (about 100 ft.) as the distance from his hand to the stone after 3 seconds of its flight. The actual path of the stone is *HCS*.

Distance Traversed in a Given Time.—Equations 10, 11, 12, 13, and 14, which are derived from a consideration of a *particular* kind of uniformly accelerated motion, namely, that of falling bodies, become perfectly *general* by substituting in them the *general* symbol *a* in place of the *particular* symbol *g* to represent the acceleration. Making this substitution, these equations, taken in order, become

$$v_t = v_o + at \tag{10a}$$

$$v = v_o + \tfrac{1}{2}at \tag{11a}$$

$$d = \bar{v}t = (v_o + \tfrac{1}{2}at)t = v_o t + \tfrac{1}{2}at^2 \tag{12a}$$

$$d = \tfrac{1}{2}at^2 \tag{13a}$$

$$v_t = \sqrt{2ad} = \sqrt{2ah} \tag{14a}$$

The equations just given apply to the motion of a car when coasting on a uniform grade, or to the motion of any body when acted upon by a *constant* accelerating force. In the case of a car on a *uniform* grade, the accelerating force is, barring friction, the component of the car's weight which is parallel to the grade (Fig. 8, Sec. 19), and is therefore *constant.*

Aside from the motion resulting from gravitational attraction, there are very few examples of uniformly accelerated motion. Such motion, however, is very roughly approximated by many bodies when *starting* from rest; *e.g.*, by a train, a steamship, a sailboat, or a street car. In all these cases the accelerating force, that is, the amount by which the applied force exceeds friction, decreases rapidly as the speed increases; consequently the acceleration decreases rapidly, and the motion is then not even approximately uniformly accelerated.

35. Atwood's Machine.—If we attempt to make an experimental study of the motion of freely falling bodies we find that the time of fall must be taken very small, or the distance fallen will be inconveniently large. Thus in so short a time as three seconds, a body falls somewhat more than 144 feet. Hence, in all devices for studying the laws of falling bodies and verifying

experimentally the equations expressing these laws, the rapidity of the motion is reduced. Thus a wheel or a marble rolling down an inclined plane experiences an acceleration much smaller than if allowed to fall freely. For in the latter case the accelerating force is the full weight of the marble or the wheel; while in the former case it is only the *component of the weight* parallel to the incline. This reduction of the acceleration makes it possible to study the motion for a period of several seconds. In the *Atwood Machine*, shown in its simplest form in Fig. 18, the reduction in the acceleration is attained in an entirely different way. A and B are two large equal masses connected by a light cord passing over a *light* wheel as shown. If a small additional mass C is placed on A, it will cause A to descend and B to ascend.

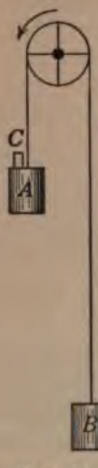

FIG. 18.

Suppose that A and B are each 150-gm. masses and that C is a 10-gm. mass. If we neglect the slight mass and opposing friction of the wheel, it is clear that the weight of C is the accelerating force that must accelerate A, B, and C—an aggregate mass equal to 31 times the mass of C; while if C were permitted to fall freely, its weight would have to accelerate itself only. Hence the acceleration under these circumstances is 1/31 of that of free fall or g, or $1/31 \times 980 = 31.6$ cm. per sec. per sec., which is about 1 ft. per sec. per sec. With this value for the acceleration, we see from Eq. 13*a* that A would "fall" only about 4.5 feet in 3 seconds. By experiment also we find that A "falls" 4.5 feet in 3 seconds, thus *verifying* Eq. 13*a*.

The above acceleration may also be calculated by means of the equation $F = Ma$, in which F is the *weight* of C in *dynes* and M is the combined *mass* of A, B, and C in *grams*. A pendulum or other device beating seconds is an essential auxiliary. If by means of an attached thread, C is removed after one second of "fall," A's velocity, since no accelerating force is then being applied, will be constant, and will have the value 31 cm. per sec. (see above); while if in another test C remains 3 seconds, A's velocity at the end of the 3 seconds will be 93 cm. per sec., as may easily be observed. This verifies the equation $v_t = at$ (Eq. 7, Sec. 26).

36. Motion of Projectiles: Initial Velocity Vertical.—If a rifle ball is fired vertically upward, it experiences a downward

force (its weight) which slows it down, giving rise to a negative acceleration. This decrease in velocity each second is of course 32.17 ft. per sec.; so that if the muzzle velocity is 1000 ft. per sec., the velocities after 1, 2, 3, 4, etc., to t seconds are, respectively, $1000-g$ (or 968), $1000-2g$, $1000-3g$, $1000-4g$ (or 872 ft. per sec.), etc., to $1000-gt$. Since the velocity of the bullet is zero when it reaches its highest position, the number of seconds (t) that the bullet will continue to rise is found by placing $1000-gt$ equal to zero and solving for t. (Compare Sec. 39.) This gives $t=31$ sec., approximately. The bullet requires just as long to fall back, so that its time of flight is 62 seconds. To get the height to which it rises, which is obviously the distance it falls in 31 seconds, let t be 31 in Eq. 13 and solve for d. We may also use the relation $v=\sqrt{2gh}$ (Eq. 14) to find h if v is known, or *vice versa*. Here $v=1000$ ft. per sec., since, neglecting air friction, the bullet, in falling, strikes the ground with the same velocity with which it was fired.

Throughout the discussion of projectiles no account will be taken of the effect of air friction, which effect is quite pronounced on very small projectiles (Sec. 39). In approximate calculations, the distance a body falls in the first second will be taken as 16 ft. instead of 16.08, and g will be taken as 32 instead of 32.17 ft. per sec. per sec. If a rifle ball is fired vertically *downward*, *e.g.*, from a balloon, with a velocity v_o, its velocity will increase by 32 ft. per sec. every second (ignoring air friction), so that t seconds later its velocity will be v_o+gt. In this case the distance traversed in the first t seconds is $v_ot+\frac{1}{2}gt^2$ (Eq. 12); while if the initial velocity is upward, the distance from the rifle to the rifle ball after t seconds is $v_ot-\frac{1}{2}gt^2$, as explained in Sec. 34.

37. Motion of Projectiles: Initial Velocity Horizontal.—If a projectile is fired horizontally, it experiences, the *instant* it leaves the muzzle A of the gun (Fig. 19), a downward pull (its weight) which gives it a *downward component* of velocity of 32 ft. per sec. for every second of flight. This causes it to follow the curved path $AB'C'$ . . . F'. If it were not for gravitational attraction, the bullet at the end of the first, second, third, . . . etc., seconds would be at the points B, C, D, . . . etc., respectively ($AB=BC=CD=1000$ ft.), instead of at B', C', etc. . . .

To find the velocity of the bullet at any time t, say when at F' 5 sec. after leaving the muzzle of the gun, we simply find the *vector sum* v' of its initial velocity and its acquired velocity, as

shown in Fig. 19 (left lower corner). The downward velocity acquired in 5 sec. would of course be gt, or 160 ft. per sec. (that is, 32×5), and we will assume 1000 ft. per sec. as the initial horizontal muzzle velocity.

It will be evident that the *horizontal component* of velocity (1000 ft. per sec.) must be *constant*, for the pull of gravity has no horizontal component to either increase or decrease the horizontal component of velocity. This, of course, is true whether the initial velocity is vertical, horizontal, or aslant. Hence, neglecting friction, it is *always* only the *vertical component* of velocity of a projectile that *changes*.

To find the distance that the bullet will "fall" in going the first 1000 ft., *i.e.*, its distance BB' (Fig. 19) from the horizontal line of firing AF, apply Eq. 13. From this equation we see that a body falls approximately 16 ft. in one second, 64 ft. in two sec., and 144 ft. (*i.e.*, 16×3^2) in 3 sec. Hence $BB' = 16$ ft., $CC' = 64$ ft.,

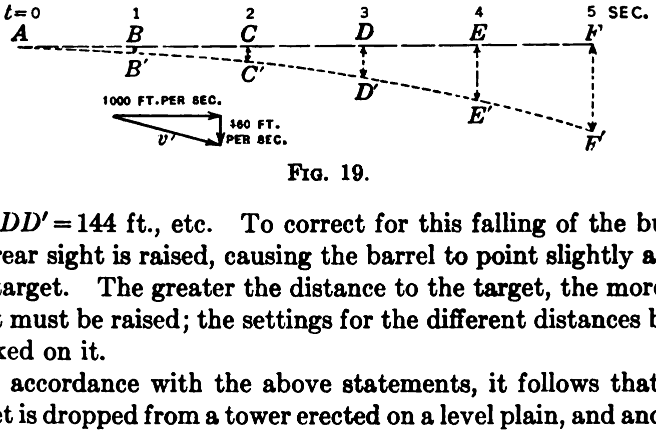

FIG. 19.

and $DD' = 144$ ft., etc. To correct for this falling of the bullet, the rear sight is raised, causing the barrel to point slightly above the target. The greater the distance to the target, the more the sight must be raised; the settings for the different distances being marked on it.

In accordance with the above statements, it follows that if a bullet is dropped from a tower erected on a level plain, and another bullet is fired horizontally from the same place at the same instant, then the two bullets will reach the ground at the same instant, whether the second one is fired at a high or low speed. This fact can be verified experimentally (Sec. 40).

38. Motion of Projectiles: Initial Velocity Inclined.—If a rifle ball is fired from a point A (Fig. 20), in a direction AQ making an angle θ with the horizontal, it describes a curved path which may be drawn as follows. Since distance is a vector, to find where the projectile will be after a time t, we simply obtain the vector sum of the distance traversed in t seconds due to its *initial velocity* and the distance traversed in t seconds of *free fall* from rest, as

was done in Sec. 34 (Fig. 17). Hence on the line AQ, which has the direction of the initial velocity, lay off the distances AB, BC, CD, DE, etc., each representing 1000 ft. (for a muzzle velocity of 1000 ft. per sec.). From B, C, D, E, etc , draw the lines BB', CC', DD', EE', etc., representing respectively the distances fallen in 1, 2, 3, 4, sec. Then here, just as in Fig. 19, we have $BB' = 16$ ft., $CC' = 64$ ft., $DD' = 144$ ft., etc. The curve $AB'C'D'E'$, etc., represents the path of the projectile. For consider any point, *e.g.*, K'. Due to its initial velocity *alone*, the projectile would go from A to K (10,000 ft.) in 10 seconds. Due to gravity *alone* it would fall a distance KK', or 1600 ft., in 10 seconds. Hence, due to *both*, it covers the distance AK', the *vector* sum of the distances AK and KK', as shown.

Note that the straight line AK' gives not only the *magnitude* but also the *direction* of the distance from A to the projectile

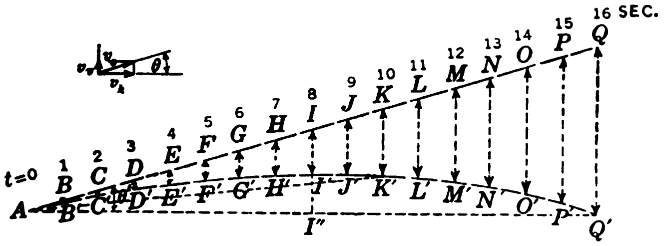

FIG. 20.

after ten seconds of flight. Note also that AK is the v_0t, and that KK' is the $\frac{1}{2}gt^2$ of Eq. 12 (Sec. 34).

39. Time of Flight and Range of a Projectile.—The *Range* is the horizontal distance AQ' (Fig. 20), or the distance from the point from which the projectile is fired to the point at which it again reaches the same level. The *Time of Flight* is the time required to traverse this distance.

To find how long the projectile will *continue to rise*, in other words, to find the time t_1 that will elapse before its vertical component of velocity (v_v) will be zero, place $v_v - gt_1 = 0$ (*i.e.*, $t_1 = v_v/g = v_o \sin\theta/g$) and solve for t_1 (compare Sec. 36). It was shown in Sec. 37 that only the vertical component of velocity changes. Since the vertical component of velocity is zero at this time t_1, the projectile must be at the middle of its path (I', Fig. 20). Therefore the time of flight.

$$T = 2t_1 \tag{15}$$

The vertical component of velocity $v_v = v_o \sin \theta$, and the horizontal component of velocity $v_h = v_o \cos \theta$ (see left upper corner Fig. 20). If $v_o = 1000$ ft. per sec., then, as the projectile leaves the gun, v_v = about 240 ft. per sec., and v_h = about 970 ft. per sec. If the angle θ is known, these two components of the velocity may be accurately found by the use of tables of sines and cosines. The graphical method may also be used. When $t = 1$ sec., *i.e.*, 1 sec. after the projectile leaves the gun (see Fig. 20), $v_v = 208$ ft. per sec. Another second later v_v is 32 ft. per sec. less, and when $t = 240/32$, or approximately 8 sec. after the gun is fired, the vertical component of velocity is zero. That is, in 8 sec. the bullet reaches the horizontal part of its path at I', at which point its vertical component of velocity is clearly zero. Since t_1 is 8 sec., the time of flight T (Eq. 15) is 16 sec.

Obviously, the range (R) is given by the equation,

$$R = v_h \times T = v_o \cos \theta \times 2t_1 = v_o \cos \theta \times 2v_o \sin \theta / g \qquad (16)$$

Here the range is 15,520 (*i.e.*, 16×970) ft. The *Maximum Height* reached, or $I'I''$, is $\frac{1}{2}gt^2$, in which t is the t_1 of Eq. 15. For at I' the path is horizontal, and it was pointed out in Sec. 37 that a bullet fired horizontally would reach the ground in the same time as would a bullet dropped from the same point. Hence $I'I'' = 16 \times 8^2 = 1024$ ft.

Effect of Air Friction on Velocity and Range.—Thus far, in the study of the motion of projectiles, we have neglected the effects of air friction; so that the resulting deductions apply strictly to a projectile traveling through a space devoid of air or any other substance, *i.e.*, through a *vacuum*. The theoretical range so found is considerably greater than the actual range, since the friction of the air constantly decreases the velocity of the projectile (see table below), and therefore causes it to strike the earth much sooner than it otherwise would. Below is given the velocity of an *Army Rifle* projectile in feet per second at various distances from the muzzle.

Distance in yds.	Velocity in ft. per sec.	Distance in yds.	Velocity in ft. per sec.
100	1780	1000	830
200	1590	1200	755
300	1420	1400	690
400	1265	1600	630
600	1044	1800	575
800	923	2000	530

The angle (θ, Fig. 20) which the barrel of the gun makes with the horizontal is called the *Angle of Elevation*. Obviously, if the angle of elevation is small, increasing it will increase the range. It can be shown by the use of calculus that the theoretical maximum range is obtained when this angle is 45°. The trigonometric proof is given below. For heavy cannon (12-in. guns), the angle of fire for maximum range is nearly the same as the theoretical, namely, 43°; while for the army rifle it is about 31°. This difference is due to the greater retarding effect of air friction upon the lighter projectile.

In firing at targets 1/4 mi. distant or less, such as is usually the case in the use of small arms, there is not a very marked difference between the theoretical and the actual path of the projectile. The maximum range of the new army rifle is about 3 miles. It may be of interest to note that its range in a vacuum (angle of elevation 45°) would be about 24 miles, and that the bullet at the middle of its flight would be about 6 miles above the earth, and would strike the earth with its original muzzle velocity.

The artillery officer who directs the firing at moving ships at a distance of 5 miles or more, especially during a strong wind, must make very rapid and accurate calculations or he will make very few "hits." Many other things concerning the flight of projectiles, which are of the utmost importance to the artillery man, must be omitted in this brief discussion.

Angle of Elevation for Maximum Range.—Since $\sin 2\theta = 2 \sin \theta \cos \theta$ (trigonometry), Eq. 16 may be written

$$R = 2v_o^2 \frac{\sin \theta \cos \theta}{g} = v_o^2 \frac{\sin 2\theta}{g}$$

Now the maximum value of the sine of an angle, namely, unity, occurs when the angle is 90°. Therefore when $2\theta = 90°$, *i.e.*, when $\theta = 45°$, $\sin 2\theta$ is a maximum; hence the range R is also a maximum, which was to be proved.

40. Spring Gun Experiment.—From the discussion given in Sec. 38, it is seen that if a target at B, or at C, or at D, or at any other point on AQ (Fig. 20), is released at the instant the trigger is pulled, it will by falling reach B' (or C', or D', etc., as the case may be) just in time to be struck by the bullet. This may be shown experimentally by the use of a spring gun, using wooden balls for both projectile and target. The target ball is held by an electrical device which automatically releases it just as the projectile ball leaves the muzzle of the gun. The two balls meet in the air whether the projectile ball is fired at a high or low velocity. If the target is placed at the same height as the spring gun,

and the latter is fired horizontally, the two balls will reach the floor at the same instant.

41. The Plotting of Curves.—The graphical method of presenting data is found very useful in all cases in which a series of several observations of the same phenomenon has been made. Coördinate or cross section paper is used for this purpose. Usually a vertical line at the left of the page is called the axis of ordinates, and a horizontal line at the bottom of the page is called the axis of abscissæ. To construct a curve, plot as abscissæ the quantity that is arbitrarily varied, and as ordinates the *corresponding* values of the particular quantity that is being studied. This can be best illustrated by an example.

To plot the results given in the table, Sec. 39, choose a suitable scale and lay off 200, 400, etc., upon the axis of abscissæ (Fig. 21) to represent

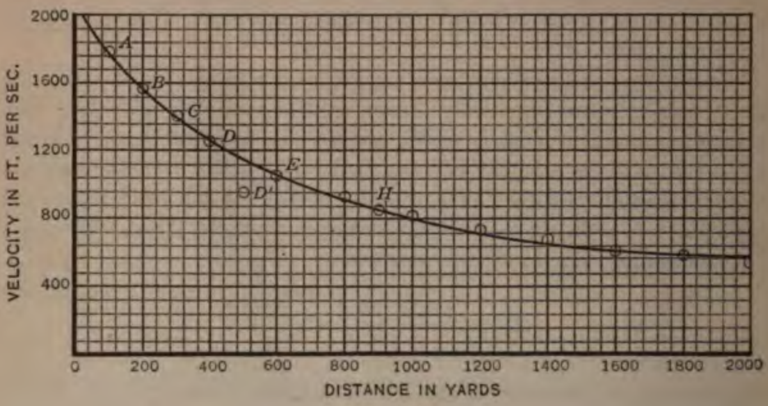

FIG. 21.

the distance (from muzzle of gun) in yards, and 400, 800, etc., on the axis of ordinates to represent the velocity of the bullet in feet per second. From the table we see that the velocity for a range of 100 yds. is 1780 ft. per sec. A point A at the center of a small circle (Fig. 21) gives this same information graphically, for the abscissa of A is 100 and its ordinate is 1780. Point B, whose abscissa is 200 and whose ordinate is 1590, fully represents the second pair of values (200 and 1590) in the table. In like manner the points C, D, etc., are plotted. Through these points a smooth curve is drawn as shown.

Use of the Curve.—It will be observed that the smooth curve passing through all of the other points does not pass through D'. The fact that a point does not fall on the curve indicates a probability of error either in taking the data or in plotting the results. In this case a defective cartridge may have been used at the 500-yd. distance. A second trial

from that same distance with a good cartridge would probably give a velocity of 1130 ft. per sec. as we would expect from the curve.

To find the velocity at a distance of 900 yds., note that the vertical line at 900 strikes the curve at H. But the ordinate of H is 850. Hence we know *without actually firing* from that distance, that the velocity of the projectile when 900 yds. from the muzzle is 850 ft. per sec. This method of finding values is called *Interpolation*. Such use of curves for detecting errors and for interpolating values makes them very valuable. They also present the data more forcibly than does the tabulated form, for which reason debaters frequently use them. In the physical laboratory and in engineering work curves are almost indispensable.

If there were also negative velocities to be plotted, *i.e.*, velocities having a direction opposite to that of the bullet, they would be designated by points at the proper distance *below* the axis of abscissæ. This axis would then be near the middle of the coördinate sheet instead of at the bottom as shown.

42. Newton's Three Laws of Motion.—Sir Isaac Newton, the great English mathematician and physicist, formulated the following fundamental laws of motion which bear his name.

1. A body at rest remains at rest, and a body in motion continues to move in the same direction and at the same speed, unless acted upon by some external force.

2. The acceleration experienced by a given mass is proportional to the applied force (accelerating force), and is in the direction of the applied force.

3. Action and reaction are equal, and oppositely directed.

The first law refers to the inert character of matter, the property of inertia by virtue of which any body resists any change in velocity, either in magnitude or direction. It is really impossible to have a body perfectly free from the effects of all external forces, but the more we eliminate these effects by reducing friction, etc., the more readily do we observe the tendency of a body to keep in motion when once started. The second law states the fact with which we have already become familiar in the discussion of the equation $F = Ma$ (Sec. 25). The third law is a statement of the fact that *whenever and wherever a force is applied there arises an equal and oppositely directed force.* This law will be further considered in the next section.

43. Action and Reaction, Inertia Force, Principle of d'Alembert.—If we press with the hand upon the top, bottom, or side of a table with a force of, say 10 lbs., we observe that the table exerts a counter push or force exactly equal to the applied force,

but oppositely directed. If the applied force is increased, the counter force, or *Reaction*, is inevitably increased. If, in order to push a boat eastward from a bank, the oarsman exerts a westward thrust (force) upon a projecting rock by means of his oar, the eastward reacting thrust of the rock that arises dents the oar and starts the boat eastward. If an eastward pull is exerted on a telephone pole, the guy wires to the westward tighten.

If a horse exerts a 300-lb. pull or force F_1 upon the rope attached to a canal boat a moment after starting, then the backward pull that the canal boat exerts upon the other end of the rope cannot possibly be either more or less than 300 lbs. Many people cling tenaciously to the erroneous belief that the forward pull of the horse must be at least *slightly* greater than the backward pull of the boat or the latter would not move. Many people also think that the winning party in a tug-of-war contest must exert a greater pull on the rope than does the losing party, which is certainly not the case. For this reason, we shall discuss very carefully the problem of the horse and canal boat. The applied force F_1 in this case overcomes two forces; one, the friction resistance, say 100 lbs., encountered by the boat in moving through the water, the other (200 lbs.), the backward pull exerted by the boat because, by virtue of its *inertia*, it *resists* having its speed increased. Note that we are here dealing with four forces. The 100 lbs. of the forward pull exerted by the horse just balances the 100-lb. backward pull of water friction on the boat; while the other 200 lbs. of forward pull or force f_1 exerted by the horse, just balances the resisting pull or force f_2 that the boat offers to having its speed increased. Obviously the accelerating force $f_1 = -f_2 = Ma$, in which M is the mass of the canal boat and a is its acceleration. The minus sign indicates that the forces are oppositely directed.

From this discussion, we arrive at the conclusion that the *forward* pull exerted *upon* any body is exactly equal in magnitude to the *backward* pull or resisting force exerted *by* the body. Thus here, if the horse had exerted a 400-lb. pull, we cannot escape the conclusion that the backward pull of the boat would have been 400 lbs.; 100 lbs. being the pull of water friction resistance as before, and 300 lbs. backward pull arising from the resistance the boat offered to having its speed increased. Since the accelerating force would be 300 lbs. in this case, instead of 200 lbs. as before, the acceleration would be 1/2 greater than before.

The above backward pull or force that any body, by virtue of its inertia, exerts in resisting change of velocity, has been very appropriately called *Inertia Force*. **The inertia force is always numerically equal to the accelerating force that gives rise to it, and is always oppositely directed.** If the canal boat were to run onto a sand bar, the friction would produce a large *negative* accelerating force, and the resistance the boat offered to decrease of speed would develop an equal *forward*, or **Driving Inertia Force,** that would carry the boat some distance onto the bar, even though the horse had ceased to pull. Had the sand bar been more abrupt, then both the negative accelerating force and the driving inertia force would have been greater than before, but they would still have been *exactly equal*.

The above fact, that all the forces exerted both *upon* and *by* any body under any possible circumstances are balanced forces, *i.e.*, that *the vector sum of all the forces exerted upon and by a body is invariably zero*, is known in mechanics as the **Principle of d'Alembert.** In common language, we frequently speak of unbalanced forces. In physics, even, it is frequently found convenient to use the term, but in such cases we are simply ignoring the inertia force. Strictly speaking, then, there is *no such thing as unbalanced forces*, if all forces, including inertia force, are taken into account. In the above case of the canal boat, the only *external* forces acting upon the boat to affect its motion are the forward pull exerted by the horse, and the backward pull exerted by the water friction. These *external* forces are clearly *unbalanced* forces. In this sense, and in this sense *only*, may we correctly speak of unbalanced forces.

44. Practical Applications of Reaction.—A horse cannot draw a heavy load on a slippery road unless sharply shod. In order to exert a *forward* pull on the vehicle, he must exert a *backward* push on the ground. A train cannot, by applying the brakes, stop quickly on a greased track because of the inability of the wheels to push *backward* on the axle, and therefore on the car, without pushing *forward* on the track. The wheels cannot, however, exert much forward push on a greasy rail.

A steamship, by means of its propellers, forces a stream of water backward. The reaction on the propellers pushes the ship forward. One of the best suggestions to give a person who is learning to swim is to tell him to *push the water backward*. The reaction forces the swimmer forward.

An aeroplane, by means of its propellers, forces a stream of air backward. The reaction on the propellers forces the aeroplane forward. The forward edge of each plane or wing is slightly higher than the rear edge. This causes the planes to give the air a downward thrust as the machine speeds horizontally through it. The reaction to this thrust lifts on the planes and supports the weight of the machine.

Suppose that an aeroplane, traveling 50 miles per hour, suddenly enters a region in which the wind is blowing 50 miles per hour in the same direction. Under these circumstances the air in contact with the planes, having no horizontal motion with respect to the planes, fails to give rise to the upward reacting thrust just mentioned, and the aeroplane suddenly plunges downward. Such regions as these, described by aeronauts as "holes in the air," are very dangerous. It is interesting to note in this connection that birds face the wind, if it is blowing hard, both in alighting and in starting, thus availing themselves of the maximum upward thrust of the air through which their wings glide.

45. Momentum, Impulse, Impact, and Conservation of Momentum.—The *Momentum* of a moving body is defined as the product of the mass of the body and its velocity, or

$$\text{Momentum} = Mv \tag{17}$$

The *impulse* of a force is the product of the force and the time during which the force acts, or

$$\text{Impulse} = Ft \tag{18}$$

An impulse is a measure of the ability of a force to produce motion or change of motion. We readily see that a force of 100 lbs. acting upon a boat for 2 sec. will produce the same amount of motion as a force of 200 lbs. acting for 1 sec. The term "impulse" is usually applied only in those cases in which the force acts for a brief time, *e.g.*, as in the case of collision or impact of two bodies, the action of dynamite or powder in blasting, the firing of a gun, etc., and the force is then called an *impulsive* force.

We shall now show that an impulse is numerically equal to the momentum change which it produces in a body, *i.e.*, $Ft = Mv$. Observe that a "bunted" ball loses momentum (mainly), while a batted ball loses momentum and then instantly acquires even

greater momentum in the opposite direction, due to the impulse applied by the bat. Obviously, the total *change* in the momentum of the ball, in case it returns toward the pitcher, is the product of the mass of the ball and the sum of its "*pitched*" and "*batted*" velocities. If a force F acts upon a certain mass M, it imparts to the mass an acceleration, determined by the equation $F=Ma$; while if this force acts for a time t, the impulse $Ft=Mat$. But the acceleration of a body multiplied by the time during which it is being accelerated gives the velocity acquired. Hence

$$Ft=Mat=Mv \tag{19}$$

It should be emphasized that v here represents the *change* in velocity produced by the impulse Ft.

We shall next show that when two free bodies are acted upon by an impulse, for example in impact or when powder explodes between them, then the change of momentum of one body is exactly equal but opposite in sign to the change in momentum of the other. *In other words the total momentum of both bodies is, taking account of sign, exactly the same before and after impact.* This law is very appropriately called the law of the *Conservation of Momentum.*

Theoretical Proof of the Conservation of Momentum.—Let us now study the effects of the impact in a rear end collision, caused by a truck A of mass M_a and velocity v_a overtaking a truck B of mass M_b and velocity v_b. Let v'_a and v'_b be the velocities after impact. During the brief interval of impact t, truck A pushes *forward* upon B with a variable force whose average value may be designated by F_b. During this *same time* t, truck B pushes backward upon A with a force equal at every instant to the forward push of A upon B (action equals reaction). Consequently the average value F_a of this backward push must equal F_b, and therefore

$$F_bt=-F_at \tag{20}$$

The minus sign in this equation indicates that the forces are oppositely directed. In fact F_a, being a *backward* push, is *negative.*

Since an impulse is equal to the change in momentum which it produces, and since the *change* in velocity of A is v'_a-v_a, and that of B is v'_b-v_b, we have

$$F_at=M_a(v'_a-v_a) \text{ and } F_bt=M_b(v'_b-v_b)$$

Hence, from Eq. 20, we have

$$M_b(v'_b - v_b) = -M_a(v'_a - v_a),$$

or

$$M_b(v'_b - v_b) + M_a(v'_a - v_a) = 0 \tag{21}$$

From the conditions of the problem, we see at once that v_b is less than v'_b, and that v_a is greater than v'_a. Accordingly, in Eq. 21, the first term, which represents the momentum change of truck B, is positive; while the second term, which represents the momentum change of truck A (momentum lost), is negative. Since these two changes are numerically equal but opposite in sign, the combined momentum of A and B is unchanged by the impact, thus proving the *Conservation* of *Momentum.*

Observe in equation 21 that the changes in velocity vary *inversely* as the masses involved. Thus if B had 3 times as great mass as A, its change (increase) in velocity would be only 1/3 as great as the change (decrease) in the velocity of A.

Briefer Proof.—The above concrete example has been used in the proof for the sake of the added clearness of illustration. We are now prepared to consider a briefer, and at the same time more general proof. In every case of impact of two bodies, whatever be their relative masses, or their relative velocities before impact, the impulsive force acting on the one, since action is equal to reaction, is equal to, but oppositely directed to that acting upon the other. Since these two *forces* are not only *equal* but also act for the *same* length of *time*, the two *impulses* are *equal*, and they are also oppositely directed. But, since an impulse is equal to the change in momentum *(Mv)* produced by it, it follows that the *momentum changes* of the two bodies are *equal* but oppositely directed, and that their *sum* is therefore zero. *In other words, the momentum before impact is equal to the momentum after impact*, thus proving the *Conservation* of *Momentum.*

Experimental Proof.—Consider two ivory balls A and B of equal mass suspended by separate cords of equal length. Let A be displaced through an arc of say 6 inches and then be released. As A strikes B it comes to rest and B swings through an equal 6-inch arc. This shows that the velocity of B immediately after impact is equal to the velocity of A immediately before impact. But A and B have equal mass, hence the total momentum is the same before and after impact, as is required by the law of the conservation of momentum.

46. The Ballistic Pendulum.—The ballistic pendulum affords a simple and accurate means of determining the velocity of a rifle ball or other projectile. It consists essentially of a heavy block of wood P (Fig. 22), of known mass M, suspended by a cord of length L. In practice, *four* suspending cords so arranged as to prevent all rotary motion are used.

As the bullet b of mass m and velocity v strikes P, it imparts to P a velocity V which causes it to rise through the arc AB, thereby raising it through the vertical height h. After impact, the mass of the pendulum is $M+m$. From the conservation of momentum we know that the momentum of the bullet before impact, or mv,

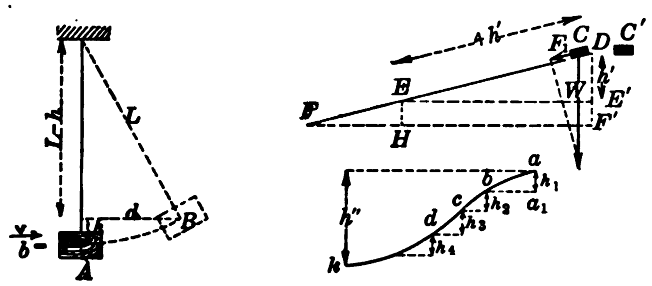

FIG. 22.

will be equal to the momentum of the pendulum (with bullet embedded) after impact, or $(M+m)V$, *i.e.*,

$$mv = (M+m)V \tag{22}$$

The values of m and M are found by weighing, and V is found from $V = \sqrt{2gh}$ (Eq. 14). For, as we shall presently prove, the velocity which enables the pendulum to swing through arc AB, or the equal velocity which it attains in returning from B to A, is that velocity which it would acquire in falling through the vertical height h. All other quantities being known, Eq. 22 may then be solved for v, the velocity of the bullet.

Velocity Dependent upon Vertical Height Only.—We shall now show that the velocity acquired by a body in descending through a given vertical height h by a frictionless path, is independent of the length or form of that path. Thus, if it were not for friction, the velocity of a sled upon reaching the foot of a hill of *varying* slope would be exactly that velocity which a body would acquire in falling through the vertical height of the hill.

In Fig. 22 (upper right corner) let DE be an incline whose slant height is, say, four times its vertical height DE' or h', *i.e.*, $DE = 4h'$. Let the body C, starting from rest, ***slide*** without friction down the incline, and let C', also starting from rest, ***fall*** without friction. Let us prove that the velocity (v_t) of C as it reaches E is equal to the velocity (v'_t) that C' acquires in *falling* to E'. Note that the vertical descent is the same for both bodies.

The component F_1 of C's weight W is the accelerating force acting upon C. From similar triangles we have

$$F_1/W = \frac{DE'}{DE} = \frac{h'}{4h'}, \text{ or } F_1 = \frac{W}{4}.$$

and therefore C's acceleration a is $g/4$. From Eq. 14*a* we have for the velocity of C at E, $v_t = \sqrt{2ad} = \sqrt{\frac{2g}{4} \times 4h} = \sqrt{2gh'}$. But from Eq. 14 we have, for the velocity v'_t of C' as it reaches E', $v'_t = \sqrt{2gh'}$; therefore $v_t = v'_t$, which was to be proved.

Further, it is obvious that the same reasoning would apply had h' been chosen larger, say equal to DF'. Accordingly, the velocity of C upon reaching F, would equal the velocity of C' upon reaching F'. This shows that the *increase* in C's velocity while going from E to F is equal to the *increase* in the velocity of C' in going through the *equal vertical distance* $E'F'$ (or EH).

Let us now consider the path $a\ b\ c\ \ldots\ k$, Fig. 22 (lower right corner), whose slope is ***not uniform***. By subdividing this path into shorter and shorter portions, in the limit each portion ab, bc, cd, etc., would be straight, and therefore aba_1, etc., become triangles similar to triangle EFH in the figure just discussed. From the discussion of triangle EFH already given, we see that the velocities acquired by a body in sliding without friction through the successive distances ab, bc, cd, etc., are equal respectively to the velocities that would be acquired by a body falling through the corresponding successive distances h_1, h_2, h_3, etc. But the sum of one series is the distance ak, while the sum of the other series is h'', the *vertical height* of ak.

Consequently the total velocity acquired by a body in sliding from a to k, or in general down any frictionless path, is equal to the velocity that would be acquired in free fall through the distance h'', or in general through the vertical height of the path.

We now see that V of Eq. 22 is given by the relation $V = \sqrt{2gh}$. If h is measured, V is known, and therefore v of Eq. 22 is determined. In practice, h is too small to be accurately measured and is therefore expressed in terms of d and L (see figure).

PROBLEMS

1. The distance by rail from a town A to a town B, 120 miles east of A, is 240 miles. The speed of a train going from A to B is 30 miles an hour for the first 120 miles, and 20 miles an hour for the remainder. Find the average speed and average velocity of the train for the run.

2. A train starts from rest at a town A and passes through a town B 5.5 miles to the eastward at full speed. The excess pull upon the drawbar above that required to overcome friction (*i.e.*, the accelerating force) is kept constant, so that the motion from A to B is uniformly accelerated. The train requires 22 minutes to make the trip. Find its average velocity and maximum velocity in mi. per min.; mi. per hr.; and ft. per sec.

3. Express the acceleration of the train (Prob. 2) in miles per hr. per min.; miles per min. per min.; and feet per sec. per sec.

4. What is the velocity of the train (Prob. 2) 15 sec. after leaving A? 2 min. after leaving A?

5. How long will it take a 2-ton pull to give a train of 40 cars, weighing 50 tons each, a velocity of 1 mi. per min. (*i.e.*, 88 ft. per sec.) on a level track? Neglect friction.

6. Compare the intensities of illumination due to an arc lamp at the two distances, 1/2 block, and 2 blocks.

7. A 50-lb. stone falls 16 ft. and sinks into the earth 1 ft. Find its negative acceleration, assuming it to be constant for this foot. Find the force required to penetrate the earth. Suggestion: Since the velocity of the stone during fall changes uniformly from zero to its "striking" velocity, and during its travel through the earth from striking velocity to zero, it follows that its average velocity in air and its average velocity in earth are the same, and that each is equal to 1/2 the striking velocity. See Sec. 33 and Sec. 45.

8. If an elevator cable pulls upward with a force 1200 lbs. on a 1000-lb. elevator, what is the upward acceleration? How far will it rise in 2 sec.? Suggestion: Find the accelerating force and express it in poundals, not pounds (see Sec. 32). Neglect friction.

9. How much would a 150-lb. man weigh standing in the above elevator if the pull on the cable were increased so as to make the acceleration the same as in problem 8?

10. A car that has a velocity of 64 feet per sec. is brought to rest in 10 sec. by applying its brakes. Find its average negative acceleration; and by comparing this acceleration with g, show graphically at what average slant a passenger standing in the car must lean back during this 10 sec.

11. If the car (Prob. 10) weighs 30 tons, what is the forward push exerted by its wheels upon the rails while it is being brought to rest?

12. Prove that the weight of a gram mass is 980.6 dynes, and that a force of 1 pound is equal to 32.17 poundals of force.

13. Reduce 2.5 tons to poundals; to dynes.

14. How far does a body travel in the first second of free fall from rest? In the second? In the third? In the fifth?

15. What is the gravitational pull of the earth upon a mass of 1 ton at the moon?

16. How far will a body fall in 7 sec.; and what will be its average and final velocities?

17. A car on a track inclined 30° to the horizontal is released. How far will it travel in the first 7 sec.; and what will be its average and final velocities (neglecting friction)? Compare results with those of problem 16.

18. How long will it take a body to fall 400 meters?

19. If a rifle ball is fired downward from a balloon with a muzzle velocity of 20,000 cm. per sec., how far will it go in 4 sec.? If fired upward, how far will it go in 4 sec.?

20. A baseball thrown vertically upward remains in the air 6 sec. How high does it go? Observe that the times of ascent and descent are equal, neglecting friction.

21. A stone is thrown upward from the top of an 80-ft. cliff with a velocity whose vertical component is 64 ft. per sec. What time will elapse before it strikes the level plain at the base of the cliff?

22. With what velocity does a body which has fallen 2000 ft. strike the ground?

23. A man 500 ft. south of a west-bound train which has a velocity of 60 miles per hour, fires a rifle ball with a muzzle velocity of 1000 ft. per sec. at a target on the train. Assuming the aim to be accurate, how much will the bullet miss the mark if the rifle sight is set for close range?

24. A stone is dropped from a certain point at the same instant that another stone is thrown vertically downward from the same point with a velocity of 20 ft. per sec. How far apart are the two stones 3 sec. later?

25. A rifle ball is fired at an angle of 30° above the horizontal with a muzzle velocity of 1200 ft. per sec. Neglecting air friction, find the range and time of flight.

26. If the rifle ball (Prob. 25) is fired horizontally from the edge of the cliff (Prob. 21), when and where will it strike the plain on the level of the base of the cliff?

27. If a 20-ton car A, having a velocity of 5 mi. an hr., collides with and is coupled to a 30-ton car B standing on the track, what will be their common velocity after impact?

28. If the above car A when it strikes B rebounds from it with a velocity of 1 mile per hour, find the velocity of B after collision. Observe that the total change of A's velocity is 6 miles per hour. Will B's change be more or less, and why?

29. A 2-gram bullet fired into a 2-kilo ballistic pendulum of length 2 meters produces a horizontal displacement $d = 10$ cm. (Fig. 22). Find the velocity of the bullet in cm. per sec. and ft. per sec.

CHAPTER IV

ROTARY MOTION

47. Kinds of Rotary Motion.—As has previously been stated (Sec. 22), a body has pure rotary motion if a line of particles, called the axis of rotation, remains stationary, and all other particles of the body move in circular paths about the axis as a center. Familiar examples are the rotation of shafts, pulleys, and flywheels. Rotary motion is of the greatest importance in connection with machinery of all kinds, since it is much more common in machines than reciprocating motion. The study of rotary motion is much simplified by observing the striking similarity in terms to those that occur in the discussion of translatory motion.

Translatory motion, as we have seen (Sec. 22), may be either uniform or accelerated; and the latter may be either uniformly accelerated or nonuniformly accelerated motion. The acceleration may also be either positive or negative. Likewise there are three kinds of rotary motion: (*a*) *uniform* rotary motion, *e.g.*, the motion of a flywheel or line shaft after it has acquired steady speed; (*b*) *nonuniformly accelerated* motion, *e.g.*, the usual motion of a flywheel when the power is first turned on (or off); and (*c*) *uniformly accelerated* rotary motion, *e.g.*, the motion which a flywheel would have if the torque (Sec. 48) furnished in starting had the proper value to cause its increase of rotary speed to be uniform.

48. Torque.—Torque may be defined as that which produces, or tends to produce, rotary motion in a body, just as force is that which produces, or tends to produce, motion of translation in a body. The magnitude of a torque is force times "lever arm" (Eq. 25), and its direction depends upon both the direction and the point of application of the force. A torque is not simply a force, for it is readily seen that any force directed either toward or away from the axis, *e.g.*, force *a* (Fig. 23*a*), has no tendency to produce rotation. A torque tending to produce rotation in a counterclockwise direction is called a positive

torque, while a torque which is oppositely directed is called negative.

Fig. 23*a* represents the grindstone shown in Fig. 23 as viewed from a point in line with the axle. The torque due to the force *a* alone is zero. The torque due to the force *b* alone is $b \times OP$ (*i.e.*, $b.OP$), and is negative. The torque due to force *c* alone is also negative, and its magnitude is $c.OE$. For the thrust *c* equals the pull c', which may be thought of as exerted upon a cord $c'P$. Evidently the pull of such a cord would be just as effective in producing rotation, at the instant shown, if attached to *E* on a crank *OE*, as if attached to *P* on the crank *OP*. Thus when we define torque as force times "lever arm," or

$$T = Fr \tag{25}$$

we must interpret the "lever arm" *r* to mean the *perpendicular* distance from the axis of rotation to the *line* of *action* of the force.

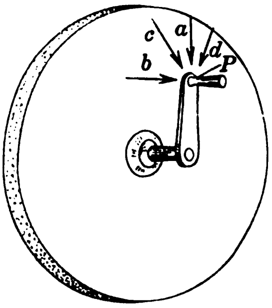

Fig. 23.

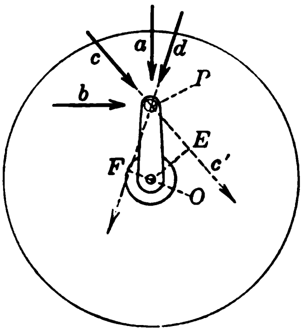

Fig. 23*a*.

The force may be expressed in dynes, poundals, pounds, etc., and the lever arm in centimeters, inches, feet, etc.; so that torque may be expressed in dyne-centimeter units, or in poundal-feet, or pound-feet units, etc. If several torques, some positive and some negative, act simultaneously upon a flywheel, the flywheel will start (or, if in motion, increase its speed) clockwise, provided the negative torques exceed the positive torques; whereas it will start, or, if in motion, increase its speed counter-clockwise, provided the positive torques are the greater. If the positive and negative torques just "balance," then the [illegible] wheel will remain at rest; or if already in motion, its [illegible] not change.

The Couple.—Two equal and oppositely directed forces which do not have the same line of action (F and F', upper sketch, Fig. 24) constitute a *Couple.* The torque developed by this couple is equal to the product of *one* of the forces, and the distance AC between them, and is entirely independent of the position (in the plane of the figure) of the pivot point about which the body rotates. The torque due to this particular couple is also counterclockwise (positive) whether the pivot point is at A, B, C, D, or at any other point. If A is the pivot point, then the force F produces no torque, while F' produces the positive torque $F' \times AC$ (*i.e.*, $F'.AC$). If B is the pivot point, then both forces produce positive torques; but, since the lever arm for each is then only $\frac{1}{2}$ AC, the total torque is the same as before. If D is the pivot point, then F' produces a negative torque, and F, a positive torque; but, since F acts upon a lever arm which is longer than that of F' by the distance AC, it follows that the sum of these two torques about D is $F.AC$ as before, and is also *positive.*

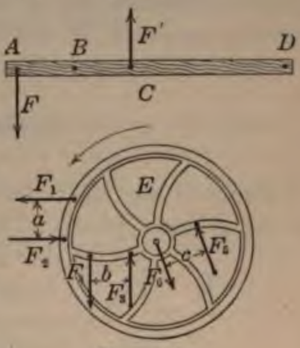

Fig. 24.

If three men A, B, and C by pushing with one hand and pulling with the other apply respectively upon the wheel E (Fig. 24) the couples represented by F_1 and F_2, F_3 and F_4, and F_5 and F_6, then each man will contribute an equal positive torque helping to rotate the wheel. For, as sketched, the forces are all equal, and the distances a, b, and c are equal; consequently the three torques are equal. But from the above discussion we see that the torques due to these three similar couples will be equal about any point in the plane of the wheel, and hence about its axis.

49. Resultant Torque, and Antiresultant Torque.—Let the forces a, b, c, and d, Fig. 23a, be respectively 20, 12, 14, and 40 pounds, and let $OP = 1$ ft., $OE = 8$ in., and $OF = 4$ in. The torque due to a is zero; that due to b is 12×1 or 12 lb.-ft., or 144 lb.-in., negative; the torque due to c is 14×8 or 112 lb.-in., negative, and that due to d is $40 \times 4 = 160$ lb.-in. positive. The sum of all these ...at is the one torque that would be just as effective in ...ation as all of these torques acting simultaneously, ...b.-ft., a negative torque. Consequently, one

force, say h, acting in the direction b, but of magnitude 8 lbs., would produce just as great a torque as would all four forces, a, b, c, and d acting together. This torque may be called the *Resultant* of the other four torques. If the force h is reversed in direction, it produces a positive torque of 8 lb.-ft., called the *Antiresultant* torque. This antiresultant torque, acting with the torques due to a, b, c, and d, would produce equilibrium. Obviously, this antiresultant torque, instead of being an 8-lb. force on a 1-ft. arm, might, for example, be a 4-lb. force on a 2-ft. arm, or a 16-lb. force on a 6-in. arm.

50. Angular Measurement.—Angles may be measured in degrees, minutes, and seconds, in revolutions, or in radians. In circular measure, an angle is found by dividing the subtended arc by the radius, that is,

$$\text{Angle} = \frac{\text{arc}}{\text{radius}}, \text{ or } \theta = \frac{\text{arc}}{r} \tag{26}$$

If the arc equals the radius, then the angle is of course unity, and is called one *Radian*. Thus angle AOC (Fig. 25) is one radian because arc ABC equals the radius r. The angle AOB, or θ, is 1/2 radian because the arc AB is 1/2 the radius r. Since the circumference of a circle is $2\pi r$, it follows that there are 2π radians in 360°, or the radian equals 57°.3. In the study of Mechanics, angles, angular velocity, and angular acceleration are almost always expressed in terms of radians instead of degrees.

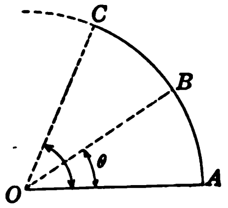

Fig. 25.

51. Angular Velocity and Angular Acceleration.—Angular velocity is the angle traversed divided by the time required; or, since the unit of time is usually the second, it is numerically the angle turned through in one second. If a certain flywheel makes 600 revolutions per min. (written 600 R.P.M.), its angular velocity

$$\omega = 10 \text{ rev. per sec., or } 62.8 \ (i.e., \ 10 \times 2\pi) \text{ radians per sec.}$$

If the rotary speed of the flywheel is constant during the one minute, the above 62.8 radians per sec. is its actual angular velocity at any time during that minute; whereas if its speed fluctuates, then 62.8 radians per sec. is simply the *average* angular velocity $\bar{\omega}$ (read "barred omega") for the minute considered.

Again, suppose that the above flywheel, starting from *rest* and uniformly increasing its *speed*, makes 600 revolutions in the first minute. Its average angular velocity $\bar{\omega}$ is 62.8 rad. per sec. as before; but, since its initial velocity is zero, its angular velocity ω_t at the close of the first minute must be twice the average, or 125.6 rad. per sec. (Compare $\bar{v}$ of Sec. 33.) Since this angular velocity is acquired in one minute, the angular acceleration (α) is given by the equation

$$\alpha = \frac{\omega_t}{t} = 125.6 \text{ radians per sec. per min.}$$

In one second the wheel will acquire 1/60 as much angular velocity as it does in 1 min.; hence we may also write

$$\alpha = 2.09 \text{ radians per sec. per sec.}$$

which means that in one second the increase in angular velocity is 2.09 radians per sec. Evidently, at a time t seconds after starting, the angular velocity $\omega_t = \alpha t$. Thus 5 seconds after starting $\omega = 10.45$ radians per sec.

To summarize (see also Secs. 52 and 57), we have, in translatory motion,

$$\text{Average velocity} = \frac{\text{distance traversed}}{\text{time required}}, \text{ or } \bar{v} = \frac{d}{t}.$$

In rotary motion

$$\text{Average angular velocity} = \frac{\text{angle traversed}}{\text{time required}}, \text{ or } \bar{\omega} = \frac{\theta}{t} \quad (27)$$

$$\text{Acceleration (trans. motion)} = \frac{\text{gain in velocity}}{\text{time required}}, \text{ or } a = \frac{v_t - v_o}{t}$$

$$\text{Angular acceleration} = \frac{\text{gain in angl. velocity}}{\text{time required}}, \text{ or } \alpha = \frac{\omega_t - \omega_o}{t} \quad (28)$$

52. Relation between Linear and Angular Velocity and Acceleration.—If, due to a constant accelerating torque, a body starts from rest with a constant angular acceleration α, and, in a time t, rotates through an angle θ and acquires an angular velocity ω, then it will be true that any mass particle in this body at a distance r from the axis travels, in this time t, a distance $d = r\theta$ (note that arc $= r\theta$, Eq. 26), acquires in this time a linear velocity $v = r\omega$, and experiences during this same time a linear acceleration $a = r\alpha$.

Proof: Dividing both sides of the equation $d = r\theta$ by

$\bar{v}=r\frac{\theta}{t}=r\bar{\omega}$. If a body starts from rest with uniform acceleration, its average velocity $\bar{v}$ is of course only half as great as its final velocity v; hence $v=2\bar{v}$. Likewise $\omega=2\bar{\omega}$. Hence, since $\bar{v}=r\bar{\omega}$, it follows that $v=r\omega$. Now $a=v/t$; therefore, dividing both sides of the equation $v=r\omega$ by t, gives $a=r\omega/t=r\alpha$. Accordingly

$$d=r\theta,\quad v=r\omega, \text{ and } a=r\alpha \tag{29}$$

If θ is given in radians, ω in radians per second, and α in radians per second per second, then if r is given in feet, d will be expressed in feet, v in feet per second, and a in feet per second per second. From Eq. 29, we see (1) that the distance which a belt travels is equal to the product of the radius (r) of the belt wheel over which it passes, and the angle θ (in radians) through which this wheel turns; (2) that the linear velocity of the belt is equal to r times the angular velocity of the wheel in radians per second, and (3) that the linear acceleration which the belt experiences in starting, is equal to r times the angular acceleration of the belt wheel expressed in radians per second per second.

Let it be required to find the angular velocity ω of the drivers of a locomotive when traveling with a known velocity v. From Eq. 29 we have $\omega=v/r$; hence, dividing the linear velocity of the locomotive expressed in *feet* per second by the radius of the driver in feet, we obtain ω in radians per second.

53. The Two Conditions of Equilibrium of a Rigid Body.— If the resultant of all of the *forces* acting upon a body is zero, the *First Condition of Equilibrium* is satisfied (Sec. 17), and the body will remain at rest, if at rest, or continue in uniform motion in a straight line if already in motion. If, in addition, the resultant of the *torques* acting upon the body is zero, the *Second Condition of Equilibrium* is satisfied, and the body will remain at rest, if at rest, or if already rotating its angular velocity will neither increase nor decrease. Forces which satisfy the first condition of equilibrium may not satisfy the second. The general case of several forces acting upon various points of the body, and in directions which do not all lie in the same plane, is too complex to discuss here. The simpler but important case of three forces all lying in the same plane will now be considered.

A body acted upon by three forces which lie in the same plane is in equilibrium if (a) the three forces when represented graphically form a closed triangle (first condition of equilibrium); and

(*b*) if the lines of action of these three forces meet in a point (second condition of equilibrium). Thus the body *A* (Fig. 26) is in equilibrium, since the three forces *a*, *b*, and *c*, form a closed triangle as shown, and they also (extended if necessary) meet at the point *E*.

The three forces a', b', and c' which act upon the body *B* (Fig. 27), when graphically represented form a closed triangle and therefore have zero resultant. Consequently they have no tendency to produce motion of translation in the body, but they do tend to produce rotation. For the forces b' and c' meet at *D*, about which point the remaining force a' clearly exerts a clockwise torque; hence the second condition of equilibrium is not fulfilled.

That forces *a*, *b*, and *c* (Fig. 26) produce no torque about *E* is

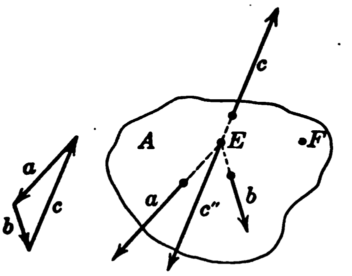

FIG. 26.

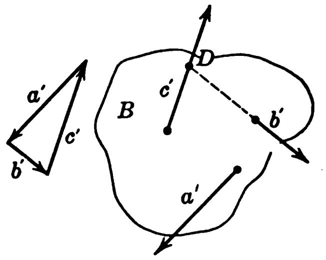

FIG. 27.

evident, since all three act directly away from *E*. It may not be equally evident that they produce no torque about any other point in *A*, such as *F*. That such is the case, however, may be easily shown. The two forces *a* and *b* have a resultant, say c'', which is equal to *c* but oppositely directed (since the three force *a*, *b*, and *c* are in equilibrium); hence *a* and *b* may be *replaced* by c'' acting downward at *E*. But obviously *c* and c'' would produce equal and opposite torques about *F*, or about any other point that may be chosen. *Hence three forces which form a closed triangle and also meet in a point have no tendency to produce either translation or rotation of a body.*

Applications to Problems.—A ladder resting upon the ground at the point *A* (Fig. 28) and leaning against a frictionless wall at *P* supports at its middle point a 200-lb. man whose weigh

represented by W. Neglecting the weight of the ladder, let us find the thrusts a and b. Since the ladder is in equilibrium, the three forces a, b, and W which act *upon* it must *meet at a point* and must also *form a closed triangle*. The thrust b must be horizontal, since the wall is frictionless, and it therefore meets W produced at C. The upward thrust of the ground on the ladder must also pass (when extended) through C; *i.e.*, it must have the direction AC. To find the magnitude of a and of b, draw W to a suitable scale, and through one end of W draw a line parallel to b, and through the other end draw a line parallel to a. The intersection of these two lines determines the magnitude of a and of b, as explained under Fig. 7, Sec. 18.

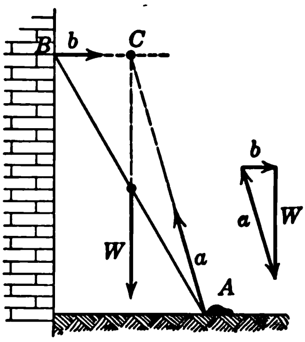

FIG. 28.

Since the crane beam in the problem at the close of this chapter is acted upon by three forces, and since it is also in equilibrium, the problem may be solved by the method of this section.

54. Moment of Inertia and Accelerating Torque.—The mass of a body may be defined as that property by virtue of which the body resists a force tending to make it change its velocity. The *Moment of Inertia* of a body, *e.g.*, of a flywheel, is that property by virtue of which the flywheel resists a torque tending to make it *change* its angular velocity. Consider a steam engine which is belted to a flywheel connected with a buzz saw, as in the case of a small saw mill. The difference between the tension on the tight belt and the slack belt, times the radius of the pulley over which the belt passes, gives the applied torque. If the *applied* torque is just sufficient to overcome the opposing torque due to friction of bearings, and the friction encountered by the saw, then the speed remains uniform; while if the applied torque exceeds this value, the angular velocity increases, and its rate of increase, that is, the angular acceleration, is proportional to this *excess* torque. If the applied torque is *less* than the resisting torque, the angular acceleration is negative, that is, the flywheel slows down, and the rate at which it slows down is proportional to the *deficiency* in torque. Compare with accelerating force, Sec. 25.

The relation between the moment of inertia I of a flywheel, the

accelerating torque, and the resulting angular acceleration α, is given by the following equation,

$$\text{Accelerating torque} = I\alpha, \textit{ i.e.}, \ T = I\alpha \qquad (30)$$

Compare with $F = Ma$ (Eq. 5, Sec. 25). If we apply a known torque and determine α experimentally, we may find the numerical value of I from Eq. 30. If the torque is expressed in dyne-centimeters (*i.e.*, the force in dynes and the lever arm in centimeters) and α in radians per second, then I will be expressed in C.G.S. units (see also Sec. 55). From Eq. 30 we see that unit *torque* will give a body of unit *moment of inertia* unit *angular* acceleration; while from Eq. 5, we see that unit *force* will give unit *mass* unit *linear* acceleration.

The moment of inertia of two similar wheels is found to be proportional to the products of the mass and the radius squared for each (Eq. 31, Sec. 55). Hence we find fly wheels made with large mass and large radius, and with the greater part of the mass in the rim, for which part the radius is *largest.*

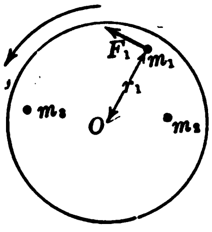

Fig. 29.

55. Value and Unit of Moment of Inertia.—We shall now determine the relation between the C.G.S. unit of moment of inertia (Sec. 54) and the mass and radius of the rotating body, say a wheel. We shall first determine the expression for the moment of inertia of a particle of mass m_1 at a distance r_1 from the axis of rotation (Fig. 29). Let us consider only this mass m_1, ignoring, for the time being, the mass of the rest of the wheel. To further simplify the discussion, let the force F_1 that produces the accelerating torque T_1, act upon m_1 itself, so that

$$T_1 = F_1r_1 = m_1ar_1 = m_1\alpha r^2{}_1$$

since $\qquad a = r_1\alpha$ (Eq. 29, Sec. 52).

But this *same* accelerating torque $= I_1\alpha$ (Eq. 30), in which I_1 is the moment of inertia of m_1 about the axis through O, and α is its angular acceleration about the same axis.
Consequently

$$I_1\alpha = m_1\alpha r_1^2, \text{ or } I_1 = m_1r_1^2 \qquad (31)$$

Likewise, the moment of inertia I_2 of m_2 (see Fig. 29) can be shown to be $m_2r_2^2$, and that of m_3 to be $m_3r_3^2$, etc. Now if we add together the moments of inertia of all the mass particles of the wheel we have for the moment of inertia of the *entire* wheel

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ . \ . \ . \ \text{etc.}$$

This may be briefly written

$$I = \Sigma\ mr^2 \tag{32}$$

in which Σmr^2 (read *sigma* mr^2) signifies a summation of mr^2 for all of the mass particles in the wheel.

If, in Eq. 31, all quantities are expressed in C.G.S. units, then m will be expressed in grams, r in centimeters, and hence I will be expressed in gm.-cm.^{2} units. If units of the F.P.S. system are used, I will be expressed in lb.-ft.^{2} units. Thus a 2000-lb. flywheel having practically all of its mass in the rim of mean radius 5 feet, would have a moment of inertia $I = Mr^2$ (approx.) $= 50{,}000$ (*i.e.*, 2000×5^2) lb.-ft.^{2} For the r of Eq. 32 is practically the same (*i.e.*, 5 ft.) for every mass particle in the wheel, and the combined mass of all these particles is M or 2000 lbs.

The moment of inertia of an emery wheel or grindstone of radius r and mass M is obviously less than Mr^2; for in this case the mass is not mainly concentrated in the "rim," since many of the mass particles move in circles of very small radius r. It can be shown by the use of higher mathematics that the moment of inertia of such disc-like bodies is

$$I = \tfrac{1}{2}Mr^2 \tag{33}$$

For a sphere of radius r and mass M

$$I = \tfrac{2}{5}Mr^2 \tag{34}$$

56. Use of the Flywheel.—The purpose of a flywheel, in general, is to "steady" the motion. Thus, in the above-mentioned case of the saw mill (Sec. 54), if the applied torque furnished by the steam engine is greater than all the resisting friction torques, this *excess* torque, or *accelerating* torque, causes the speed of the flywheel to increase; while if the saw strikes a tough knot, so that the friction torques exceed the applied torque, then the flywheel helps the engine to run the saw,

and in so doing is slowed down. Indeed the flywheel, when its speed is *increasing*, is *storing up energy*, which is again handed on to the saw when its speed *decreases*.

It is a matter of common observation that a heavy wheel, when being set in motion with the hand, offers an opposing *backward* inertia torque; while if we attempt to slow down its motion, it offers an opposing *forward* inertia torque, or *Driving Inertia Torque*. It is just this *driving inertia torque, developed by the flywheel when slowing down, that helps the engine to run the saw* through the tough knot. Compare this with the *driving inertia force* that pushes the canal boat onto the sand bar (Sec. 43). If one were to shell some corn with the ordinary hand corn sheller, both with and without the flywheel attached, he would be very forcibly impressed with the fact that, at times, the flywheel assists with a driving torque.

In the case of "four cycle" gas engines (Chap. XVIII) which have one working stroke to three *idle* strokes (*i.e.*, the three strokes during which the gas is not pushing upon the piston), it must be clear that the flywheel runs not only the machinery, but also the engine itself, during these three strokes. Doing this work, *i.e.*, supplying the driving torque during the three idle strokes, necessarily slows down the flywheel, but this lost speed is regained during the next stroke, or working stroke, when the explosion occurs. If the flywheel is too light, this fluctuation in speed is objectionably great. Since, in the case of steam engines, *every* stroke is a working stroke, lighter flywheels suffice than for gas engines of the same horse power and speed.

The flywheel of a high speed gas engine need not have so great moment of inertia as is required for a lower speed engine furnishing the same horse power. In each case, to be sure, the flywheel "carries" the load during the three idle strokes, but the time for these three idle strokes is shorter for the high speed engine. (Flywheel design will be considered in Sec. 76).

57. Formulas for Translatory and Rotary Motion Compared.—Below will be found a collection of formulas applied to translatory motion, and opposite them the corresponding formulas for rotary motion. The similarities and differences in these two sets of formulas should be observed. All of these formulas should be thoroughly understood, and most of them may be memorized with profit.

Translatory Motion	*Rotary Motion*
$\bar{v}=\frac{d}{t}$	$\bar{\omega}=\frac{\theta}{t}$
$v_t=at$ or, v_o+at	$\omega_t=\alpha t$ or, $\omega_o+\alpha t$
$\bar{v}=v_o+\frac{1}{2}at$	$\omega=\omega_o+\frac{1}{2}\alpha t$
$a\,(\text{or } g)=\frac{v_t-v_o}{t}$ or $\frac{v_t}{t}$	$\alpha=\frac{\omega_t-\omega^o}{t}$ or $\frac{\omega_t}{t}$
$d=\bar{v}t$	$\theta=\bar{\omega}t$
$d=\frac{1}{2}gt^2$	$\theta=\frac{1}{2}\alpha t^2$
$d=v_ot+\frac{1}{2}gt^2$	$\theta=\omega_ot+\frac{1}{2}\alpha t^2$
$F=Ma$	$T=I\alpha$
F is *accelerating* force.	T is *accelerating* torque.
Kinetic energy $=\frac{1}{2}Mv^2$	Kinetic energy $=\frac{1}{2}I\omega^2$

(Energy is discussed in Chap. VI.)

PROBLEMS

1. Reduce 2.5 revolutions (*a*) to radians; (*b*) to degrees. Express the angle between north and northeast in (*c*) radians; (*d*) degrees; and (*e*) revolutions.

2. A shaft makes 1800 R.P.M. Find ω in radians per sec.; in degrees per sec.

3. Through how many degrees will a shaft rotate in 3 min., if $\bar{\omega}=20$ radians per sec.?

4. A flywheel, starting from rest with uniformly accelerated angular motion, makes 15 revolutions in the first 10 sec. What is its average angular velocity (*a*) in revolutions per sec.? (*b*) In radians per sec.? (*c*) In degrees per sec.? (*d*) What is its velocity at the close of the first 10 sec.?

5. What is the angular acceleration of the flywheel of (problem 4) (*a*) in radians per sec. per sec.? (*b*) In radians per sec. per min.?

6. A belt which travels at the rate of 30 ft. per sec. drives a pulley whose radius is 3 in. What is the angular velocity for the pulley?

7. A small emery wheel acquires full speed (1800 R.P.M.) 5 sec. after starting. Assuming the angular acceleration to be constant, find its value for this 5 sec.

8. Through what angle does the emery wheel rotate (Prob. 7) in the first 5 sec.?

9. Find the total torque produced by the forces *a*, *b*, *c*, and *d* (Sec. 49) if *a* and *b* are both reversed in direction.

10. A locomotive has a velocity of 30 miles per hr. one minute after starting. (*a*) What is its average acceleration for this minute? (*b*) What is the average angular acceleration of its drivers, which are 6 ft. in diameter?

11. A crane (Fig. 7, Sec. 18) is lifting a load of 2400 lbs. Find the thrust of the beam *B* against the post *A* and the pull on cable *C* due to this

load, if B is 30 ft. in length and inclines 30° to the vertical, and if C is attached to B at a point 10 feet from O, and to A at a point 20 feet above the foot of B. Use the graphical method and compare with the ladder problem, Sec. 53.

12. The arms AO and OB of the bell crank (Fig. 30) are equal. Find the pull F, and also the thrust of O on its bearings.

13. Find the required pull and thrust (Prob. 12) if F has the direction BC. Compare ladder problem, Sec. 53.

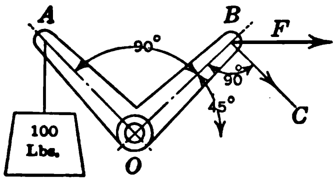

Fig. 30.

14. The belt which drives a 1600-lb. flywheel, whose rim has an average radius of 2 feet (assume mass to be all in the rim), passes over a pulley of 1-ft. radius on the same shaft as the flywheel. The average pull of the tight belt exceeds that of the slack belt by 100 lbs. Neglecting friction, how long will it take the flywheel to acquire a velocity of 600 R.P.M. First find I, then α, etc.

CHAPTER V

UNIFORM CIRCULAR MOTION, SIMPLE HARMONIC MOTION

58. Central and Centrifugal Forces, and Radial Acceleration.—If a body moves in a circular path with uniform speed, it is said to have *Uniform Circular Motion.* If a stone, held by a string, is whirled round and round in a horizontal circular path, it has approximately uniform circular motion. In order to compel the stone to follow the curved path, a certain inward pull must be exerted upon the string by the hand. This pull is termed the *Centripetal* or *Central* force. The opposing pull or force exerted by the stone by virtue of its inertia (which inertia in accordance with Newton's first law tends to make it move in a straight line tangent to the circle), is exactly equal to this central force in magnitude, and is termed the *Centrifugal* force.

If the string breaks, both the central and centrifugal forces disappear, and the stone flies off in a straight line tangent to its path at that instant. The pull upon the string causes the stone to change its velocity (not in magnitude but in *direction*) and is therefore an accelerating force and equal to Ma, in which M is the mass of the stone and a, its acceleration. Hence to find the pull upon the string it will be necessary to weigh the stone to get M, and also to compute the acceleration a. Observe that the applied *accelerating* force is the pull of the string; while the centrifugal force is really the inertia force that arises due to the resistance the stone offers to having its velocity changed (in direction).

Here, as in all possible cases that may arise, the *accelerating* force and the *inertia force* are equal in magnitude but *oppositely* directed, and they disappear simultaneously (Sec. 43). The simultaneous disappearance of the central and centrifugal forces at the instant the string breaks, is in complete accord with the behavior of all reactions. Thus, so long as we push down upon a table, we experience the upward reacting thrust; but the instant we cease to push, the reacting thrust disappears.

Centrifugal force has many important applications, for example, in the cream separator (Sec. 60), the centrifugal gov-

ernor (Sec. 63), the centrifugal pump, and centrifugal blower (Sec. 150). It is this force which causes too rapidly revolving flywheels and emery wheels to "burst" (Sec. 59), and it is also this force which necessitates the raising of the outer rail on curves in a railroad track (Sec. 62). The centrifugal clothes dryer used in laundries, and the machine for separating molasses from sugar, used in sugar refineries, both operate by virtue of this principle. The centrifugal force due to the velocity of the earth in its orbit prevents the earth from "falling" to the sun (Sec. 29), while the centrifugal force due to its rotation about its axis causes the earth to flatten slightly at the poles and bulge at the

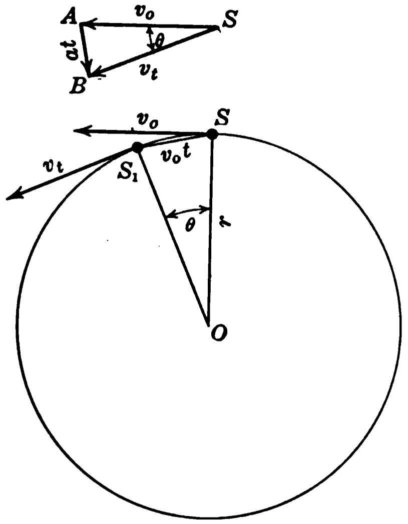

FIG. 31.

equator. The polar diameter is about 27 miles less than the equatorial diameter.

To find the *Radial Acceleration* a, construct a circle (Fig. 31) whose radius r represents the length of the string. Let S represent the stone at a certain instant ($t = 0$), at which instant it is moving west with a velocity v_o. After a time t (here t is chosen about 1/2 sec.), the stone is at S_1, and its velocity v_t is the same in magnitude as before, but is directed slightly south of west. Its velocity has evidently changed, and if this change is divided by the time t in which the change occurred, the result is by definition (Sec. 24) the acceleration a.

This change in velocity, or the velocity acquired, is readily found by drawing from S (Fig. 31, upper sketch) two vectors, SA and SB, to represent v_o and v_t respectively, and then connecting A and B. Obviously the acquired velocity is that velocity which added (vectorially) to v_o gives v_t; consequently it is represented by the line AB. Acquired velocity, however, is always given by the product of acceleration and time, or at; hence, the velocity $AB = at$.

The triangles OSS_1 and SAB are similar, since their sides are perpendicular each to each; and if θ is very small, arc SS_1 may be considered equal to chord SS_1. But SS_1 is the distance the stone travels in the time t, or $v_o t$. Hence, from similar triangles,

$$\frac{at}{v_o} = \frac{v_o t}{r}, \text{ or } a = \frac{v_o^2}{r} \tag{35}$$

Since $F = Ma$, the central force, usually designated as F_c, is given by the equation

$$F_c = \frac{Mv^2}{r} \tag{36}$$

As already stated, the centrifugal force and the central force are equal in magnitude but oppositely directed, hence, F_c (Eq. 36) may stand for either. If M is the mass of the stone S in pounds, v its velocity in feet per second, and r is the length of the string in feet; then F_c is the pull on the string in *poundals*, not pounds (see Secs. 25 and 32). If M is given in grams, v in centimeters per second, and r in centimeters, then F_c is the pull in *dynes*, not grams of force. By means of this equation we may compute the forces brought into play in the operation of the centrifugal clothes dryer, cream separator, steam engine governor, or in the case of a fast train rounding a curve.

In many cases it is found more convenient to use a formula involving angular velocity in revolutions per second instead of linear velocity. If a body, *e.g.* a wheel, makes n revolutions per second, its "rim" velocity, or the distance traversed in one second by a point on the rim of the wheel, is n circumferences or $2\pi rn$. Substituting this value for v in Eq. 36 we have

$$F_c = \frac{M(2\pi rn)^2}{r} = 4\pi^2 n^2 rM \text{ or } \omega^2 rM \tag{37}$$

For, since one revolution is 2π radians, $\omega = 2\pi n$, and $\omega^2 = 4\pi^2 n^2$.

Central Force Radial.—That F_c is radial is apparent in the above case, since the force must act in the direction of the string. That this is equally true in the case of a flywheel or cream separator, or in all cases of uniform circular motion, may be seen from a discussion of Fig. 32. For if the central force F_c acting upon a particle P which is moving to the left in the circle, had the direction a, there would be a component of this force, a', acting in the direction of the motion, and hence tending to increase the velocity; if, on the other hand, F_c acted in the direction b, there would be a component of the force, b', acting in such a direction as to decrease the velocity. But if P has *uniform* circular motion, its velocity must neither increase nor decrease; hence neither of these components, a' and b', can be present, and F_c must therefore be radial.

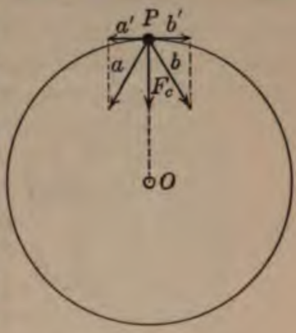

Fig. 32.

That the acceleration is radial can be shown in another way. As point S_1 (Fig. 31) is taken closer and closer to S (*i.e.*, as t is chosen smaller and smaller) v_t becomes more nearly parallel to v_o, and AB (see upper sketch, Fig. 31) becomes more nearly perpendicular to v_o. In the limit, as S_1 approaches S, AB becomes perpendicular to v_o, and therefore parallel to r. But the acceleration has the direction AB, hence it is radial. It should also be emphasized that the acceleration is *linear* (not angular), and is therefore usually expressed either in feet per second per second or in centimeters per second per second (Sec. 24).

59. Bursting of Emery Wheels and Flywheels.—The central force F_c required to cause the material near the rim of a revolving emery wheel to follow its circular path, is usually enormous. If the speed is increased until F_c becomes greater than the strength of the material can withstand, then the material pulls apart, and we say that the emery wheel "bursts." It is evident that it does not burst in the same sense that it would if a charge of powder were exploded at its center. In the latter case the particles would fly off *radially;* while in the former they fly off *tangentially*. Indeed, the instant the material cracks so that the central force disappears, the centrifugal force also disappears

(Sec. 58), and each piece moves off in a straight line in the direction in which it happens to be moving at that instant.

60. The Cream Separator.—The *essentials* of a cream separator are, a bowl A (Fig. 33), attached to a shaft B, and surrounded by two stationary jackets C and E. When B is rapidly revolved by means of the "worm" gear, as shown, the fresh milk, entering at G, soon acquires the rotary motion of the bowl, and, due to its inertia which tends to make it move in a straight line, it crowds toward the outside of the bowl with a force F_c.

Both the cream and the milk particles tend to crowd outward from the center of the bowl, but the milk particles being heavier than cream particles of the *same size*, experience the greater force, and a separation takes place. In the figure the cross-hatched portion c represents the cream, and the space between this and the bowl, marked m, represents the milk. Small holes marked a permit the cream to fly outward into the stationary jacket E, from which it flows through the tube F into the cream receptacle, the holes marked b, farther from the center of the bowl than holes a, permit the skim-milk to fly outward into the stationary jacket C, from which it flows through the tube D into the milk receptacle.

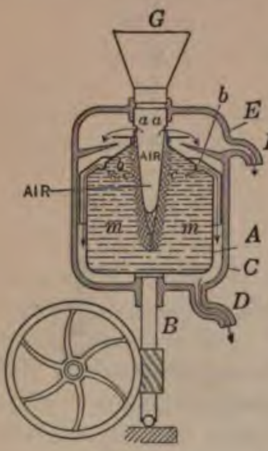

FIG. 33.

The bowls of many of the commercial separators contain numerous separating chambers designed to make them more effective. This simple form, however, illustrates the features common to all. With a good cream separator about 98 or 99 per cent. of the butter fat is obtained; *i.e.*, 1 to 2 per cent. remains in the skim-milk. In the case of "cold setting" or gravity separation and skimming as usually practised, 5 per cent. or more remains in the skim-milk.

61. Efficiency of Cream Separator.—In fresh milk, the cream is distributed throughout the liquid in the form of finely divided particles. If allowed to stand for several hours the cream particles, being slightly lighter than the milk particles, slowly rise to the surface. Thus a separation of the cream from the milk takes place, and, since it is due to gravitational force, it is termed "gravita-

tional" separation. Calling the mass of one of these cream particles m_1 and the mass of an *equal volume* of milk m_2, the pull of the earth (in dynes, Sec. 32) on the cream particles is m_1g and the pull on the milk particles is m_2g. The *difference* between these two pulls, m_2g-m_1g, or $g(m_2-m_1)$ constitutes the *separating* force. This slight separating force is sufficient to cause the cream particle to travel from the bottom of a vessel to the top, a distance of one foot or so in the course of a few hours.

In the case of the centrifugal separator, the force with which m_2 crowds toward the outside of the bowl is $4\pi^2n^2rm_2$ (Eq. 37) while for the cream particle it is $4\pi^2n^2rm_1$. The difference between these two forces, or $4\pi^2n^2r\ (m_2-m_1)$, is, of course, the separating force which causes the cream particle to travel *toward* the center. The ratio of this separating force to the separating force in the case of gravity separation is sometimes called the separator efficiency. Hence

$$\text{Efficiency}=\frac{4\pi^2n^2r(m_2-m_1)}{g(m_2-m_1)}=\frac{4\pi^2n^2r}{g} \tag{38}$$

In the above equation, if the gram and the centimeter are used throughout as units of mass and length respectively, the separating force will be expressed in dynes; while if pounds and feet are used, the force is expressed in *poundals*, not *pounds* (see Sec. 58). The word efficiency is used in several distinctly different ways—the more usual meaning brought out in Sec. 85, being quite different from that here given.

62. Elevation of the Outer Rail on Curves in a Railroad Track.—Let B (Fig. 34) represent a curve in the railroad track ABC. Suppose that for a short distance this curve is practically a circle of radius r_1 with center of curvature at E. Let it be required to find the "proper elevation" d of the outer rail in order that a car, when passing that particular part of the curve with a velocity v_1, shall press squarely against the track, so that its "weight," so-called, shall rest *equally* on both rails.

On a level, straight track, the thrust of the car against the track is simply the weight of the car, and is vertical; whereas on a curve, the thrust T_1 (lower sketch, Fig. 34), is the resultant of the weight of the car W and the centrifugal force F_c which the car develops in rounding the curve (Eq. 36, Sec. 58). These forces should all be considered as acting on the center of mass O of the car (Sec. 95). If the velocity of the car is such that

Mv_1^2/r (*i.e.*, F_c), has the value shown, then the total thrust T_1 will be perpendicular to the track, and consequently the thrust will be the same on both rails.

If the car were to pass the curve at a velocity twice as great as that just mentioned (or $2v_1$), the centrifugal force would be quadrupled, and would therefore be represented by the line OH. This force, combined with W, would give a resultant thrust T' directed toward the outer rail. The inner rail would then bear no weight, while the thrust T' on the outer rail would be about one-half greater than the entire weight W of the car, as the figure

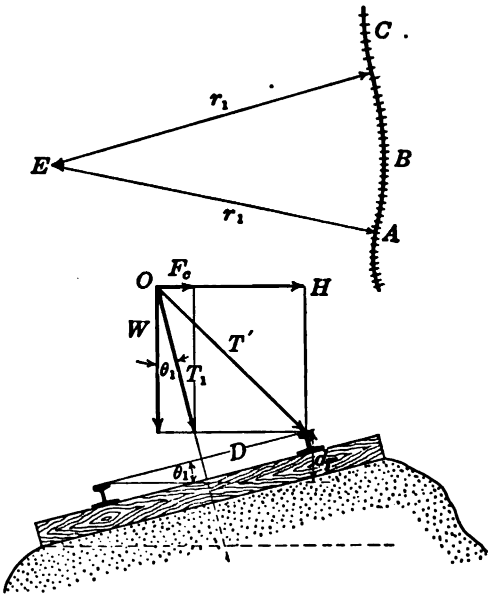

Fig. 34.

shows. The least further increase in velocity would cause the car to overturn. This *theoretical limiting velocity* could never be reached in practice, because either the wheel flanges or the rail would give way under the enormous sidewise thrust. Indeed whenever the above-mentioned velocity v_1, which may be called the "proper" velocity, is exceeded, the wheel flanges push out on the outer rail. If this sidewise push is excessive, a defective flange may give way and cause a wreck. Hence the "proper" velocity should not be much exceeded.

From the figure it may be seen that

$$\tan\theta_1=\frac{F_c}{W}=\frac{Mv_1^2/r_1}{Mg}=\frac{v_1^2}{r_1g} \tag{39}$$

From the figure we also have

$$\frac{d_1}{D}=\sin\theta_1,\ \text{or}\ d_1=D\sin\theta_1 \tag{40}$$

Observe that the two angles marked θ_1 are equal (sides perpendicular each to each). Knowing the values of v_1, g, and r_1 we may determine $\tan\theta_1$ from Eq. 39. Having found the value of $\tan\theta_1$, we may obtain θ_1 by the use of a table of tangents. If the width of the track D is also known, the proper elevation d_1 of the outer rail may be found from Eq. 40. All quantities involved in Eqs. 39 and 40 must be expressed either in F.P.S. units throughout, or else in C.G.S. units throughout. Observe that for radius r_1 we use v_1, T_1, θ_1, and d_1 respectively for the "proper" velocity, thrust, angle, and elevation.

In practice the curvature is not made uniform, but decreases gradually on both sides of the place of greatest curvature until the track becomes straight; while the elevation of the outer rail likewise gradually decreases until it becomes zero, where the straight track is reached. This construction eliminates the violent lurching of the car, which would occur if the transition from the straight track to the circular curve were sudden.

63. The Centrifugal Governor.—The essential features of the centrifugal governor, or Watt's governor, used on steam engines, are shown in the simplest form in Fig. 35. The vertical shaft S, which is driven by the steam engine, has attached to its upper end two arms c and d supporting the two metal balls A and B as shown.

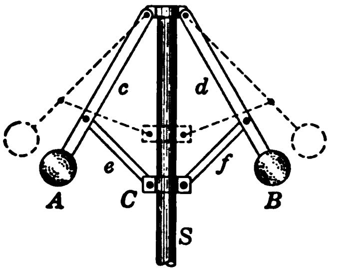

FIG. 35.

It will readily be seen that the weight of the balls tends to bring them nearer to the shaft, while the centrifugal force tends to make them move farther from the shaft. If, then, the speed of the engine becomes slightly greater than normal, the balls move farther out, c and d rise (see dotted position), and by means of rods e and f cause collar C to rise. By

means of suitable connecting levers, this upward motion of C partially closes the throttle valve. The supply of steam being reduced, the speed of the engine drops to normal. If, on the other hand, due to a sudden increase in load, the speed of the engine drops *below* normal, then the balls, arms c and d, and collar C, all lower. This lowering of C opens the throttle wider than normal, thereby supplying more steam to the engine, and restoring the normal speed.

In some engines, when the speed becomes too low, the governor automatically adjusts the inlet valve so that steam is admitted during a greater fraction of the stroke. This raises the average steam pressure on the piston, and the normal speed is regained. This subject will be further considered under "cut off point" in the chapter on the steam engine.

63a. The Gyroscope.—Just as a body in linear motion resists change in direction (Sec. 58), so a rotating flywheel resists any change in direction of rotation, *i.e.*, it resists any shifting in direction of its axis. By virtue of this principle, a rapidly rotating flywheel, properly mounted, will greatly reduce the rolling of a ship, as has been shown by tests. This principle may also yet be successfully applied in securing greater stability for aeroplanes.

The Gyroscope in its simplest form is shown in Fig. 35*a*. This device, until recent years, was merely an interesting, perplexing scientific toy. The wheel W rotates as indicated by arrow c at a high speed and with very little friction on the axle AB. If, now, the end of the axle A is rested upon the supporting point P, the end B, which is without support, does not drop in the direction d as it would if the wheel were not rotating, but moves horizontally round and round the point of support as indicated by arrow e.

The mathematical treatment of the gyroscope is very difficult; so that we shall here simply state a few facts with regard to its motion. The angular velocity ω of the wheel W is a vector, and may be represented at a given instant by the arrow ω_o, called a *rotor*. Observe that if W were a right-handed screw, a rotation in the direction indicated by arrow c would advance the screw in the direction of arrow ω_o.

Following this same *convention* we see that the torque produced by the weight of the wheel would tend to produce rotation, *i.e.*, would produce an angular acceleration about the horizontal axis indicated by the rotor α, and further that the direction of this angular acceleration would be properly represented by placing the arrow head on the end of α away from the reader. Note that rotor α lies in the axis of torque. The rotation of B in the direction of arrow e (horizontal) with a constant angular velocity ω' about a vertical axis through P, is, by this same con-

vention (right-handed screw) properly represented by the rotor ω'. From the figure, we see that ω_o lies in the *axis of spin*, and α in the *axis of torque*. The vertical axis in which lies ω' is called the *Axis of Precession*. The change in direction of the axis AB is called *Precession*.

As an aid to the memory, using the right hand, place the middle finger at right angles to the forefinger and the thumb at right angles to both. Next point the forefinger in the direction of rotor ω_o and the middle finger in the direction of α. It will then be found that the thumb points in the direction of the rotor ω' (*i.e.*, down, not up).

Cause of Precession.—Since rotors are vectors, they may be added graphically. In the figure, ω_o represents the angular velocity of W at a given instant. Its angular velocity ω_t a short time t later would be given by the equation $\omega_t = \omega_o + \omega_a$, in which ω_a is the angular velocity acquired in the short time t. But angular velocity acquired (gained) is αt (Eq. 28, Sec. 51). Therefore $\omega_t = \omega_o + \alpha t$ as shown graphically in Fig. 35a, in which the rotor αt is drawn from the arrow point of rotor ω_o. The resultant is the closing side, or the new angular velocity ω_t, which

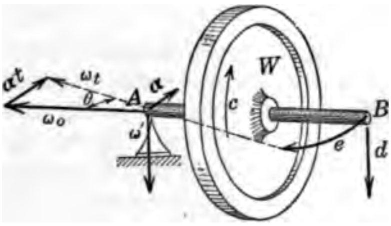

FIG. 35a.

differs from ω_o only in direction. In other words, during this short time t, the axis has changed in direction through the angle θ, and B has moved in the direction e. Clearly θ is the angle of precession in this time t, and θ/t is the precessional angular velocity ω'.

Compare this change in *direction* (not in magnitude) of rotary motion with the change in direction of linear motion (centrifugal force, Sec. 58). Observe in the vector diagram given in Fig. 35a that ω_o, αt, and ω_t correspond respectively to v_o, at, and v_t of the vector diagram shown in Fig. 31.

The *Reeling* (precession of axis of rotation) of a top when its axis is inclined, is due to this gyroscopic action. In fact if B is considerably higher than A (Fig. 35a) when A is placed upon P, the gyroscope becomes *essentially* a reeling top.

Due to the rotation of the earth (centrifugal force), the equatorial diameter is 27 miles greater than the polar diameter. Since the axis of the earth inclines to the normal to the plane of its orbit around the sun ("Plane of the Ecliptic") by an angle of 23°.5, the gravitational pull of

the sun (and also the moon) on this equatorial protuberance produces a torque about an axis perpendicular to the earth's axis of spin, just as the weight of the top (when inclined) produces a torque about an axis lying on the floor and at right angles to the spindle of the top (axis of spin).

Thus the earth reels like a great top once in about 26,000 years. The earth's axis if extended would sweep, each 26,000 years, around a circle of 23°.5 radius with a point in the sky in the direction normal to the plane of the ecliptic as center of this circle. Consequently, 13,000 years from now the earth's axis will point in a direction 47° from our present pole star, Polaris. This reeling of the earth causes the Precession of the Equinoxes around the ecliptic once in 26,000 years.

Monorail Car.—One of the most wonderful recent mechanical achievements is the successful operation of a car which runs on a track consisting of only one rail. By a clever adaptation of the gyroscopic principle of precession of two wheels having opposite rotation (the "Gyrostat"), the car is balanced, whether in motion or at rest. In rounding a curve, the "Gyrostat" causes the car to "lean in" just the right amount (Sec. 62).

If the passengers move to one side of the car, that side of the car rises, paradoxical though it may seem, and the equilibrium is maintained. As the passengers move to the side, a "table" presses on the axis of the wheel, which axis is transverse to the car, and through the friction developed by the rotation of the axis, against the table, the end of the axis is caused to creep forward (or backward) thus developing a torque about a vertical axis, and precession about an axis at right angles to both of these, namely, an axis lengthwise of the car. This precession gives rise to the torque that raises higher the heavier loaded side of the car. For an extended discussion of the gyroscope and numerous illustrations and practical applications, consult Spinney's Textbook of Physics, or Franklin and MacNutt's Mechanics and Heat.

64. Simple Harmonic Motion.—Simple harmonic motion (S.H.M.) is a very important kind of motion because it is quite closely approximated by many vibrating bodies. Thus if a mass, suspended by a spiral spring, is displaced from its equilibrium position and then released, it will vibrate up and down for some time, and its motion will be simple harmonic motion. Other examples are the vibratory motions of strings, and reeds in musical instruments, the vibratory motion of the air (called *sound*) which is produced by strings or other vibrating bodies, and the motion of the simple pendulum.

The vibrations of the string of a musical instrument consist, as a rule, of a combination of vibrations of the string as a *whole*,

and vibrations of certain *portions* or segments. Consequently the motion of a vibrating string is usually a combination of several simple harmonic motions. We shall here restrict ourselves to the study of the simpler case of uncombined S.H.M.

The piston of a steam engine executes *approximately* S.H.M.; while in the motion of the *shadow* of the crank pin cast upon a level floor by the sun when over head, we have a *perfect* example of S.H.M. Observe that the motion of the crank pin itself is not S.H.M., but uniform circular motion. An exact notion of what S.H.M. is, and a simple deduction of its important laws, are most readily obtained from the following definition, which, it will be seen, accords with the statement just made with regard to the crank pin. *S.H.M. is the projection of uniform circular motion upon a diameter of the circle described by the moving body.*

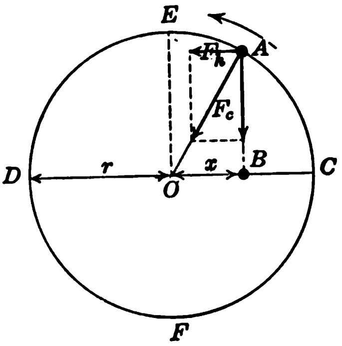

FIG. 36.

To illustrate the meaning of the above definition, let A (Fig. 36) be a body traveling with uniform speed in the circular path as shown. Let DC be any chosen diameter, say a horizontal diameter. From A drop a perpendicular on DC. The foot B of this perpendicular is the "*projection*" of A. Now as A moves farther toward D, B moves to the left at such a rate as always to keep directly *below* A. As A moves from D back through F to C, B constantly keeps directly *above* A. Under these conditions the motion of B is S.H.M.

In the position shown it will be evident that B, in order to keep under A, need not move so fast as A. When A reaches E, however, B will be at O and will then have its *maximum* speed, which will be equal to A's speed. As A and hence B approach D, the speed of B decreases to zero. In case of the vibrating mass supported by a spring (mentioned at the beginning of the section) it is evident that its velocity would be zero at both ends of its vibration and a maximum at the middle, just as we have here shown to be the case with B.

65. Acceleration and Force of Restitution in S.H.M.—If the two bodies A and B move as described in Sec. 64, it is clear that

they both have at any and every instant the same horizontal velocities. Thus, in the position shown in Fig. 36, we see that B's velocity (horizontal) must be equal to the horizontal *component* of A's velocity. An instant later, A's horizontal component of velocity will have *increased*, and since B always keeps directly below (or above) A, we see that B's velocity must have *increased* by the same amount. In other words, the *rate of change of horizontal velocity*, or the *horizontal acceleration*, is the same for both bodies.

Similar reasoning shows that as A passes from E to D, and consequently B passes from O to D, the leftward velocity *decreases* at the same rate for both bodies. As A passes from D to F and then from F to C, we see that B passes from D to O with ever increasing velocity, and then from O to C with decreasing velocity. To summarize, we may state that at every instant the *horizontal components* of A's *velocity and acceleration are equal, respectively, to the actual (also horizontal) velocity and acceleration of B at that same instant.*

We have seen that whenever B moves toward O, its velocity increases, while as it moves away from O, its velocity decreases; *i.e.*, its acceleration is always *toward* O. To impart to B such motion, obviously requires an accelerating force always pulling B *toward* O. We shall presently show that this force, called the *Force of Restitution* F_r, is directly proportional to the distance that B is from O. This distance is called the *Displacement* x.

The central force required to cause A to follow its circular path is

$$F_c = 4\pi^2 n^2 r M$$

and the horizontal component of this, or $-F_h$, has the value shown in Fig. 36. Note that if the vector x, directed to the *right*, is positive, then F_h, when directed to the left, is negative. Now F_h is the accelerating force that gives A its horizontal acceleration, while F_r is the accelerating force that gives B its horizontal acceleration; but these two horizontal accelerations have been shown to be *always equal*. Hence if A and B are of equal mass M, it follows from $F = Ma$ (Eq. 5) that $F_r = F_h$.

From similar triangles (Fig. 36) we have

$$-F_h/F_c = x/r, \text{ i.e., } -F_h \text{ or } -F_r = \frac{x}{r}F_c = 4\pi^2 n^2 M x \qquad (41)$$

Eq. 41 shows that the *force of restitution*, acting upon B at any instant, is proportional to the displacement of B at that same instant. Accordingly B's accelerating force, and hence its acceleration or rate of *change* of velocity, is a maximum when at C or D, at which points its velocity is zero, and a minimum (in fact zero) at O, at which point B has its maximum velocity. The minus sign indicates that the force of restitution is *always* oppositely directed to the displacement. Thus when B is toward the left from O, x is negative, but F_k is then positive.

If, then, a body is supported by a spring or otherwise, in such a manner that the force required to displace it varies directly as the displacement, we know at once that the body will execute S.H.M. if displaced and then released. Thus it can easily be shown, either mathematically (Sec. 67) or experimentally, that the force required to displace a pendulum bob is proportional to the displacement, provided the latter is small. Hence we know that when the bob is released it will vibrate to and fro in S.H.M.

66. Period in S.H.M.—Solving Eq. 41 for n gives

$$n=\frac{1}{2\pi}\sqrt{\frac{-F_r}{Mx}}$$

If a body makes n vibrations per second, its period of vibration, or the time P required for one complete vibration (a swing to-and-fro), is $1/n$; hence

$$P=2\pi\sqrt{-\frac{Mx}{F_r}} \tag{42}$$

Eq. 42 gives the period of vibration for any body executing S.H.M., *i.e.*, for any body for which the *force of restitution is proportional to the displacement* x, and in such a direction as to *oppose* the displacement. In this equation, M is the mass of the vibrating body in *grams*, P the period of vibration in seconds, and F_r the force of restitution in *dynes*, *when the displacement* is x centimeters. See remark on units below Eq. 36, Sec 58 and also Sec. 32. Since x and F_r always differ in sign, the expression under the radical sign is intrinsically positive.

If a heavy mass suspended by a spiral spring requires a force of 1 kilogram to pull it downward, say, 1 cm. from its equilibrium position, it will require a force of 2 kilograms to displace it (either downward or upward) 2 cm. from its equilibrium posi-

tion. This shows that the force of restitution is proportional to the displacement; hence we know that if the mass is pulled down and suddenly released, it will vibrate up and down and execute S.H.M. Here F_r or $2 \times 1000 \times 980 = 1{,}960{,}000$ dynes when $x =$ 2 cm. Suppose that the mass is 3 kilograms. We may then find its period of vibration, *without timing it*, by substituting these values in Eq. 42. Thus, neglecting the mass of the spring,

$$P = 2\pi\sqrt{\frac{3000 \times 2}{1960000}} = 0.346 \text{ sec.}$$

67. The Simple Gravity Pendulum.—The following discussion applies, to a very close degree of approximation, to the physical simple pendulum having a small bob B (Fig. 37) suspended by a light cord or wire. The length L of the pendulum is the distance from the center of the bob to the point of suspension.

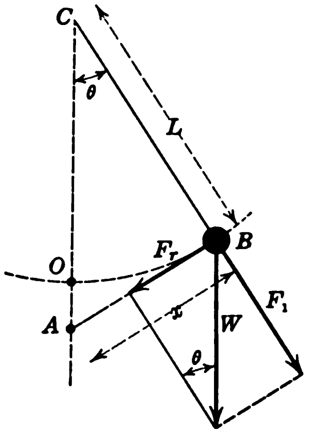

FIG. 37.

Consider the force upon the bob at some particular point in its path. Its weight, W, or Mg (Fig. 37), may be resolved into two components, F_1 in the direction of the suspending wire, and F_r in the direction of motion, *i.e*,. toward A. For small values of θ, A approaches O, so that CA may be called equal to CO, *i.e.*, equal to L, and F_r may be called the force of restitution. From similar triangles,

$$\frac{F_r}{W} = \frac{-x}{CA} \text{ or approx. } \frac{F_r}{Mg} = \frac{-x}{L} \text{ or } F_r = \frac{-Mgx}{L} \tag{43}$$

Eq. 43 shows that the force of restitution F_r is proportional to the displacement, and oppositely directed. Hence the pendulum executes S.H.M.; and we may therefore substitute the value of F_r from Eq. 43 in Eq. 42, and obtain the period P of the pendulum,

$$P = 2\pi\sqrt{\frac{-Mx}{\frac{-Mgx}{L}}} = 2\pi\sqrt{\frac{L}{g}} \tag{44}$$

The maximum value of x, *i.e.*, the distance from O to the bob when at the end of the swing, is called the *Amplitude* of vibration. Since x and M cancel out in Eq. 44, the period of a pendulum is seen to be independent of either its mass or its amplitude. The latter is true only for small amplitudes. If x is large, CA and CO are not approximately equal, as is assumed in the above derivation. A pendulum vibrates somewhat more slowly if the amplitude is large than if it is small, since CA appreciably exceeds L when θ is large, thus making F_r smaller than given by Eq. 43.

68. The Torsion Pendulum.—The torsion pendulum usually consists of a heavy disc suspended from its center by a steel wire, and hence free to rotate in a horizontal plane. When the disc is rotated from its equilibrium position through an angle θ, it is found that the resisting torque is proportional to the angle θ. In this case, the *torque of restitution*, or the returning torque, is (*a*) *proportional* to the displacement angle θ, and (*b*) *opposes* the displacement. These are the two conditions for S.H.M. of rotation. In the case of the balance wheel of a watch, the torque of restitution due to the hair spring, is proportional to the angle through which the balance wheel is rotated from its equilibrium position. Hence the balance wheel of a watch executes S.H.M., and therefore its period is independent of the amplitude of its rotary vibration.

PROBLEMS

1. If a 2-lb. mass is whirled around 240 times per minute by means of a cord 4 ft. in length (*a*) what is the pull on the cord? (*b*) What is the radial acceleration experienced by the mass?

2. A mass of 1 kilogram is whirled around 180 times per minute by means of a cord 1 meter in length. What is the pull on the cord? (*a*) in dynes? (*b*) in grams force? (*c*) What is the radial acceleration?

3. How many times as large does the central force become when the velocity (Prob. 2) is doubled? When r, the length of the cord, is doubled, the number of revolutions per second, and also the mass, remaining the same?

4. An emery wheel 12 in. in diameter makes 2400 R.P.M. Find the force (in poundals, and also in pounds), acting upon each pound mass of the rim of the wheel tending to "burst" it.

5. At a point where the radius of curvature r (Fig. 34) is 2000 ft., what is the "proper" elevation of the outer rail for a train rounding the curve at a velocity of 30 miles per hr., *i.e.*, 44 ft. per sec.? Distance between rails is 4 ft. 8 in.

6. An occupant of a ferris wheel 20 ft. from its axis observes that he

apparently has no weight when at the highest point. Find his linear velocity and radial acceleration, and also the angular velocity of the wheel.

7. Find the maximum velocity of the occupant of a 20-ft. swing if the pull he exerts upon the swing at the instant the ropes are vertical is one-half more than his weight.

8. The diameter of a cream separator bowl is 20 cm. Find its "efficiency" when making 4800 R.P.M.

9. A 4000-gm. mass, when suspended by a spring, causes the spring to elongate 2 cm. What will be the period of vibration of the mass if set vibrating vertically? Neglect the mass of the spring.

10. A sprinter passing a turn in the path, where the radius of curvature is 60 ft., at a speed of 10 yds. per sec., leans in from the vertical by an angle θ. Find $\tan \theta$.

11. What is the period of a pendulum (in Lat. 45°) which has a length of 20 cm.? 100 cm.?

12. What is the length of a pendulum (Lat. 45°) that beats seconds, *i.e.*, whose full period of vibration is 2 sec.?

13. A pendulum 30 ft. in length has a period of 6.0655 sec. at London. What is the value of g there?

14. A pendulum whose length is 10 meters makes 567.47 complete vibrations per hour at Paris. Find the value of g at Paris.

107

CHAPTER VI

WORK, ENERGY AND POWER

69. Work.—Work is defined as the production of motion against a resisting force. The work done *by* a force in moving a body is measured by the product of the force, and the distance the body moves, provided the *motion* is in the *direction* of the *force* (see Sec. 71). Hence work W may *always* be expressed by the equation

$$W = Fd \tag{45}$$

Thus the work done by a team in harrowing an acre of ground is equal to the product of the average force required to pull the harrow, and the distance the harrow moves. To harrow two acres would require twice as much work, because the distance involved would obviously be twice as great. If the applied force is not sufficient to move the body, it does no work upon the body. Thus if a man pushes upon a truck, it does not matter how hard he pushes, nor how long, nor how tired be becomes; he does no work upon the truck unless it moves in response to the push.

In case F and d are oppositely directed, *i.e.*, in case the body, due to previous motion or any other cause, moves a distance d *against* the force, then work is said to be done *by* the body *against* the force. Thus if a stone is thrown upward, it rises a certain height because of its initial velocity, and in rising it does work (Fd) *against* the force of gravity. As it falls back the force of gravity does work (Fd) upon the stone in accelerating it.

From the above discussion, we see that work may be applied in three general ways; viz., (*a*) to move a body against friction, (*b*) to move it against some force other than friction, *e.g.*, as in lifting a body, and (*c*) to accelerate a body, *i.e.*, to impart velocity to it. Observe that in all three cases the applied force does work *against some equal opposing force.* In case (*a*) it is the friction force F_f, in case (*b*) the weight W (or a component of the weight), and in case (*c*) the inertia force F_1, against which the applied force does work.

As a train starts upgrade from a station and traverses a distance d, the pull F_1 upon the drawbar of the locomotive does work in each of these three ways. Calling the average total friction force on the train F_f, the component of the weight of the train which tends to make it run down grade F_w (see Fig. 8, Sec. 19), and the average inertia force or resistance which the train offers to being accelerated F_i, we have

$$\text{Total work } F_1d = F_fd + F_wd + F_id \tag{46}$$

If, at the above distance d from the station, the drawbar of the locomotive becomes uncoupled from the train while going full speed up grade, and if the train comes to rest after going a distance d', it is clear that the *driving inertia force* F'_i of the train (Sec. 43) does work F'_id' in pushing the train up the grade against F_f and F_w, so that the work

$$F_i' d' = F_f d' + F_w d' \tag{47}$$

Observe that d and d' and also F_i and F'_i would, in general, be quite different in value, while the values of F_f and F_w would be practically the same before and after uncoupling; hence these same symbols are retained in Eq. 47.

70. Units of Work.—Since force may be expressed in dynes, grams, poundals, pounds, or tons, and distance in centimeters, inches, or feet, it follows that work, which is *force times distance*, may be expressed in dyne-centimeters or *ergs*, gram-centimeters, foot-poundals, foot-pounds, foot-tons, etc. Thus, if a locomotive maintains a 1-ton pull on the drawbar for a distance of one mile. the work done is 5280 ft.-tons, or 10,560,000 ft.-lbs. If a 20-lb, mass is raised a vertical distance of 5 ft., the work done against gravitational attraction is 100 ft.-lbs. If a force of 60 dynes moves a body 4 cm., it does 240 ergs (dyne-centimeters) of work. In scientific investigations, the erg is the unit usually employed; in engineering calculations, on the other hand, the unit is the foot-pound. The work done by an electric current is usually computed in *joules*. One joule is 10^7 ergs.

In changing from one work unit to another, it must be observed that work contains *two* factors. For example, let it be required to express the above 100 ft.-lbs. of work in terms of ergs. This may be done in two ways: (1) by reducing the 20-lb. force to dynes and the 5 ft. to centimeters, and then multiplying the two results together; or (2) by finding the number of ergs in a foot-

pound and then multiplying this number by 100. The foot-pound is larger than the erg for *two* reasons: first, 1 foot = 30.48 centimeters, and second, the pound being approximately 453 grams, and the gram force being 980.6 dynes, it follows that the pound force = 445,000 dynes. The foot-pound is therefore $30.48 \times 445{,}000$ or 13,563,000 ergs. Therefore 100 ft.-lbs. of work is 1,356,300,000 or 1.356×10^9 ergs.

71. Work Done if the Line of Motion is not in the Direction of the Applied Force.—In Sec. 69 it was shown that work = Fd provided F and d have either the same direction or opposite directions, *i.e.*, provided the angle between the applied force and the direction of motion is either zero or 180°. If this angle is zero, then work is done *by* the force; while if it is 180°, work is done *against* the force. If this angle is 90°, no work is done either by or against the force. Thus if a team is pulling a wagon westward, it is perfectly obvious that a man, walking along side the wagon and pushing north upon it, neither helps nor hinders the team.

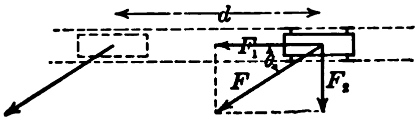

FIG. 38.

If he pushes directly forward, the above angle is zero, and in traveling a distance d while pushing with a force F he *helps* the team by an amount of work Fd; while if he pulls back the angle is 180°, and he *adds* Fd to the work the team must do.

If he pulls slightly to the south of west with a force F (Fig. 38, top view of wagon) he does an amount of work which is less than Fd. Resolving F into components F_1 and F_2, respectively parallel and perpendicular to the line of motion, we see that F_2 simply tends to overturn the wagon, while F_1 is fully effective in helping the team. The work done by F is then F_1d, but $F_1 = F \cos\theta$, hence

$$W = F_1d = Fd\cos\theta \tag{48}$$

As θ approaches 90°, $\cos\theta$, and hence the work done, approaches zero. As θ decreases, *i.e.*, as the man pulls more nearly west, $\cos\theta$ approaches its maximum value, unity (when θ = zero), and the maximum work (Fd) is obtained. Since $\cos 180° = -1$, we

see that when F is a backward pull on the wagon, then $W = -Fd$. The negative sign indicates that the work instead of being done by the man, is added work done by the team.

72. Work Done by a Torque.—If the force F (Fig. 39) pushes the crank through an arc AB, the work done is force times distance, or $W = F \times AB$. But by definition

$$\theta = \frac{\text{arc}}{r} = \frac{AB}{r}, \text{ from which } AB = r\theta;$$

hence

$$W = F \times AB = Fr\theta$$

But since torque (T) equals force times radius,

$$W = T\theta \tag{49}$$

In rotary motion, it is usually more convenient to compute work by means of Eq. 49 than by means of Eq. 45. If F is expressed in pounds and r in feet, *i.e.*, if the torque is expressed in pound-feet, and θ in radians, then $T\theta$ gives the work done in foot-pounds. Thus, for example, if F is 10 lbs., r is 2 ft., and θ is 0.6 radians, the work done is 12 ft.-lbs. If $T\theta$ is expressed in C.G.S. units (dyne, centimeter, and radian), the resulting work is given in ergs.

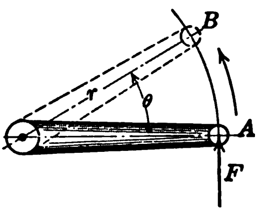

Fig. 39.

73. Energy—Potential and Kinetic.—The energy of a body may be defined as the ability of the body to do work. The *potential energy* of a body is its ability to do work by virtue of its *position* or *condition*. The *Kinetic Energy* of a body is its ability to do work by virtue of its *motion*.

The weights of a clock have potential energy equal to the work they can do in running the clock while they descend. Likewise the main spring of a clock or watch, when wound, has potential energy equal to the work it can do as it unwinds. The water in a mill pond has potential energy. Powder and coal have potential energy before ignition. A bended bow has potential energy. When the string of the bow is released and the arrow is in flight, the energy then possessed by the arrow is kinetic. Any mass in motion has kinetic energy.

The immense amount of kinetic energy possessed by a rapidly moving train is appreciated only in case of a derailment or a

collision. The kinetic energy of a cannon projectile enables it to do work in piercing heavy steel armor plate even after a flight of several miles, during all of which flight it does work against the air friction upon it. The work done upon the armor plate of the target ship is Fd; in which F is the enormous force (average value) required to push the projectile into the plate, and d is the distance to which it penetrates.

74. Transformation and Conservation of Energy.—Energy may be transformed from potential to kinetic energy and *vice versa*, or from kinetic energy into heat, or by a suitable heat engine, *e.g.*, the steam engine, from heat to kinetic energy; but whatever transformation it experiences, in a technical sense, none is lost. In practice, energy is lost, as far as useful work is concerned, in the operation of all machines, through friction of bearings, etc. This energy spent in overcoming friction is not actually lost, but is transformed into *heat* energy which cannot be profitably reconverted into mechanical energy. In all cases of energy transformation, the energy in the new form is exactly equal in magnitude to the energy in the old form. This fact, that energy can neither be created nor destroyed, is referred to as the law of the *Conservation of Energy*. This law is of great importance, as will appear from time to time. It condemns as visionary all *perpetual motion machines* purporting to furnish power without having a *source* of energy. Further, since it is impossible to entirely eliminate friction, a perpetual motion machine neither using nor furnishing power is seen to be an impossibility. The kinetic energy of the moving parts of such a machine would soon be transformed by friction into heat, and no longer exist as visible motion.

The conservation of energy is one of the well-established laws of Physics, and is frequently used as a basis in the derivation of equations, and in various lines of reasoning such as just given with regard to perpetual motion machines. From the conservation of energy, we see that to give a body a certain amount of energy, whether potential or kinetic, an exactly equivalent amount of work must be done on the body.

We may now state in slightly different form than that used in Sec. 69, the fact that the work done upon a body may be used in three ways: (*a*) to move the body against friction; (*b*) to give the body potential energy; and (*c*) to give the body kinetic energy. These three amounts of work done by the locomotive upon the

train (Sec. 69) are represented respectively by the three terms of the right-hand member of Eq. 46. Since $F_t d$ is the work done by the locomotive in accelerating the train, *i.e.*, in giving it its velocity and hence its kinetic energy, it follows, from the conservation of energy, that $F_t d$ is the kinetic energy of the train just as it reaches the point at the distance d from the station. Hence, when uncoupled at this point, this kinetic energy does an equal amount of work $F'_t d'$ in forcing the train on up the grade. Eq. 47 shows that this work is used partly $(F_f d')$ in driving the train on against friction, and partly $(F_w d')$ in giving the train more potential energy.

It should be emphasized, that in the transformation of kinetic energy into potential energy, and *vice versa, work is always done.* To illustrate, suppose that a gun of length d feet fires a projectile of weight W pounds vertically to a height h feet. Designating by F the average force (in pounds) with which the powder, upon exploding, pushes upon the projectile, and ignoring all *friction effects* (see Dissipation of Energy, Sec. 77) we have Fd foot-pounds for the work done in giving the projectile its kinetic energy, and Wh foot-pounds (force times distance) for the work done by the kinetic energy of the projectile in raising itself to the height h, in which position its potential energy (E_p) is a maximum and has the value Wh foot-pounds. This maximum potential energy (E_p max.) is the ability the projectile has to do work by virtue of its elevated position, and it does this work Wh (force times distance) while descending, in causing the velocity of the projectile to increase, thereby increasing its kinetic energy. This kinetic energy (E_k) at the instant of striking is of course a maximum (E_k max.), and, by the conservation of energy, it must be equal to the work Wh done by gravitational attraction in giving it this energy.

To summarize, we have, in accordance with the conservation of energy, the following successive energy transformations:
Fd (work done by powder) $= E$ max. (at muzzle) $=$ work Wh (done while rising) $= E_p$ max. or Wh (at highest point) $=$ work Wh (done while descending) $= E_k$ max. (at striking).

As the projectile rises, its kinetic energy decreases, while its potential energy increases; but, from the conservation of energy, we see that at any instant, $E_p + E_k = Wh = E_p$ max. Thus, when the projectile is at a height $\frac{1}{3}h$, it is evident that $E_p = \frac{1}{3}Wh$; hence, at that same instant, it must be that $E_k = \frac{2}{3}Wh$. If h

were 10,000 ft., and d, 10 ft., then F would be 1000 times the weight of the projectile (since $Fd=Wh$). Likewise, if a 1-ton pile driver falls 20 ft. (19 ft. before striking) and drives the pile 1 ft., the average force on the pile is, barring friction effects, 20 tons. The above discussion applies to the similar energy transformations that occur in the operation of a pile driver, and in the vibration of a pendulum.

75. Value of Potential and Kinetic Energy in Work Units.—From the preceding sections, we see that the potential energy, or the kinetic energy possessed by a body, is equal to the work (Fd) required to give it that energy. Accordingly, the equation expressing the potential energy, or the kinetic energy of a body is very simply obtained by properly expressing this work (Fd). In deriving the equation for potential energy, it is customary to take for this work, the work (Wh) done in raising a mass M a certain distance against gravitational force; while for the kinetic energy equation, use is made of the work done by gravitational force on a mass M in falling a certain distance. This is done for two reasons: first, because gravitational potential energy is the kind of potential energy with which we have to deal very largely in calculations, while the kinetic energy of falling bodies is of prime importance; and second, because of the fact that the gravitational force upon a body, *i.e.*, its weight, is sensibly constant regardless of change of height or velocity of the body, which fact very much simplifies the derivations.

The *Potential* energy of a mass M, when raised a height h (Fig. 40), is equal to the work done in raising it, or force times distance. Here the force is W or Mg, and the distance is h, so that

$$E_p=Mgh \tag{50}$$

Since Mg expresses the force either in dynes or poundals (Sec. 32) and h is the distance either in centimeters or feet, depending upon which system is used, the work, and hence the potential energy, is expressed in either ergs or foot-poundals. If the work is wanted in foot-pounds, the weight must be expressed in pounds and the distance in feet. The potential energy is then given by

$$E_p=Mh \tag{50a}$$

Note that a mass of M pounds weighs M pounds, not Mg pounds (Sec. 32).

The *Kinetic energy* of a moving body would naturally be expected to depend upon the mass of the body and upon the rapidity of its motion, *i.e.*, upon its velocity. Suppose that the body of mass M (Fig. 40) falls the distance h. Its kinetic energy after having fallen that height must, according to the law of conservation of energy, be equal to the work done upon it by gravity while falling, or force Mg times the distance h. Its kinetic energy is then Mgh, which, by Eq. 50, is just the potential energy that it has lost during its fall. Substituting for h its value for falling bodies given in Eq. 13, Sec. 34, namely, $h=\frac{1}{2}gt^2$, gives

$$E_k(=Mgh)=Mg\tfrac{1}{2}gt^2=\tfrac{1}{2}M(gt)^2=\tfrac{1}{2}Mv^2 \tag{51}$$

If the English system is used, since the weight or force is expressed in *poundals*, the result obtained by substituting the mass and velocity of the moving body, in Eq. 51, is expressed in *foot-poundals*, not *foot-pounds*. If M is the mass of the body in grams, then Mg is the force in dynes, and if h is expressed in centimeters, Mgh, and hence the kinetic energy $\frac{1}{2}Mv^2$, is expressed in dyne-centimeters or *ergs*.

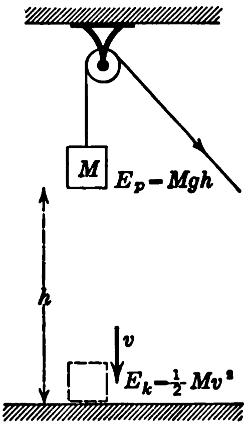

FIG. 40.

76. Energy of a Rotating Body.—Any mass particle of a rotating body, *e.g.*, a flywheel, has the kinetic energy $\frac{1}{2}mv^2$, in which m is the mass of the particle and v its velocity. Hence the kinetic energy of the entire wheel is the sum of all the quantities $\frac{1}{2}mv^2$ for each and all of its mass particles. Now the particles near the rim of the flywheel have much higher velocities and hence much greater amounts of kinetic energy than those near the axis, so that the actual summation of the kinetic energy for all particles cannot be effected without the use of higher mathematics. We readily see, however, that two wheels of equal mass M, having equal angular velocity ω, will possess different amounts of kinetic energy if the mass is mainly in the rim of one and in the hub of the other. Here, as in so many other cases, a very simple method of deriving the expression for the kinetic energy comes from the use of the law of the conservation of energy.

From this law we know that the kinetic energy E_k of the flywheel, when it has acquired the angular velocity ω, must be

equal to the work $T\theta$ (Eq. 49, Sec. 72) done by the applied torque in giving it this kinetic energy, *i.e.*, in imparting to it this angular velocity ω. Hence $E_k = T\theta$, which, by a few simple substitutions, may be brought into a form involving only the moment of inertia I of the wheel, and its angular velocity ω. From Eq. 30, Sec. 54, $T = I\alpha$,

also $\alpha = \frac{\omega}{t}$ (Sec. 51), $\theta = \bar{\omega}t$ (Eq. 27, Sec. 51), and $\bar{\omega} = \frac{\omega}{2}$

Since the wheel starts from rest with uniform acceleration, its average angular velocity $\bar{\omega}$ must be one-half its maximum angular velocity ω, as explained in Sec. 52. Making successively these substitutions, we have

$$E_k = T\theta = I\alpha\theta = \tfrac{1}{2}I\omega^2 \tag{52}$$

If we use C.G.S. units exclusively, then $T\theta$ (Eq. 52) gives the work in ergs (Sec. 72) required to *produce* the kinetic energy $\frac{1}{2}I\omega^2$, which energy must therefore *also* be expressed in ergs. I is then, of course, expressed in C.G.S. units or gm.-cm.^{2} units (Sec. 55), and ω in radians per second. If we use the F.P.S. system throughout, then $T\theta$ is expressed in foot-poundals (Sec. 72), $\frac{1}{2}I\omega^2$ in foot-poundals, ω in radians per second, and I in lb.-ft.^{2} units (Sec. 55).

Let us now apply Eq. 52 to find the kinetic energy of the 1-ton flywheel mentioned in Sec. 55, when $\omega = 20$ radians per sec., *i.e.*, when the flywheel is making slightly more than 3 revolutions per second. The moment of inertia of the wheel was found in Sec. 55 to be 50,000 lb.-ft.^{2}, whence, from Eq. 52, we have $E_k = \frac{1}{2}\ 50{,}000 \times 20^2 = 10{,}000{,}000$ foot-poundals or 310,000 ft.-lbs. Dividing this energy (310,000 ft.-lbs.) by 550 (550 ft.-lbs. per sec. is one horse power, Sec. 82) gives 562, which shows that the above flywheel, when rotating at the rate of 20 radians per second, has enough kinetic energy to furnish 1 horse power (H.P.) in driving the machinery for 562 seconds, or nearly 10 minutes, before coming to rest.

In case the angular velocity of a flywheel, connected with a gas engine, decreases from ω_1 just after an explosion stroke, to ω_2 just before the next explosion stroke, then the energy E_k which it gives up in carrying the load during the three idle strokes (Sec. 56) is

$$E_k = \tfrac{1}{2}I\omega_1^2 - \tfrac{1}{2}I\omega_2^2, \text{ or } \tfrac{1}{2}I(\omega_1^2 - \omega_2^2) \tag{53}$$

If the wheel makes 2 revolutions per sec., *i.e.*, if the piston makes 4 strokes per sec., then the 3 idle strokes will last 3/4 second; so that if the engine were a 10-H.P. engine, the work W which the flywheel would have to do in this 3/4 second would be $550\times 10\times 3/4$ or 4125 ft.-lbs. Evidently this work W equals E of Eq. 53, or

$$W=\tfrac{1}{2}I(\omega_1^2-\omega_2^2) \tag{54}$$

Eq. 54 is usually employed in computing the proper moment of inertia I for a flywheel working under certain known conditions. Thus, if we know the horse power of a certain gas engine, the average angular velocity $\bar{\omega}$ of its flywheel shaft, and the permissible speed variation $\omega_1-\omega_2$, we can compute both W (in foot-poundals) and $\omega_1^2-\omega_2^2$; then, substituting these two quantities in Eq. 54, we may solve for I. Having found the value of I in lb.-ft.^{2} units, we may, by using the equation $I=Mr^2$ (Sec. 55), choose a certain value for the radius r of the flywheel and then solve for its mass M; or we may choose a value for M and then find the proper value for r in order to make the wheel meet the above requirements.

If a small car and a hoop of equal mass are permitted to run down the same incline, it will be found that upon reaching the bottom of the incline the velocity of the hoop will be about 7/10 that of the car. Suppose that these velocities are 7 ft. per sec. for the hoop and 10 ft. per sec. for the car. The potential energy at the top of the incline was the same for both bodies, hence the kinetic energy upon reaching the bottom must be the same for both (conservation of energy). The hoop has kinetic energy of both translation and rotation, while the car, neglecting the slight rotational energy of its wheels, has only energy of translation. Consequently we have

$$\tfrac{1}{2}M(7)^2+\tfrac{1}{2}I\omega^2=\tfrac{1}{2}M(10)^2$$

in which the left member is the energy of the hoop, and the right member that of the car. Solving, we find that half of the energy of the hoop is rotational energy, that is, *experiment* shows that $\tfrac{1}{2}I\omega^2=\tfrac{1}{2}Mv^2$ for the hoop.

Mathematical Proof.—Since $I=Mr^2$ (the mass M of the hoop considered to be all in its "rim" of radius r (see below, Eq. 32, Sec. 55), and since $v=r\omega$ (Eq. 29, Sec. 52), we have

$$\tfrac{1}{2}I\omega^2=\tfrac{1}{2}Mr^2\omega^2=\tfrac{1}{2}Mv^2$$

which was to be proved.

A sphere, or a wheel with a massive hub, would travel more nearly as fast as the car, because in such case the mass would not be all concentrated in the "rim," and consequently the moment of inertia, and therefore the rotational energy, would be less than for the hoop.

77. Dissipation of Energy.—The fact that the energy of a body, whether potential or kinetic, always tends to disappear as such, is a matter of common observation, and is referred to as the principle of the *Dissipation of Energy*. Thus a body, for example a stone, in an elevated position has potential energy. If released, the stone falls, and at the instant of striking the ground its energy is kinetic. An instant later the stone lies motionless upon the ground, both its potential energy and kinetic energy having disappeared.

The results of many carefully performed experiments lead to the conviction that in the above case no energy has been lost (see conservation of energy); but that, due to air friction while falling, and friction against the ground as it strikes, the stone has slightly warmed itself, the air, and the ground; and that the amount of heat energy so developed is exactly equal to the original potential energy of the stone. This example illustrates the *general trend* of energy change throughout nature; viz., the *potential energy of a body tends to change to kinetic energy, and its kinetic energy tends to change into heat energy*. The relation between heat and other forms of energy will be further considered in the study of heat, but it might here be mentioned that 778 ft.-lbs. of work used in stirring 1 pound of water will warm it 1° F. Attention is also called to the fact that the hands may be warmed by rubbing them together, and that primitive man lighted his fires by vigorously rubbing one piece of wood against another.

A vibrating pendulum, a rotating flywheel, or a moving train soon loses its motion if no power is applied. These are good examples of the dissipation of energy. In all such cases, the potential energy or the kinetic energy of the body is transformed into *heat* through the *work done by the body against friction*.

78. Sliding Friction.—If one body is forced to slide upon another, the rubbing together of the two surfaces gives rise to a resisting force which always opposes the motion and is called *friction*. It may also be called the *force of friction*. Either surface may be that of a solid, a liquid, or a gas. Thus in drawing a sled on a cement walk, the friction is between two solids, steel and cement. In the passage of a boat through water, the friction is be-

tween a solid and a liquid, *i.e.*, between the sides and bottom of the boat and the water. In the case of the aeroplane, there is friction between the canvas planes or wings and the air through which they glide. If the wind in the higher regions of the atmosphere has either a different velocity or a different direction than the surface wind, there will be friction between them. In all cases, the *work* (*Fd*) *done against friction* is the product of the frictional force and the distance of sliding, and is transformed into heat energy (Sec. 77). Bending a piece of wire back and forth rapidly, heats it because of the *Internal Friction* between its molecules, which are thereby forced to slide past each other. Internal friction in liquids causes them to become heated when stirred, and also gives rise to *viscosity*. The greater viscosity or molecular friction of syrups makes them flow much more slowly than water.

A smooth board or iron plate appears rough under the microscope due to innumerable slight irregularities. The *cause* of friction is the fitting together or interlocking of these irregularities of one surface with those of the other over which it slides. It is easily observed that it takes a greater force to *start* the sliding of a body than to *maintain* it. The former force must overcome the backward drag of *Static Friction;* the latter, that of *Kinetic Friction.* The greater value of static friction is probably due to the better interlocking of the irregularities of the two surfaces when at rest than when in motion relatively to each other. This view is supported by the fact that when the velocity of sliding is very small the kinetic friction differs very little from the static.

The so-called "*Laws of Friction*" are: (*a*) the friction is directly proportional to the force pressing the surfaces together; (*b*) it is independent of the area of the surfaces in contact; and (*c*) it is independent of the velocity of sliding. These laws are approximately true between wide limits. Thus the force required to draw a sled will be approximately doubled by doubling the load, will be very little affected by change in the length of runner (within reasonable limits), and will remain about the same though the velocity is varied from 1 mile per hour or less, to several times that value.

To reduce the waste of power and also the wearing of machinery due to friction, lubricating oils are used. The film of oil between the two rubbing surfaces prevents their coming into such intimate contact, and thus prevents, in a large measure, the

interlocking of the above-mentioned irregularities. During the motion, the particles of oil in this film glide over each other with very little friction, and the total friction is thus reduced by substituting, in part, liquid friction for sliding friction. The resistance which a shaft bearing offers to the rotation of the shaft, is evidently sliding friction, and is therefore reduced by proper oiling.

In general, friction is greater between two surfaces of the same material than it is between those of different materials. Thus bearings for steel shafts are sometimes made of brass, and frequently of babbitt, to reduce friction. Babbitt metal is an alloy of tin with copper and antimony, as a rule. Sometimes lead is added. On the other hand, iron brake shoes are used on iron wheels to obtain a large amount of friction, and pulleys are faced with leather to prevent belt slippage.

The wasteful effects of friction are usually apparent, but the beneficial effects are probably not so generally appreciated. If it were not for friction, it would be impossible to transmit power by means of belts, or to walk upon a smooth surface. Furthermore, all machinery and all structures which are held together by nails, screws, or by bolts (unless riveted), would fall to pieces instantly if all friction were eliminated.

79. Coefficient of Friction.—The *Coefficient* of *Kinetic Friction* is defined as the *ratio* of the force required to move a body slowly and with uniform velocity *along* a plane, to the *force* that presses it *against* the plane. Thus, if a force of 30 lbs. applied in a horizontal direction is just sufficient to move a body of mass 100 lbs. slowly and with uniform velocity over a level surface, then the coefficient of kinetic friction of that particular body upon that particular surface is 30/100 or 0.3.

A very simple piece of apparatus for finding the coefficient of friction is shown in Fig. 41. B is a board, say of oak, which may be inclined at such an angle that the block C, say of walnut, will slide slowly down the plane due to its weight. Let this angle be θ. Resolving W, the weight of the block C, into two components, one component F_1 urging it *along* the *plane*, and the other F_2 pressing it *against* the plane, we have by *definition* F_1/F_2 as the coefficient of friction. F_1/F_2, however, is also $\tan \theta$, hence for this type of apparatus the

$$\text{Coeff. of friction} = F_1/F_2 = \tan \theta.$$

From the figure it is seen that h/d is also $\tan \theta$; so that if in this

particular case $h/d = 1/3$, the coefficient of friction for walnut on oak is 0.33 for the particular specimens tested.

The coefficient of friction of metal on metal is, as a rule, somewhat greater than 0.2 for smooth, dry surfaces. Oiling may reduce this to as low as 0.04.

If the coefficient of friction between the locomotive drivers and the rail is 0.2, then the maximum pull, or "tractive effort," which the locomotive can exert upon the drawbar, is about 0.2 of the weight carried by the drivers. Any attempt to exceed this, results in the familiar spinning of the drivers. For the same reason, the maximum resistance to the motion of a car that can be obtained by setting the brakes, is about 0.2 of the weight of the car. Any attempt to exceed this force results in sliding, with the production of the so-called "flat" wheel.

The *Coefficient* of *Static* Friction is defined as the *ratio* between the force required to *start* a body to slide, and the force pressing it

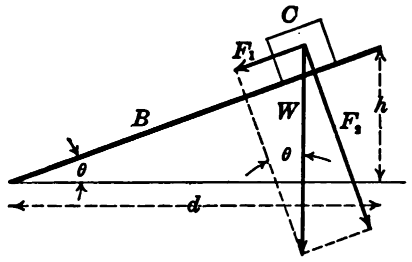

FIG. 41.

against the plane. Since it requires a greater force to *start* sliding than to *maintain* it, the coefficient of static friction is larger than the coefficient of kinetic friction for the same materials. The probable reason for this difference is the better interlocking of the surfaces in the case of static friction (Sec. 78).

80. Rolling Friction.—It is a matter of common knowledge that to draw a 1000-lb. sled, having steel runners, along a steel track would require a much greater force than to draw a 1000-lb. truck, having steel wheels, along the same track. In the former case sliding friction must be overcome; in the latter case, rolling friction. The fact that rolling friction is so much smaller than sliding friction has led to the quite common use of ball bearings in machinery. Thus the wheel of a bicycle or of an automobile supports the axle by means of a train of very hard steel balls of uniform size, which are free to roll round and round in a groove

on the inside of the hub as the wheel turns. The axle rests with a similar groove upon these balls and is thereby prevented from direct rubbing (sliding friction) against the hub. Recent American practice favors rollers instead of balls for automobile "anti-friction" bearings. By means of ball bearings, the coefficient of friction, so-called, may be reduced to about 1/2 per cent.

In drawing the above truck on the steel track, the resistance encountered is due to the fact that the steel wheel makes a slight depression in the rail, and is itself slightly flattened by the weight. Since the material in the rail is *not* perfectly elastic, the minute "hill" in front of the wheel is larger than the one behind it. The wheel is constantly crushing down a small "hill" A in front of it (shown greatly exaggerated in Fig. 42), and the energy required to do this is always greater than the energy applied by the small "hill" B that is springing up behind it.

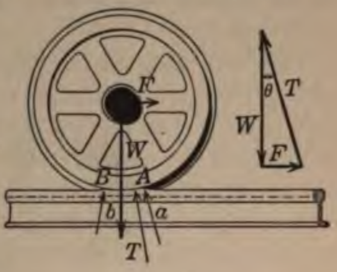

FIG. 42.

Since the thrust a, due to "hill" A, is greater than the thrust b, due to B, the general upward thrust of the rail against the wheel inclines *very* slightly backward from the vertical as shown. If the weight W, and the pull F necessary to make the wheel roll, are both known, the thrust T can easily be determined For, since the wheel is in equilibrium, the three forces W, F, and T, acting upon it must form a closed vector triangle. If, then, W and F are drawn to scale as shown, the closing side T of the triangle represents the required thrust. In the case of car wheels on a steel track, F is about 1 per cent. of W, so that the angle θ is really much smaller than shown. In the case of a rubber wheel rolling on a steel rail, the depression of the rail would be practically zero; but in this case there would be a "bump" on the wheel itself just in front of the flat portion, which would have to be crushed down as the wheel advanced. To be sure, the springing out of the rubber "bump" just behind the flat portion would help the wheel forward just as the rising of the minute hill on the rail just behind the wheel would help it forward (in case the rail is depressed). Since rubber is not perfectly elastic, the energy required to crush the one "bump" is greater than that obtained from the other formed by the rubber in springing out

again behind the wheel. The difference between these two amounts of energy is the energy used in overcoming rolling friction.

If the wheel and the rail are made of very hard steel, friction is reduced, because the depression made is less; but the danger of accidents from the breaking of brittle rails is increased. In the case of a wagon being drawn on the level along a soft spongy road, the conditions are the same as those just discussed, except that the "hill" is more marked in front of the wheel, and the rising of the hill behind the wheel is extremely sluggish indeed. For this reason, rolling friction is a vastly greater factor in wagon traffic than in railway traffic, and for the same reason, slight grades, which would be prohibitive in railway traffic, are in wagon traffic of small importance as compared with the character of the road bed.

The friction upon the axle of the car is simply sliding friction, but the amount of energy required to overcome it is very much less than if the sliding were directly upon the rail itself, by means of a shoe, for example. If the diameter of the axle is 1/10 that of the wheel, the distance of sliding between the axle and the hub is clearly 1/10 the distance traversed by the car. Hence we see that the work required to overcome this friction is only 1/10 as much as it would be if the sliding were directly upon the rail, and if oil were sufficiently cheap to maintain as good lubrication between rail and shoe as is maintained on axles.

81. Power.—Power is defined as the rate of doing work; consequently average power is the work done divided by the time required to do the work, or, proper units being chosen,

$$P = W/t \tag{55}$$

If the work done in t seconds is divided by t, the result is the work done in one second. Hence power is *numerically* the work done per unit time (usually the second). Thus if a man lifts a 50-lb. weight to a height of 6 ft. in 2 sec., he does 300 ft.-lbs. of work. Dividing this amount of work by the time required to do it gives the power or 150 ft.-lbs. per sec. Also multiplying the force, 50 lbs., by the velocity, 3 ft. per sec., gives likewise 150 ft.-lbs. per sec. For, since distance $d = vt$, we have

$$P = \frac{W}{t} = \frac{Fd}{t} = \frac{Fvt}{t} = Fv \tag{56}$$

or power is equal to the force applied multiplied by the velocity of motion of the body to which it is applied, provided the motion is in the direction of the force. Thus, multiplying the pull on the drawbar of a locomotive in *pounds*, by the velocity of the locomotive in *feet per second*, gives at any instant the power developed by the locomotive in *foot-pounds per second.*

82. Units of Power.—Since power is the rate of doing work, it must be expressed in terms of work units and time units, *e.g.*, ergs per second, foot-pounds per second, foot-pounds per minute, etc. The *horse power* (H.P.) is one of the large power units in common use.

$$1 \text{ H.P.} = 550 \text{ ft.-lbs. per sec.} = 33{,}000 \text{ ft.-lbs. per min.}$$

Since the pound force or pound weight increases with g, it follows that the horse power becomes a larger unit with increase of g. Strictly, the standard H.P. is 550 ft.-lbs. per sec. at latitude 45° ($g = 980.6$). At latitude 60°, *e.g.*, in central Sweden and Norway, g is about 1/10 per cent. greater than at latitude 45°, so that the H.P. there used is about 1/10 per cent. larger unit than the standard H.P., unless corrected. Such correction is not made in practice, because it is small in comparison with the fluctuations in power that occur during a test of an engine or motor.

If a 140-lb. man ascends a stairway at the rate of 4 ft. (vertically) per sec., the work done per second, *i.e.*, the power he develops, is 560 ft.-lbs. per sec., or slightly more than 1 H.P.

If a span of horses, pulling a loaded wagon weighing 2 tons up a hill rising 1 ft. in 10, travels at the rate of 5 ft. per sec., then, since the load rises 1/2 ft. per sec., the power developed by the two horses in working against gravity alone is,

$$4000 \times 0.5 \text{ ft.-lbs. per sec., or } 3.63 \text{ H.P.}$$

Considering also the work done against friction, it will be seen that each horse would probably have to develop more than 2 H.P. The above unit (550 ft.-lbs. per sec.) expresses the power which a horse can develop for long periods of time, *e.g.*, for a day. It is a rather high value for the average horse. On the other hand, for very short periods (1/2 min. or so), a horse may develop 6 or 8 H.P. This accounts in part for the fact that a 30-H.P. automobile, stalled in the sand, may readily be drawn by a 4-horse team. It may be mentioned in passing that the French H.P. of 75 kilogram-meters per sec. is 541 ft.-lbs. per sec.

Other units of power are the watt (one joule per sec.), and the kilowatt (1000 watts). These units are used extensively in expressing electrical power. The H.P. equals approximately 746 watts, or in round numbers, 3/4 kilowatt.

From Eq. 55 we see that work equals power times time. A span of horses working at normal rate for ten hours does 20 H.P.-hours of work. A good steam engine will do 1 H.P.-hour of work for every 1.5 lbs. of coal burned. If the lighting of a certain building requires 2 kilowatts (K.W.) then the energy used in five hours is 10 K.W.-hours. This energy is recorded by the watt-hour meter, commonly called a recording wattmeter, and costs usually about ten cents per K.W.-hour. A 32-candle-power "carbon" lamp (*i.e.*, a lamp whose filament is made of carbon) requires about 100 watts, while a "tungsten" lamp having the same candle power requires only about 40 watts. Observe in this connection that it is not *power* that is bought or sold, but *energy*, which is the product of the power and the time.

83. Prony Brake.—Various devices have been used to test the power of steam engines and motors. With some of them the test may be made while the engine is doing its regular work, while others require that the regular work cease during the test. The Prony Brake, in fact all brakes, are of the latter class, and are known as *absorption dynamometers*. The former devices are termed *transmission dynamometers*.

Since $W = T\theta$ (Eq. 49), and $\theta = \omega t$,

$$P = W/t = T\ \omega t/t = T\omega \tag{57}$$

Hence to find the power of a motor, for example, it is merely necessary to find what torque it exerts, and then multiply this by its angular velocity ω, or $2\pi n$, in which n is the number of revolutions per second as determined by a speed indicator held against the end of the motor shaft. A strap pressed against the pulley of the motor shaft would be pulled in the direction of rotation with a certain force F. If r is the radius of the pulley, then Fr gives the torque of Eq. 57. Multiplying this torque by ω, as above found, would give the power of the motor in foot-pounds per second, provided n is given in revolutions per second, F in pounds, and r in feet. Dividing this result by 550 would then give the power of the motor in H.P. If n were given in revolutions per *minute* (R.P.M.), it would be necessary to divide by 33,000 instead of by 550.

A simple form of the *Prony Brake*, suitable for testing small motors or engines, is shown in Fig. 43. The pulley A of the motor shaft is clamped between two pieces of wood, B and C, as shown. The end D of C is attached to a spring balance E. As the pulley turns, it tends to rotate the brake with it, but is prevented by the upward pull F exerted by E on D. The force, say F_1, required to make the surface of the pulley slide past the wood, times the radius r_1 of the pulley, gives the driving torque F_1r_1 tending to rotate the brake in a clockwise direction. Since the brake does not rotate, we see that the opposing torque, that is, the above pull F times its lever arm r, or Fr, must equal the torque F_1r_1. Accordingly the former torque (Fr), which is easily found, may be used in Eq. 57.

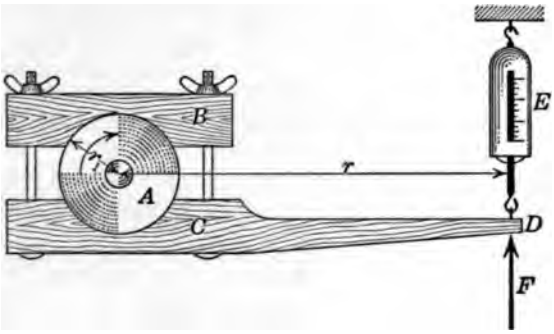

FIG. 43.

If B and C are lightly clamped together, this torque will be very small, making the power small (Eq. 57); while if clamped too tightly, the motor may be so greatly slowed down that the power is again too small. The proper way to make the test is to gradually tighten the clamp until the electrical instruments show that the motor is using its rated amount of electrical power, and then take simultaneous readings of E and the speed indicator. From these readings the H.P. of the motor is found as above outlined. Likewise in testing a steam engine, the clamp should be tightened until both the speed and the steam consumption are normal.

In testing large engines or motors with the Prony Brake, D rests on a platform scale, and pulley A, in some cases, has a rim projecting inward which enables it to hold water when revolving,

due to the centrifugal force thereby developed. Water applied in this or some other way prevents undue heating. The clamp also differs slightly from that shown.

A convenient form of brake for testing small motors is the *Strap Brake*. A leather strap attached to one spring balance is passed down around the motor pulley and then up and attached to another spring balance. Evidently when the motor is running, the two spring balances will register different forces. The difference between these two forces multiplied by the radius of the pulley, is the opposing torque. But this torque is equal to the driving torque. This driving torque, multiplied by the angular velocity ω, gives the power (Eq. 57).

PROBLEMS

1. How much work is required to pump a tank full of water from a 40-ft. well, the tank being 10 ft. long, 5 ft. wide, and 8 ft. deep, and resting upon a platform 20 ft. above the ground? The pipe enters at the bottom of the tank. Assume that half of the work is done against friction, the other half against the force of gravity. 1 cu. ft. of water weighs 62.4 lbs. Sketch first.

2. A horse drawing a sled exerts a pull of 120 lbs. upon the sled at an angle of 20° with the road bed. How much work is required to draw the sled 1/4 mile? $\text{Cos. } 20° = 0.94$.

3. A 10-lb. force applied to an 18-in. crank turns it through 4000°. How much work is done?

4. A plow that makes 12 furrow widths to the rod, *i.e.*, which makes 16.5-in. furrows, requires an average pull of 300 lbs. How much work, expressed in ft.-lbs., is done in plowing one acre?

5. What is the potential energy of a 20-kilogram mass when raised 3 ft.? Express the result in ft.-lbs. and also in ergs.

6. What is the kinetic energy of a 200-lb. projectile when its velocity is 1600 ft. per sec.?

7. If a force of 1961.2 dynes causes an 8-gm. mass to slide slowly and with uniform velocity over a level surface, what is the coefficient of friction?

8. A sled and rider, weighing 100 lbs., reaches the foot of a hill 64 ft. high with a velocity of 50 ft. per sec. How much work must have been done against friction on the hill?

9. At the foot of the hill (Prob. 8) is a level expanse of ice. Neglecting air friction, how far will the sled (vel. 50 ft. per sec.) travel on this ice before coming to rest, assuming the coefficient of friction to be 0.03?

10. How much coal would be required per acre in plowing the land (Prob. 4) with a steam plow? Assume that 6 lbs. of the coal burned can do 1 H.P.-hour of work, and that half of this work is done in pulling the engine, and the other half in pulling the plow.

11. A 200-lb. car A and a 50-lb. car B when at rest on the same level

track are connected by a stretched spring whose average tension for 3 seconds is 2 lbs. greater than that necessary to overcome the friction of running the cars. Find the momentum and the kinetic energy of each car at the close of the 3-sec. interval.

12. What is the average H.P. developed by the powder, if the projectile (Problem 6) takes 0.02 sec. to reach the muzzle, *i.e.*, if the pressure produced by the powder acts upon the projectile for 0.02 sec.?

13. What is the average force pushing the projectile (Prob. 6) if the cannon is 20 ft. in length?

14. A runaway team, pulling 200 lbs., develops 10 H.P. How fast must they travel?

15. How fast must a 400-lb. bear climb a tree in order to develop 2 H.P.?

16. What is the kinetic energy of a 3-ton flywheel when making 180 R.P.M., if the average diameter of its rim is 12 ft.? Assume the mass to be all in the rim.

17. What is the cost of fuel for a locomotive for each ton of freight that it hauls 1000 miles? Assume that the average pull per ton of the loaded train is 30 lbs., that the train itself weighs as much as its load, and that the locomotive develops 1 H.P.-hr. from each 4 lbs. of coal. The coal costs $4.00 per ton.

18. A horse, drawing a sulky and occupant at the rate of 1 mile in 2 min., exerts a 10-lb. pull upon the sulky. How much more power must the horse furnish than if it were to travel at the same rate without sulky or rider?

19. A steam engine being tested with a Prony Brake makes 300 R.P.M. and exerts at the end of the brake arm, 4 ft. from the axis, a force of 500 lbs. Find its H.P.

20. Assuming that 20 per cent. of the energy can be utilized, how many H.P. can be obtained from a 20-ft. waterfall in a river whose average width, depth, and velocity at a certain point, are respectively 50 ft., 4 ft., and 5 ft. per sec.?

21. It is desired to reduce the speed fluctuation between successive explosions of the 10-H.P. gas engine (Sec. 76) to 1 per cent. of the average speed. If the average radius of the rim of the flywheel is 3 ft., how heavy must the flywheel be? Assume the mass to be all in the rim. Also assume in Eq. 54 that ω_1 is 1/2 per cent. greater than $\bar{\omega}$, and that ω_2 is 1/2 per cent. less than $\bar{\omega}$.

CHAPTER VII

MACHINES

84. Machine Defined.—A machine is usually a device for transmitting power, though it is sometimes (*e.g.*, the dynamo) a device for transforming one kind of energy into another. Many machines are simply devices by means of which a force, applied at one point, gives rise at some other point to a second force which, in general, differs from the first force both in magnitude and direction. The force applied to the machine is called the *Working Force*, and the force against which the machine works is called the *Resisting Force.*

It is at once apparent that whatever power is required to overcome friction in the *machine itself*, is power lost in transmission. Nevertheless, transmission of power through the machine may be profitable. Thus, in shelling corn with a corn sheller, the power required to separate the kernels, to mutilate the cobs more or less, and to overcome friction of the bearings, must be furnished by the applied power; while if the corn were shelled directly by hand, only the power required to separate the kernels would have to be applied. Since power is force times velocity (Eq. 56), it is readily seen that a person's hand can apply a great deal more power to a crank than it can if pressed directly on the kernels. For both the force and the velocity may easily be much greater in case the crank is used. Again, though a block and tackle may transmit only 60 per cent. of the applied power, it is profitable to use it in lifting heavy masses that could not be lifted directly by hand. In the case of the threshing machine, the power applied by the belt from the steam engine is transmitted by the threshing machine to the cylinder, to the blower, and to numerous other parts of the machine.

We shall here study only what are known as the *Simple Machines*. The most complicated machines consist almost entirely in a grouping together of the various simple machines described in the following sections. The study of the simple machines consists mainly in learning the meaning of the efficiency

and the two mechanical advantages of each machine, and in finding their numerical values from data given. Hence the necessity for first having a clear definition of each of these three terms.

85. Mechanical Advantage and Efficiency.—The *Actual Mechanical Advantage* of a machine is the ratio of the resisting or opposing force F_o, to the force F_a applied to the machine, or

$$\text{Act. Mech. Adv.} = F_o/F_a$$

The *Theoretical Mechanical Advantage* is the ratio of the distance d through which F_a acts, to the distance D through which F_o acts, or

$$\text{Theor. Mech. Adv.} = d/D$$

The *Efficiency* (E) of a machine is the ratio of the useful work W (*i.e.*, F_oD) done *by* the machine, to the total work W_a (*i.e.*, F_ad) done *upon* the machine, or

$$E = \frac{F_oD}{F_ad} \qquad (57a)$$

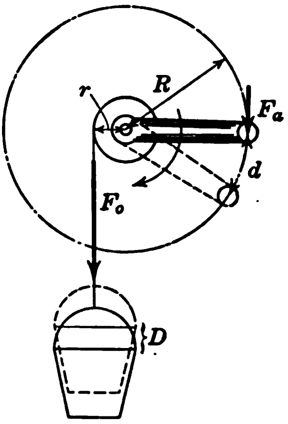

FIG. 44.

To illustrate the meaning of the above terms, consider the common windlass for drawing water from a well (Fig. 44). Let the crank, whose length (R) is 2 ft., rotate the drum of 6-in. radius (r) upon which winds the rope that pulls up the bucket of water. The hand, applying the force F_a through the distance d, does the work F_ad *upon* the machine; while the bucket, resisting with a force F_o (its weight) through a distance D, has an amount of work F_oD done upon it *by* the machine (the windlass).

From inspection we see that, since $R = 4r$, d must equal $4D$, and the theoretical mechanical advantage is therefore 4. While the theoretical mechanical advantage may be found from the *dimensions* as here done, the actual mechanical advantage must always be found from *actual experiment*. If the hand must apply a 10-lb. force to lift a 30-lb. bucket, the actual mechanical advantage is 3. If the hand applying this 10-lb. force moves 2 ft., the bucket would rise 6 inches or 1/2 ft., and the work done *upon*

the machine would be 20 ft.-lbs.; while that done *by* the machine would be 15 ft.-lbs. ($30 \times 1/2$). The efficiency (Eq. 57*a*) would then be 15/20, or 75 per cent.

Observe that the efficiency is also equal to the ratio of the two mechanical advantages, the actual to the theoretical. This is *always true*. For, since there is friction, the work done by the machine is less than that done upon it; *i.e.*, the efficiency F_oD/F_ad, or ϵ, is less than one. $F_oD/F_ad = E$ may be put in the form

$$F_o/F_a = E \times d/D \tag{58}$$

The left member of this equation is the actual mechanical advantage, while the right member is E times the theoretical mechanical advantage (note that E is never more than unity); whence the efficiency E is the ratio of the two mechanical advantages, which was to be proved. If it were possible to *entirely eliminate* friction, then the work done "*upon*" and "*by*" the machine would be equal (from the conservation of energy), and therefore E would be unity. Consequently the efficiency would be 100 per cent., and the theoertical mechanical advantage d/D would be equal to the actual mechanical advantage F_o/F_a. In other words, the theoretical mechanical advantage is the ratio that we would find for F_o/F_a *from the dimensions of the machine*, neglecting friction. This condition of zero friction is closely approximated in some machines.

86. The Simple Machines.—The *Simple Machines* are devices used, as a rule, to secure a *large* force by the application of a *smaller* force. These machines are the lever, the pulley, the wheel and axle, the inclined plane, the wedge, and the screw. Throughout the discussion of the simple machines the symbols F_a, F_o, d, and D will be employed in the same sense as in Sec. 85. It may be well to now reread the last three sentences of Sec. 84. Observe that the theoretical mechanical advantage of any simple machine, or any *combination* of simple machines for that matter, is d/D. Thus, if in the use of any combination of levers and pulleys, it is observed that the hand must move 20 ft. to raise the load 1 ft., we know at once that the theoretical mechanical advantage is 20.

87. The Lever.—The lever is a very important and much used simple machine. Indeed, as will be shown later, all simple machines may be divided into two types: the lever type and the

inclined plane type. Though the lever is usually a straight bar free to rotate about a support P, called the fulcrum or pivot point, it may take any form. Thus a bar bent at right angles and having the pivot at the angle as shown at N(Fig. 45), is a form of lever that is very widely used for changing a vertical motion or force to a horizontal one and *vice versa.*

There are three general classes of levers, sometimes called 1st class, 2nd class, and 3rd class, depending upon the relative positions of the fulcrum or pivot P, and the points A and B, at which are applied F_a and F_o respectively (see Fig. 45). In the class shown at K, P is between the other two points; in the class shown at L, F_o is between; and in the class shown at M, F_a is between. In all three cases, the applied torque about P is $F_a \times AP$, and, since the lever is in equilibrium (neglecting its

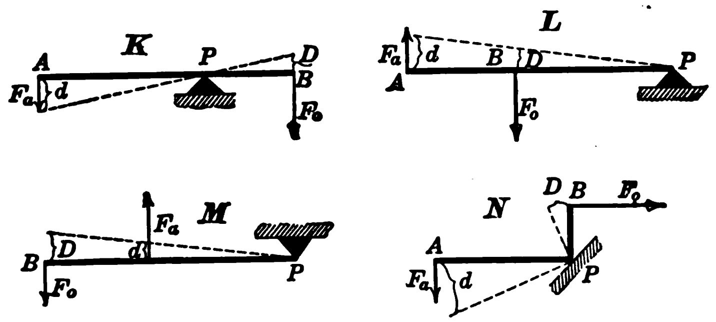

Fig. 45.

weight and also neglecting friction), this torque must equal the opposing torque due to F_o, or $F_o \times BP$. Hence $F_a \times AP = F_o \times BP$, from which, noting that for zero friction the two mechanical advantages are equal (see close of Sec. 85), we have

$$\text{Theor. Mech. Adv.} = \frac{F_o}{F_a} = \frac{AP}{BP} \tag{59}$$

The theoretical mechanical advantage may be found in another way. Let the force F_a move point A a distance d (all three classes). The point B will then move a distance D, and from similar triangles the theoretical mechanical advantage d/D is seen to be equal to AP/BP, just as in Eq. 59. By measuring AP and BP, the theoretical mechanical advantage is known. Thus if in any case AP equals $3 \times BP$, it is known at once and without testing, that,

neglecting friction, 10 lbs. applied at A will lift 30 lbs. resting at B. Friction in levers is small, so that the actual mechanical advantage is almost equal to the theoretical, and the efficiency is therefore nearly 100 per cent.

Obviously, in using a crowbar to tear down a building, the resisting force F_o is not in general a weight or load. Nevertheless, since the simple machines are very commonly used in raising weights, it has become customary to speak of F_o as the "load," or the weight lifted, and F_a as the "force," although both are of course forces. "Resistance" seems preferable to "load" and we shall call BP (for all three classes) the "resistance arm," and AP the "force arm." The latter is sometimes called the "power arm," but this seems objectionable inasmuch as we are dealing with force, not power.

From the figure, it will be seen that the force arm may be either *equal* to, *greater* than, or *less* than the resistance arm in levers of the type shown at K; while in the type shown at L, it is either *equal* to, or *greater* than the resistance arm; and in the type shown at M, it is either *equal* to, or *less* than, the resistance arm. Consequently the theoretical mechanical advantage (AP/BP) may have for the first-mentioned type (K) *any value;* for the next type (L), one or *more* than one; and for the last type (M), its value is one or *less* than one. Observe that the theoretical mechanical advantage is always given by the ratio of the force arm to the resistance arm (AP/BP), whatever the type of lever may be. The lever arm of a force is always measured from the *pivot point.*

The crowbar, in prying up a stone, may be used as a lever either as shown at K or at L. A fish pole is used as a lever of the type shown at M, if the hand holding the large end of the pole remains at rest, while the other hand moves up or down. A pump handle is usually a lever of the type shown at K. The forearm is used as a lever of type M when bending the arm, and type K when straightening it. A pair of scissors, a pair of nutcrackers, and a pair of tweezers represent, respectively, classes K, L, and M.

88. The Pulley.—The theoretical mechanical advantage of the pulley when used as shown in Fig. 46 is unity. For evidently F_a must equal F_o (neglecting friction) in order to make the two torques equal. But the theoretical mechanical advantage, if we neglect friction, is F_o/F_a (see last three sentences of Sec. 85). From an actual test in raising a load, it will be found that

F_a exceeds F_o, hence the actual mechanical advantage is less than one. Again, if F_a moves its rope downward a distance d, the weight W will rise an equal distance D, and d/D, or the theoretical mechanical advantage, from this viewpoint is also seen to be one.

Such a pulley does not move up or down, and is called a fixed pulley. Observe that this pulley may be looked upon as a lever of the class shown at K (Fig. 45) with equal arms r and r'. Although with such a pulley F_o is less than the applied force F_a, the greater ease of pulling downward instead of upward more than compensates for the loss of force.

The *movable pulley* is shown in Fig. 47. With this arrangement

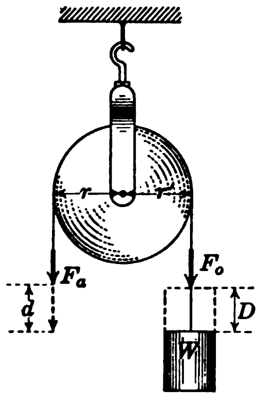

Fig. 46.

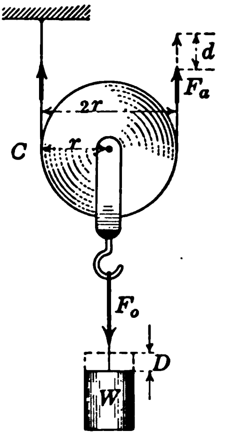

Fig. 47.

the pulley rises with the lifted weight. Since both ropes A and B must be equally tight (ignoring friction), $F_o = 2F_a$, or F_o/F_a, the theoretical mechanical advantage, is 2. This may be seen in another way by considering point C as the fulcrum for an instant, and $2r$ as the lever arm for F_a, and only r as the lever arm for F_o. It is also evident that if rope B is pulled up 1 ft. the weight W will rise only 1/2 ft., *i.e.*, d/D, the *theoretical mechanical advantage*, is 2.

A group of several fixed and movable pulleys arranged as shown in Fig. 48 with a rope passing over each pulley is called a ***Block and Tackle.*** In practice, the pulleys A and B are placed side by side on the same axle above; in like manner C and D are

placed on one axle below. The slightly different arrangement shown in the sketch is for the purpose of showing more clearly the separate parts of the rope. The rope *abcde* is continuous, one end being attached to the ring E and the other end being held by the hand.

If the applied force F_a on rope a is say 10 lbs., and the pulleys are absolutely frictionless, then the parts of the rope b, c, d, and e would all be equally tight, and hence each would exert an upward lift on W of 10 lbs., giving a total of 40 lbs. The theoretical mechanical advantage is then (neglecting friction), $F_o/F_a = 40/10 = 4$, or the number of *supporting* ropes. Again, if W is raised 1 ft. (D), each rope b, c, d, and e will have 1 ft. of slack, so that a will have to be pulled down a distance 4 ft. (d) to take up all of the slack. In other words, the hand must move 4 ft. to raise W 1 ft. Hence the *theoretical* mechanical advantage from this viewpoint is 4 (*i.e.* $d/D = 4$). Observe that here, with a theoretical mechanical advantage of 4, the weight moves 1/4 as far, and hence 1/4 as *fast* as the hand. This general fact concerning simple machines is epitomized in the following statement: "What is gained in force is lost in speed, and *vice versa.*"

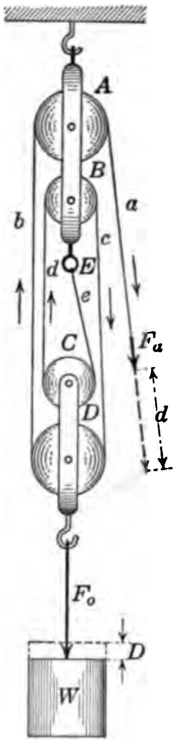

FIG. 48.

If friction causes each pulley A, B, C, and D to require 1 lb. pull to make it revolve, then if the pull applied to a were 10 lbs., the tension on b would be only 9 lbs.; on c, 8 lbs.; on d, 7 lbs.; and on e, 6 lbs. The total lift exerted on W, *i.e.*, F_o, would therefore be $9+8+7+6$, or 30 lbs.; hence the *actual* mechanical advantage F_o/F_a would be 3. Since the efficiency is the *ratio* of the *actual* to the *theoretical mechanical* advantage, it is here 3/4, or 75 per cent. The efficiency may readily be found in another way. If the hand moves downward a distance of 4 ft. while exerting a force of 10 lbs., then the work done *upon* the machine is 40 ft.-lbs., but it has been shown that, due to friction, this force can raise only 30 lbs. one ft., *i.e.*, the work done *by* the machine is only 30 ft.-lbs. The *efficiency* is then $\frac{30 \text{ ft.-lbs.}}{40 \text{ ft.-lbs.}} = 75$ per cent. as above. A considerably higher efficiency than this may

be obtained if the rope is very flexible, and if the pulley bearings are smooth and well oiled.

89. The Wheel and Axle.—The *Wheel and Axle* (Fig. 49) consists of a large wheel A of radius R rigidly attached to an axle B of radius r. A rope a is attached to the rim of the wheel and wound around it a few turns. Another rope, attached to the axle, is secured to the weight W that is to be lifted.

Viewed as a lever with the axis as pivot, the theoretical mechanical advantage is clearly the ratio of the two lever arms, or R/r. If this ratio is, say 5, the rope a will have to be pulled down a distance (d) of 5 ft. to lift the weight a distance (D) of 1 ft., giving a *theoretical* mechanical advantage (d/D) of 5. If from a test, the load lifted is only 4 times as great as the applied force, then the *actual* mechanical advantage is 4, and the efficiency (by Eq. 58) is 4/5 or 80 per cent.

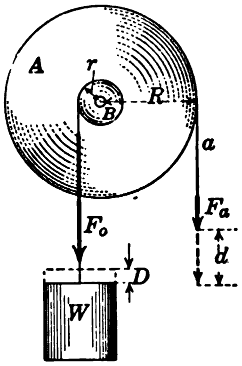

Fig. 49.

Observe that the wheel and axle and the windlass (Fig. 44) are exactly alike in principle. It may also be added that practically the only difference between the capstan and the windlass is that the drum is vertical in the capstan and horizontal in the windlass.

90. The Inclined Plane.—Let a rope, pulling with a force F_a, draw the block E of weight W up the *Inclined Plane* AC (Fig. 50). Resolving W into two components (Sec. 19), the one (F_1) normal, the other (F_2) parallel to the plane, and noting that F_a equals F_2 (if we ignore friction), we have for the theoretical mechanical advantage

$$W/F_a = W/F_2 = \frac{1}{F_2/W} = 1/\sin\theta$$

Again, if F_a draws the block from A to C, it lifts the block only the vertical height BC, and the theoretical mechanical advantage, d/D, is AC/BC, or $1/\sin\theta$, as before. Observe that

$$\text{Theo. M. Adv.} = \frac{AC}{BC} = \frac{\text{slant height}}{\text{vertical height}}$$

The less steep the grade, the greater the theoretical mechanical advantage, but the block must be drawn so much the farther in order to raise it a given vertical distance.

If the pull F_a urging the block up the incline, is horizontal, then, as the block travels from A to C (Fig. 51), F_a acts in its *own direction* through distance AB (*i.e.*, d) and the weight W is raised the distance BC (*i.e.*, D). Hence in *this case*

$$\text{Theo. M. Adv.} = \frac{AB}{BC} = \frac{\text{hor. distance}}{\text{vert. height}} = \cot\theta = 1/\tan\theta$$

The equation just given may be derived in another way. From Fig. 51 we see that the pull on the rope, or F'_a, must be of such magnitude that its component F_a parallel to AC shall equal the force F_a of Fig. 50. Drawing F_3 equal to F'_a but in the opposite direction, we have

$$\text{Theo. M. Adv.} = \frac{W}{F'_a} = \frac{W}{F_3} = \cot\theta = \frac{AB}{BC}$$

The inclined plane is frequently used for raising wagon loads and car loads of material, for example, at locomotive coaling

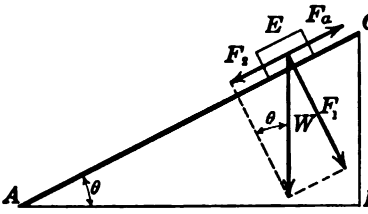

Fig. 50.

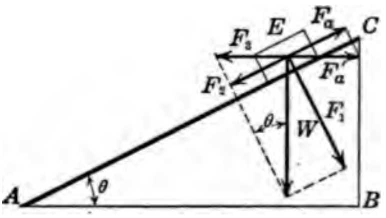

Fig. 51.

stations, and for many other purposes. A train in ascending a mountain utilizes the inclined plane, by winding this way and that to avoid too steep an incline. On a grade rising 1 ft. in 50, the locomotive must exert upon the drawbar a pull equal to 1/50 part of the weight of the train in addition to the force required to overcome friction.

91. The Wedge.—In Fig. 52 the wedge is shown as used in raising the corner of a building. F_a represents the force exerted upon the head of the wedge by the hammer, and F_o the weight of the corner of the building. If F_a acts through the distance d (the length of the wedge), *i.e.*, if F_a drives the wedge "home," then the building will be lifted a distance D (the thickness of the wedge), and F_o will resist through a distance D. Hence

$$\text{Theo. M. Adv.} = \frac{d}{D} = \frac{\text{length of wedge}}{\text{thickness of wedge}}$$

If the hand exerts a force of 20 lbs. upon a sledge hammer through a distance of 40 inches, and the hammer drives the wedge 1 inch, *i.e.*, F_a acts through 1 inch, then F_a (average value) equals 20×40 or 800 lbs. For, in accordance with the conservation of energy, the work done (force times distance) in giving the hammer its motion must be equal to the work it does upon the wedge, and, since the distance the wedge moves in stopping the hammer is 1/40 as great as the distance the hand moves in starting it, the force involved must be 40 times as great, or 800 lbs. as already found. If the wedge is 1 in. thick and 8 in. long it could, neglecting friction, lift 8×800 or 6400 lbs. In practice, friction is very great in the case of the wedge, so that the weight lifted would be very much less than 6400 lbs., say 1600 lbs. Accordingly, if the weight resting upon this particular wedge were 1600 lbs., then

Fig. 52.

each blow of the hammer would drive the wedge 1 inch and raise the building 1/8 in.

The actual mechanical advantage of the wedge would then be 1600 lbs. ÷ 800 lbs. or 2, the theoretical mechanical advantage 8 in. ÷ 1 in. or 8, and consequently the efficiency would be 2 ÷ 8 or 25 per cent. For *wedge* and *hammer* combined, the actual mechanical advantage would be 1600 lbs. ÷ 20 lbs. or 80, and the theoretical mechanical advantage, 40 in. ÷ 1/8 in., or 320. Observe that the latter ratio (320) is the distance that the hand (not the wedge) moves, divided by the distance that the building is raised. Thus we see that the great value of the mechanical advantage is due to the great force developed in *suddenly stopping the hammer* when it strikes the wedge, rather than to the wedge

itself. A wedge would be of little or no value, if used directly, that is, if pushed "home" by the hand.

If the weight on the wedge were 5 times as great (5×1600 lbs.) it would require 5 times as much force to drive it, and the hammer would be stopped more suddenly in furnishing this force. In fact, the same blow would drive the wedge 1/5 as far as before, or 1/5 inch.

92. The Screw.—The screw consists of a rod, usually of metal, having upon its surface a uniform spiral groove and ridge, the *thread*. It is a simple device by which a torque may develop a very great force in the direction of the length of the screw. For example, by using a wrench to turn the nut on a bolt which passes through two beams, the bolt draws the two beams forcibly together. The principle of the screw will be readily understood from a discussion of the jackscrew, a device much used for exerting very great forces, such as in raising buildings.

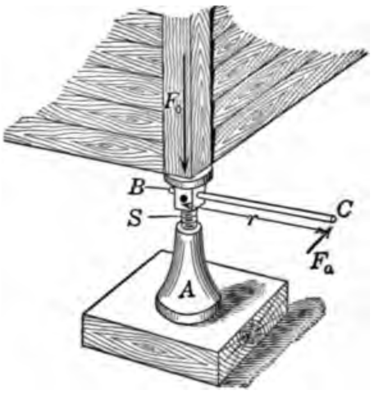

Fig. 53.

The *Jackscrew* (Fig. 53) consists of a screw S, free to turn in a threaded hole in the base A, and having at its upper end a hole through which the rod BC may be thrust as shown. Consider a force F_a applied at C at right angles to the paper and directed inward (*i.e.*, away from the reader). Let it be required to find the weight F_o that the head of the jackscrew will lift. The distance which the screws rise for each revolution is called the pitch p of the screw. Evidently for each revolution of the point C, the weight lifted, *i.e.*, the corner of the building, rises a distance p. In doing this, however, the force F_a applied to C acts through a distance $2\pi r$. Hence

$$\text{Theo. M. Adv.} = (d/D) = \frac{2\pi r}{p}$$

In the jackscrew, friction is large, consequently the *actual* mechanical advantage is much less than the *theoretical*. The actual mechanical advantage would be found by dividing the

weight of the corner of the building (*i.e.*, F_o) by the force F_a necessary to make C revolve.

Both the wedge and the jackscrew involve the principle of the inclined plane. This is obvious in the case of the wedge. In the case of the jackscrew, the thread in the base is really a spiral inclined plane up which the load virtually slides. The long rod BC makes the mechanical advantage much greater than it is for the inclined plane. Observe that all other simple machines involve the principle of the lever. Thus there are two types of simple machines, the *inclined-plane type* and the *lever type*.

93. The Chain Hoist or Differential Pulley.—The *Chain Hoist* or *Differential Pulley* (Fig. 54) is a very convenient and

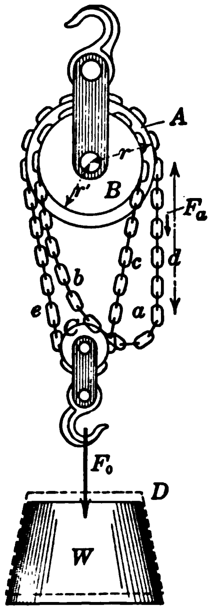

FIG. 54.

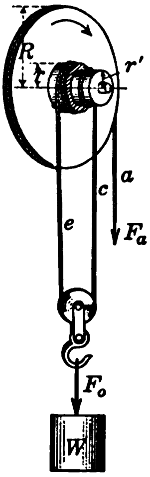

FIG. 55.

simple device for lifting heavy machinery or other heavy objects. It consists of three pulleys A, B, and C, connected by an endless chain of which the portions c and e bear the weight and a and b hang loose. The two upper pulleys A and B, which differ slightly in radius, are rigidly fastened together, and each has cogs which mesh with the links of the chain. Designating the radius of A by r and that of B by r', let us find the expression for the theoretical mechanical advantage.

Evidently if rope a is pulled down by F_a a distance $2\pi r$ (*i.e.*, d), A will make one revolution, e will be *wound* upon A a distance $2\pi r$, and c will be *unwound* from B a distance $2\pi r'$. Now the latter distance is slightly smaller than the former, so that the total length of e and c is shortened, causing pulley C, and consequently the load W, to rise the distance D. The above shortening is $2\pi r - 2\pi r'$, or $2\pi(r-r')$, and C rises only 1/2 this distance. Hence

$$\text{Theo. M. Adv.} = d/D = \frac{2\pi r}{\pi(r-r')} = \frac{2r}{(r-r')} \qquad (60)$$

Eq. 60 shows that if r and r' are made nearly equal, then D becomes very small and the mechanical advantage, very large. In practice, a ratio of 9 to 10 works very well, *i.e.*, having, for example, 18 cogs on B and 20 on A. In such case, the above-mentioned shortening would be two links per revolution (*i.e.*, per 20 links of pull), and the rise D would be one link, giving a theoretical mechanical advantage of 20/1 or 20.

In the chain hoist there is sufficient friction to hold the load even though the hand releases chain a. This is a great convenience and safeguard in handling valuable machinery. Likewise in the case of either the wedge or the jackscrew, friction is great enough to enable the machine to support the load though the applied force F_a is withdrawn. This convenience compensates for the low efficiency which, we have seen, is the direct result of a large amount of friction.

The *Differential Wheel* and *Axle* is very similar in principle to the chain hoist. It differs from the wheel and axle shown in Fig. 49, in that the axle has a larger radius at one end than at the other.

If the force F_a (Fig. 55) pulls rope a downward a distance (d) of $2\pi R$ (R being the radius of the large wheel), then, exactly as in the chain hoist, rope e is wound onto the large part of the axle a distance $2\pi r$ and rope c is unwound from the smaller part of the axle a distance $2\pi r'$. The shortening of ropes c and e is $2\pi r - 2\pi r'$ or $2\pi(r-r')$, and the weight rises a distance (D) equal to 1/2 of this distance, or $\pi(r-r')$. We thus have

$$\text{Theo. M. Adv.} = \frac{d}{D} = \frac{2\pi R}{\pi(r-r')} = \frac{2R}{r-r'} \qquad (61)$$

94. Center of Gravity.—The Center of Gravity (C.G.) of a body may be defined as that *point at which the entire weight of the body may be considered to be concentrated, so far as the torque developed by its weight is concerned.* This is equivalent to the

statement that the C.G. of a body is the point at which the body may be supported in any position without tending to rotate due to its weight. For its entire weight acts at its C.G., and hence, under these circumstances, at its point of support, and therefore develops no torque. The conditions that obtain when a body is supported at its C.G. will now be discussed.

Let Fig. 56 represent a board whose C.G. is at X. Bore a small hole at X and insert a rod as an axis. Through X pass a vertical plane at right angles to the plane of the paper as indicated by the line AX. Now the positive torque due to a mass particle m_1 is its weight m_1g times its lever arm r_1. Proceeding in the same way with m_2 and all other particles to the left of the line AX, and adding all of these minute torques, we obtain the total positive torque about X. In the same way we find the total negative torque about the same point due to m_3, m_4, etc. Since the body balances if supported at X, the total positive torque must equal the total negative torque, and for this reason, the *entire weight* behaves as a *single downward pull* W acting at its C.G. This concept greatly simplifies all discussions and problems relating to the C.G. of bodies, and will be frequently used. For example, if the rod is withdrawn from X and inserted at A, we see at once that the downward pull W, and the reacting upward pull of the supporting rod, will produce no torque, since they lie in the same straight line. If, however, the rod is inserted at B, the negative torque would be Wr, in which r is the *horizontal* distance between X and B. If free to do so, the board would rotate until B and X were in the same vertical line. In other words, a body always tends to *rotate* so that its *C.G. is directly below the point of support.*

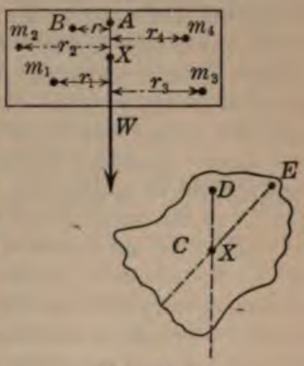

FIG. 56.

This tendency suggests a very simple means of finding the C.G. of an irregular body, such as C (Fig. 56). Supporting the body at some point as D, determine the plumb line (shown dotted). Next, supporting it at E, determine another plumb line. The intersection X of these two lines is the C.G. of the body. Why?

Effect of C.G. on Levers.—If the center of gravity of the lever

AB (sketch K, Fig. 45) of weight W, is to the left of P a distance r, then the weight of the lever produces a positive torque Wr, which torque added to that due to F_a, which is also positive, must equal that due to F_o, which is negative. Thus we see that in ignoring the weight of the lever we introduce into Eq. 59 a slight error. This error is negligibly small in the case of lifting a heavy load with a light lever. In any given case it can be seen at a glance whether this torque due to the weight of the lever helps or opposes F_a, remembering that all torques should be computed from the fulcrum P.

95. Center of Mass.—The center of mass (C.M.) of any body is ordinarily almost absolutely coincident with its C.G. Indeed the two terms are frequently used interchangeably. That the two points may differ widely under some circumstances, may be seen by considering two bodies of equal mass, one on the surface of the earth, the other 1000 miles above the surface. Since the two masses are equal, their common center of mass would be half way between the bodies, or 500 miles above the earth. Although the two *masses* are equal, the *weight* of the lower body would be roughly 3/2 times that of the upper one (inverse square law), and the center of gravity of the two, which is really the "center of weight," would be nearer the lower body. In fact, since the weight of the lower body is 3/2 times that of the upper one, its "lever arm," measured from it to the C.G., would be 2/3 as great as for the upper body. The C.G. would therefore be 400 miles above the earth, or 100 miles lower than the center of mass. As a rule, however, the C.M. of a body is practically coincident with its C.G.

Center of Population.—The center of population of a country is very closely analogous to the center of mass of a body, and is also a matter of sufficient interest to warrant a brief discussion. To simplify the discussion, let us use an illustration. Suppose that we have found that the center of population of the cities (only) of the United States is at Cincinnati. Through Cincinnati draw a north and south line A, and an east and west line B. Now multiply the population of each city east of line A by its distance from A and find the sum of these products. Call this sum S_1. Next find the similar sum, say S_2, for all cities west of A. Then $S_1 = S_2$. Proceed in exactly the same way for all cities north of line B, obtaining S_3; and finally for all cities south of B, obtaining S_4. Then $S_3 = S_4$.

It may be of interest to know that the center of population of the United States, counting all inhabitants of both city and country, was very close to Washington, D. C., in 1800. It has moved steadily westward, keeping close to the 39th parallel of latitude, until in 1900 it was in Indiana at a point almost directly south of Indianapolis and west of Cincinnati.

The mass particles of a body bear the same relation to its center of mass as does the population of the various cities to the center of population of them all. The subject is further complicated, however, by the fact that we are dealing with three dimensions in the case of a solid body, so that the distances must be measured from three intersecting planes (compare the corner of a box) instead of from two intersecting lines.

If a rod of negligible weight connecting a 4-lb. ball M and a 1-lb. ball m (Fig. 57) receives a blow F_a at a point 1/5 of its length from the larger ball, which point is the center of mass, it will be given motion of translation, but no rotation. For, since the two "lever arms" (distance from ball to C.M.) are *inversely* proportional to their respective masses, the balls, due to their inertia, produce equal (but opposing) torques about their common C.M. when experiencing equal accelerations. But if the balls experience equal acceleration, the rod does not rotate. If such a body were thrown, the two balls would revolve about their common center of mass, which point would trace a smooth curve. We may extend this idea to any body of any form. That is to say, any free body is not caused to rotate by a force directed toward (or away from) its center of mass.

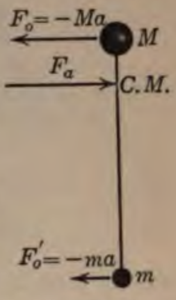

FIG. 57.

Let us again look at the problem in a slightly different way: Evidently the two torques about the point (C.M.) which receives the blow (F_a) must be equal and opposite. These torques are produced by the *inertia* forces F_o and F'_o which M and m, respectively, develop in opposing acceleration. Since F'_o acts upon 4 times as long a lever arm (measured from C.M.) as does F_o, it must be 1/4 as large as that force to produce an equal torque, and it will therefore impart to the 1-lb. mass m an acceleration exactly equal to that imparted by F_o to the 4-lb. mass M. If, however, the balls experience equal accelerations the rod will not rotate.

The mass of the earth is about 80 times that of the moon, so that the moon's "lever arm" (about their common C.M.) is 80 times as long as that of the earth, and the C.M. of the two bodies is therefore at a point 1/81 of the distance between them (about 3000 mi.), measured from the earth's center toward the moon. Since the radius of the earth is about 4000 miles, we see that the C.M. of the earth and the moon is about 1000 miles below the surface of the earth on the side toward the moon. This point travels once a year around the sun in a *smooth* elliptical path; while the earth and the moon, revolving about it (the C.M.), have very complicated *irregular* paths.

96. Stable, Unstable, and Neutral Equilibrium.—The *Equilibrium* of a body is *Stable* if a slight rotation in any direction

raises its center of gravity; *Unstable* if such rotation lowers its C.G.; and *Neutral* if it neither raises nor lowers it. The cone, placed on a level plane, beautifully illustrates these three kinds of equilibrium.

When standing upon its base, the cone represents stable equilibrium, for tipping it in any direction must raise its C.G. To overturn it with A as pivot point, its C.G. must rise a distance h as shown (Fig. 58), and the work required (in foot-pounds) would be the entire weight of the cone in pounds times h in feet, since its weight may be considered to be concentrated at its C.G. If the cone is inverted and balanced upon its apex, its equilibrium is unstable; for the least displacement in any direction would lower its center of gravity and it would fall. Finally, the cone (also the cylinder) lying on its side is in neutral equilibrium, for rolling it about on a level plane neither raises nor lowers its C.G.

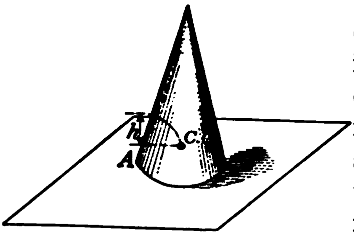

Fig. 58.

The equilibrium of a rocking chair is stable if the C.G. of the chair and occupant is below the center of curvature of the rockers. For in such case rocking either forward or backward raises the C.G. Accordingly a chair with sharply curved rockers is very apt to upset, since the center of curvature is then low. To guard against this, a short portion of the back end of the rockers is usually made straight, or better still, given a slight reverse curvature.

Equilibrium on an Inclined Plane.—To avoid circumlocution in the present discussion let us coin the phrase "Line of Centers" to indicate the plumb line through the C.G. of a body. If the plane (Fig. 58) is inclined, the cone will be in stable equilibrium so long as the line of centers falls within its base. The instant the plane is tipped sufficiently to cause the line of centers to fall without its base, the cone overturns.

A loaded wagon on a hillside is in stable equilibrium so long as the line of centers (Fig. 59) falls within the wheel base. Because of lurching caused by the uneven road bed, it is unsafe to approach very closely to this theoretical limit. A load of hay is more apt to upset on a hillside than is a load of coal, for two reasons. The C.G. is higher than in the case of the coal, and

also the yielding of the hay causes the C.G. to shift toward the lower side, as from C to D, so that the line of centers becomes DE (Fig. 59).

If the line of centers falls well within the base, a body is not easily upset, whether on an incline or on a level surface. Manufacturers recognize this fact in making broad bases for vases, lamps, portable machines, etc. Ballast is placed deep in the hold of a ship in order to lower its C.G. and thereby make it more stable in a rough sea.

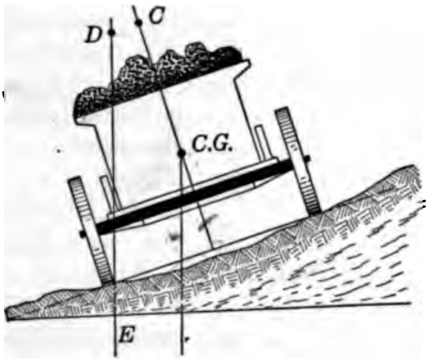

FIG. 59.

97. Weighing Machines.—The weighing of a body is the process of comparing the pull of the earth upon that body with the pull of the earth upon a standard mass, *e.g.*, the kilogram or the pound, or some fraction of these, as the gram or the ounce, etc. This comparison is not made directly with the pull of the earth upon the standard kilogram mass kept at Paris, or with the standard pound mass kept at London, but with more or less accurate copies of these, which may be called *secondary standards*. We shall here discuss briefly the beam balance, steelyard, spring balance, and platform scale. Each of these weighing devices, except the spring balance, consists essentially of one or more levers, and in the discussion of each a thorough understanding of the lever will be presupposed.

The Beam Balance consists essentially of a horizontal lever or beam, resting at its middle point on a "knife-edge" pivot of agate or steel, and supporting a scalepan at each end, also on

knife-edges. Usually a vertical pointer is rigidly attached to the beam. The lower end of the pointer, moving over a scale, serves to indicate whether the load in one scalepan is slightly greater than that in the other. The body to be weighed is placed, say, in the left pan, and enough standard masses from a set of "weights" are placed in the right pan to "balance" it. If too much weight is placed in the right pan, the right end of the beam will dip. Obviously if the balance is *sensitive*, a very slight excess weight will produce sufficient dip, and consequently sufficient motion of the pointer to be detected. The *Sensitiveness* of the balance depends upon two factors, the position of the C.G. of the beam and pointer, and the relative positions of the three knife-edges.

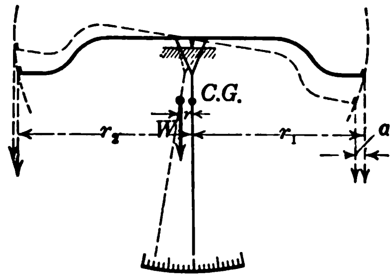

Fig. 60.

These factors will now be discussed in connection with Fig. 60, which is an exaggerated diagrammatic sketch of the beam and pointer only.

If the C.G. of the beam and pointer is far below the central knife-edge as shown, then a slight dip of the right end of the beam will cause the C.G. to move to the left a comparatively large distance r, and therefore give rise to a rather large *opposing restoring* torque equal to the weight W of the beam and pointer times its lever arm r, or a torque Wr. Observe, as stated in Sec. 94, that so far as the torque due to the weight of the beam and pointer is concerned, their entire weight may be considered to be at their C.G. From the figure, we see that if the C.G. were only 1/2 as far below the knife-edge, then r would be 1/2 as great, and 1/2 as great excess weight in the right pan would, as far as this factor is concerned, produce the same dip, and hence the same deflection of the pointer as before. Accordingly, a sensitive balance is designed so that the C.G. is a very short distance below the central knife-edge, and the smaller this distance, the more sensitive the balance.

Let us now consider the second factor in determining the sensitiveness of a beam balance. If the end knife-edges are much lower than the middle one, as in the figure, then the slight dip shortens the lever arm r_1 upon which the right pan acts by an

amount a while at the same time the length of r_2 is very slightly increased. Consequently, under these circumstances, a comparatively large restoring torque arises, and therefore a comparatively large excess weight in the right pan will be required to produce a perceptible dip of the beam or deflection of the pointer. Hence sensitive balances have the three knife-edges in a straight line, or very nearly so.

We shall now slightly digress in observing that if the three knife-edges represent in position the three holes in a two-horse "evener," and if the horse at each end of the evener be represented in the figure as pulling downward, then the "ambitious" horse would have the greater load, for, as just pointed out, the lower end has the shorter arm. If the horses are represented as pulling upward in the figure, then the horse that is ahead pulls on the *longer* lever arm and hence has the lighter load. This is

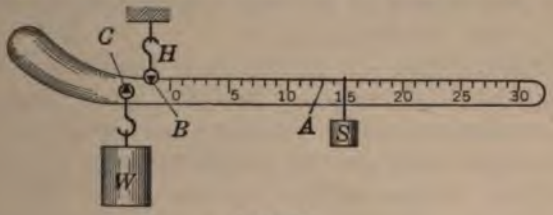

FIG. 61.

the usual condition; since the middle hole in the evener is usually slightly farther forward than the end ones.

The *Steelyard* consists of a metal bar A (Fig. 61), supported in a horizontal position on the knife-edge B near the heavy end, and provided with a sliding weight S, and a hook hanging on knife-edge C for supporting the load W to be weighed. The supporting hook H is frequently simply held in the hand. In weighing, the slider is moved farther out, thus increasing its lever arm, until it "balances" the load. The weight of the load is then read from the position of the slider on the scale.

The scale may be determined as follows: Remove W and slide S back and forth until a "balance" is secured. Mark this position of the slider as the zero of the scale. Next put in the place of W a mass of known weight, say 10 lbs., and when a balance is again secured mark the new position of the slider "10 lbs." Lay off the distance between these two positions into ten equal spaces and subdivide as desired the pound divisions thus formed. The

pound divisions should all be of the same length. For, if moving S one division to the right enables it to balance 1 lb. more at W, then moving it twice as far would double the *additional* torque due to S, and hence enable it to balance 2 lbs. more at W. The same scale may be extended to the right end of the bar.

The steelyard is made more sensitive by having its C.G. a very small distance below the supporting knife-edge B, for reasons already explained in the discussion of the beam balance. This is accomplished by having the heavy end of the bar bent slightly upward, thereby raising its C.G.

The *Spring Balance* consists essentially of a spiral steel spring, having at its lower end a hook for holding the load to be weighed. Near the lower end of the spring a small index moves past a scale, and indicates by its position the weight of the load. Since the spring obeys Hooke's Law (Sec. 107), that is, since its elongation is directly proportional to the load, a scale of equal divisions is used just as with the steelyard.

The Platform Scale.—In the platform scale two results must be accomplished; first, a small "weight" must "balance" the load of several tons; and second, the condition of balance must not depend upon what part of the platform the load is placed. The *first* result is accomplished by the use of the *Compound Lever*. A *Compound Lever* consists of a combination of two or more levers so connected that one lever is actuated by a second, the second by a third, and so on. It is easily seen that the mechanical advantage of a compound lever is equal to the *product* of the mechanical advantages of its component levers taken separately. Thus, if there are three component levers whose mechanical advantages are respectively x, y, and z, then the mechanical advantage of the compound lever formed by combining them is xyz. The *second* result, namely, the independence of the position of the load on the platform, is attained by so arranging the levers that the mechanical advantage is the same for all four corners, and therefore for *all points* of the platform.

The system of levers (only) of a common type of platform scale is shown in Fig. 62 as viewed cornerwise from an elevated position. The four levers EA, ED, FB and GC are beneath the platform (indicated by dotted lines). These levers are supported by the foundation on the knife-edges A, B, C, D, and they, in turn, support the platform on the knife-edges A', B', C', and D'. The point E is connected by means of the vertical rod EH with

the horizontal lever IJ, which lever is supported at I as indicated. Finally, J is connected by means of the vertical rod JK with the short arm KL of the horizontal lever (scalebeam) KM. A "weight," which if placed on the hanger N would "balance" 1000 lbs. on the platform, is stamped 1000 lbs. To facilitate "balancing," the "slider" S (compare the steelyard) may be slid along the suitably graduated arm LM. If the "dead load" on N balances the platform when empty, then an additional pound mass on N will balance 1000 lbs. mass resting on the platform, *provided* the mechanical advantage of the entire system of levers is 1000.

If $AA' = BB' = CC' = DD'$ and if also $FA = FB = GC = GD$, then

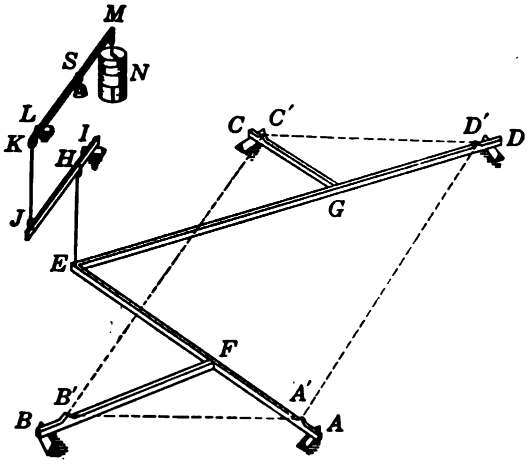

Fig. 62.

the downward force at E, and hence the reading of the scalebeam above, will not depend upon where the load is placed on the platform. This independence of the position of the load will be easily seen by assigning numerical values to the above distances. Let the first four distances each be 6 in. and the second four be each 6 ft., and let EA and ED be each 18 ft., then if E rises 1 in. (equals d), F and G will each rise 1/3 in., and A', B', C', and D' each 1/12 times 1/3, or 1/36 in. (*i.e.*, D). Consequently, the mechanical advantage d/D is $1 \div 1/36$ or 36, and has the same value for all four knife-edges A', B', C' and D', showing that the recorded weight is *independent of the position of the load on the platform*, which was to be proved.

If $JI = 3HI$, then, since the mechanical advantage obtained by lifting at E is 36, the mechanical advantage at J will be 3 times 36 or 108. Finally, if $LM = 10LK$, then a downward pull at N has a mechanical advantage of 10 times 108, or 1080. In other words, 1 lb. at N will balance 1080 lbs. placed *anywhere* on the platform.

In small platform scales, E connects directly to the scalebeam above. In practice, the knife-edges are supported (or support the load, as the case may be) by means of links, which permit them to yield in response to sudden sidewise jarring, and thus preserves their sharpness and hence the accuracy of the scale.

PROBLEMS

1. It is found that with a certain machine the applied force moves 20 ft. to raise the weight 6 in. What weight will 100 lbs. applied force lift, assuming friction to be zero? If the efficiency is 60 per cent. what will the 100 lbs. lift? What is the *theoretical* mechanical advantage of the machine? What is its actual mechanical advantage?

2. If the distance AB (sketch L, Fig. 45) is 36 in., BP is 6 in., and F_a is 100 lbs., what is F_o? That is, what weight can be lifted at B?

3. A 6-ft. lever is used: (*a*) as shown in sketch K (Fig. 45), and again (*b*) as shown in sketch L, PB being 1 ft. in each case. Find the applied force necessary to lift 1000 lbs. at B for each case. Explain why the answers differ.

4. What is the theor. m. adv. of the block and tackle (Fig. 48)? What would it be if inverted, in which position pulley A would be below and rope a would be pulled upward?

5. Sketch a block and tackle giving a theor. m. adv. of 3, of 6, and of 7.

6. What applied force would raise 1000 lbs. by using a wheel and axle, if the diameter of the wheel were 4 ft., and that of the axle 6 in., (*a*) neglecting friction, (*b*) assuming 90 per cent. efficiency?

7. A hammer drives a wedge, which is 2 in. thick and 1 ft. in length, a distance of 1/2 in. each stroke. The wedge supports a weight of 1 ton and the hand exerts upon the hammer an average force of 20 lbs. through a distance of 3 ft. each stroke. What is the theor. m. adv. of the wedge? Of both wedge and hammer?

8. Find the theoretical and also the actual mechanical advantage of a jackscrew of 30 per cent. efficiency, whose screw has 10 threads to the inch and is turned by a rod giving a 2-ft. lever arm.

9. Neglecting friction, what pull will take a 200-ton train up a 1 per cent. grade (*i.e.*, 1 ft. rise in 100 ft.)?

10. What is the value of the actual m. adv., and also what is the efficiency of the combination mentioned in problem 7?

11. If the jackscrew (Prob. 8) is placed under the lever at A (sketch L, Fig. 45), what lift can be exerted at B (of the lever) by applying a 50-lb. pull at the end of the jackscrew lever? Let lever arm BP be 2 ft. and BA, 3 ft.

12. What H.P. does the locomotive (Prob. 9) develop in pulling the train, if its velocity is 40 ft. per sec., and if the work done against friction equals that done against gravity?

13. In a certain chain hoist the two upper pulleys, which are rigidly fastened together, have respectively 22 and 24 cogs. (*a*) What is its theoretical mechanical advantage? (*b*) What load could a 150-lb. man lift with it, assuming an efficiency of 30 per cent.?

14. Let two levers, which we shall designate as G and H, be represented respectively by the sketches K and L (Fig. 45), except that G is shifted to the right so that B of G comes under A of H, thus forming a compound lever. Neglecting friction (*a*) what downward force at A of lever G will lift 1 ton at B of lever H, provided AB and PB are respectively 6 ft. and 1 ft. for both levers? (*b*) What is the theoretical mechanical advantage of G, and of H, and also of both combined? Sketch first.

15. A barrel is rolled up an incline 20 ft. in length and 6 ft. in vertical height by means of a rope which is fastened at the top of the incline, then passes over the barrel, and returns from the upper side of the barrel in a direction parallel to the incline. What theoretical mechanical advantage is obtained by a man who pulls on the return rope?

16. A man, standing in a bucket, pulls himself out of a well by means of a rope attached to the bucket and then passing over a pulley above and returning to his hand. What theoretical mechanical advantage does he have?

17. The drum of an ordinary capstan for house moving is 16 in. in diameter, and the sweep, to which is hitched a horse pulling 200 lbs., is 12 ft. long. Find the pull on the cable, assuming no friction in the drum bearings.

18. If in the lever BP, sketch M, Fig. 45, $AB = AP$, what weight can be lifted at B if the block and tackle shown in Fig. 48 lifts on A of the lever, and if the pull on rope a of the block and tackle is 100 lbs.? Neglect friction.

19. The weight of a 24-ft. timber is to be borne equally by three men who are carrying it. One man is at one end of the timber while the other two lift by means of a crossbar thrust under the timber. How far from the end should the crossbar be placed?

20. If the lever AB (sketch K, Fig. 45) be a plank 20 ft. long and weighing 100 lbs., and if PB be 2 ft., what downward force at A will lift 1000 lbs. at B, (*a*) if we consider the weight of the plank? (*b*) If we neglect it?

21. A 20-ft. plank which weighs 120 lbs. lies across a box 4 ft. in width, with one end A projecting 7 ft. beyond the box. How near to the end A of the plank can a 60-lb. boy approach without upsetting the plank? How near to the other end may be approach?

22. How far from the end of the timber should the crossbar be placed (Prob. 19) if there are two men lifting on each end of it; one man lifting on the end of the timber as before?

23. In Fig. 62, let BB' (etc.) equal 4 in., BF (etc.) equal 5 ft., AE (and DE) equal 15 ft., $HI = 5$ in., $JI = 20$ in., $LK = 1.5$ in., and $LM = 30$ in. What weight at N will balance 2 tons on the platform of the scale?

PART II

PROPERTIES OF MATTER

CHAPTER VIII

THE THREE STATES OF MATTER AND THE GENERAL PROPERTIES OF MATTER

98. The Three States of Matter.—Matter exists in three different states or forms: either as a *solid*, as a *liquid*, or as a *gas*. Liquids and gases have many properties in common and are sometimes classed together as *fluids*.

We are familiar with the general characteristics which distinguish one form of matter from another. *Solids resist change of size or shape;* that is, they resist compression or extension, and distortion (change of shape). Solids therefore have rigidity, a property which is not possessed by fluids. *Liquids* resist compression, but do not appreciably resist distortion or extension. For these reasons a quantity of liquid assumes the form of the containing vessel. *Gases* are easily compressed, offer no resistance to distortion, and tend to expand indefinitely. Thus a trace of gas introduced into a vacuous space, for example, the exhausted receiver of an air pump, will immediately expand and fill the entire space. Most substances change from the solid to the liquid state when sufficiently heated; thus ice changes to water, and iron and other metals melt when heated. If still further heated, most substances change from the liquid state to the gaseous state; thus, when sufficiently heated, water changes to steam, and molten metals vaporize. Indeed, practically all substances may exist either in the solid, the liquid, or the gaseous state, depending upon the *temperature* and in some cases upon the pressure to which the substance is subjected.

We commonly speak of a substance as being a *solid*, a *liquid*, or a *gas*, depending upon its state at *ordinary temperatures*. Thus metals (except mercury), minerals, wood, etc., are *solids;* mercury, water and kerosene are *liquids;* and air and hydrogen are *gases*. Mercury may be readily either vaporized or frozen, and air can be changed to a liquid, and this *liquid air has been frozen to a solid*. Some substances, *e.g.*, those which are paste-like or jelly-like, are on the borderline and may be called semifluids, or

semisolids. It is interesting to note that mercury and bromine are the only *elements* which are liquid at ordinary temperature.

99. Structure of Matter.—All matter, whatever its form, is supposed to be composed of minute particles called *molecules*. Thus iron (Fe) is composed of iron molecules, chlorine (Cl) of chlorine molecules, and iron chloride ($FeCl_2$) of iron chloride molecules. These molecules are composed of atoms—like atoms in the case of an element, for example, iron, and unlike atoms in the case of compounds. Thus, the iron chloride molecule ($FeCl_2$) consists of one atom of iron (Fe) and two atoms of chlorine (Cl).

Molecular Freedom.—In the case of a *solid*, the molecules that compose it do not easily move with respect to each other. This gives the solid *rigidity* which causes it to resist any force tending to make it change its shape. In *liquids*, the molecules glide readily over each other, so that a liquid immediately assumes the shape of the containing vessel. In *gases*, the molecules have even greater freedom than in liquids, and they also tend to separate so as to permeate the entire available space as mentioned in the preceding section.

Divisibility of Matter.—Any portion of any substance may be divided and subdivided almost without limit by mechanical means, but so long as the *molecule remains intact*, the substance is unchanged chemically. Thus common salt (NaCl), which is a compound of the metal sodium (Na) and chlorine, may be ground finer and finer until it is in the form of a very fine dust, and still preserve the salty taste. This powdered salt may be used for curing meats, and *chemically* it behaves in every way like the unpowdered salt. If, however, through some chemical change the molecule is broken up into its separate atoms, namely, sodium and chlorine, it no longer exists as *salt*, nor has it the characteristics of salt. Hence we may say that the molecule is the smallest portion of a substance which can exist and *retain* its original *chemical characteristics*. Certain phenomena indicate that the molecule is very small—probably a small fraction of one-millionth of an inch in diameter.

The Kinetic Theory of Matter.—According to this theory, which is generally accepted, the molecules of any substance, whether in the solid, the liquid, or the gaseous state, are in continual to-and-fro vibration. In solids, the molecule must remain in one place and vibrate; in liquids and gases it may wander about while

maintaining its vibration. This *vibratory* motion of translation is supposed to give rise to the diffusion of liquids and gases (Secs. 112 and 131).

Form certain experimental facts, a discussion of which is beyond the scope of this work, the average distance through which a hydrogen molecule vibrates, or its "*mean free path*," is estimated to be about 7/1,000,000 inch, if the hydrogen is under ordinary atmospheric pressure, and at the temperature of melting ice. This distance is smaller for the molecules of other gases, and presumably very much smaller in the case of liquids and solids. As a body is heated, these vibrations become more violent. This subject will be further discussed under "The Nature of Heat" (Sec. 160), and the "Kinetic Theory of Gases" (Sec. 171).

Brownian Motion.—About 80 years ago, Robert Brown discovered that small (microscopic) particles of either organic or inorganic matter, held in suspension in a liquid, exhibited slight but rapid to-and-fro movements. In accordance with the kinetic theory of matter, these movements may be attributed to molecular bombardment of the particles.

100. Conservation of Matter.—In spite of prolonged research to prove the contrary, it still seems to be an established fact that matter can be neither created nor destroyed. If several chemicals are recombined to form a new compound, it will be found that the weight, and therefore the mass, of the compound so formed, is the same as before combination. When a substance is burned, the combined mass of the substance and the oxygen used in combustion is exactly equal to the combined mass of ash and the gaseous products of combustion. When water freezes, its density changes, but its mass does not change. Matter then, like energy, may be transformed but neither destroyed nor created.

101. General Properties of Matter.—There are certain properties, common to all three forms of matter, which are termed *General Properties.* Important among these are *mass, volume, density, gravitational attraction, intermolecular attraction, and elasticity.*

As a rule, any portion of matter has a definite mass and a definite volume. Dividing the mass of a body by its volume gives its *Density, i.e.,*

$$d=\frac{M}{V}$$

In the case of a solid of regular form, its volume may be determined from measurement of its dimensions. Its mass, whatever its shape, would be obtained by weighing. (For the method of obtaining the density of an irregular solid see Sec. 122.) Below are given the densities of several substances in the C.G.S. system, *i.e.*, in grams per cubic centimeter. The density of water is practically 1 gm. per cm.^{3}, or, in the British system, 62.4 lbs. per cu. ft. Densities are usually expressed in the C.G.S. system.

AVERAGE DENSITIES OF A FEW SUBSTANCES

Solids (gm. per cm.^{3})		Liquids (gm. per cm.^{3})		Gases (gm. per cm.^{3})	
Gold	19.30	*Mercury*	13.60	Chlorine	0.0032
Lead	11.36	Bromine	3.15	Carbon dioxide	0.002
Silver	10.53	Glycerine	1.26	Oxygen	0.0014
Iron	7.80	Milk (whole)	1.028 to 1.035	*Air*	0.0013
Marble	2.75			Nitrogen	0.00125
Aluminum	2.60	*Sea-water*	1.025	Marsh gas	0.0007
Ice	0.917	Water, 4° C.	1.00	Steam, 100° C.	0.0006
Cork	0.25	Cream, about	1.00	Hydrogen	0.00009
		Alcohol	0.80		

In general, metals are very dense, as the table shows. Liquids are less dense, and gases have very small densities. Ice floats in water, from which it appears that the density of water decreases when it changes to the solid state. Paraffine, on the contrary, becomes more dense when it solidifies. The densities of different specimens of the same substance usually differ slightly. The approximate values of those in *italics* should be memorized. With the exception of steam, the densities given for the gases refer in each case to the density of the gas when at 0° C. and under *standard atmospheric* pressure (Sec. 136).

Solids, liquids, and gases all have weight, which shows that *gravitational attraction* acts between them and the earth. The other two general properties, *intermolecular attraction* and *elasticity*, will be discussed in the following sections.

102. Intermolecular Attraction and the Phenomena to Which it gives Rise.—It requires a very great force to pull a metal bar in two, because of the *Intermolecular Attraction* of its molecules. If, however, the ends of the bars are now carefully squared and then firmly pressed together, it will be found upon removing the pressure that a very slight force will separate them. This experiment shows that this molecular force, which is called *Cohesion*,

and which gives a metal or any other substance its tensile strength, acts through very small distances. Two freshly cleaned surfaces of lead cohere rather strongly after being pressed firmly together. The fact that lead is a soft metal, permits the two surfaces to be forced into more intimate contact, so that the molecular forces come into play.

By gently hammering gold foil into a tooth cavity, the dentist produces a solid gold filling. Gold is not only a fairly soft metal but it also does not readily tarnish. Because of these two properties, the molecules of the successive layers of foil are very readily brought into intimate contact, and therefore unite.

Welding.—In welding together two pieces of iron, both pieces are heated to make them soft, and they are then hammered together to make them unite. The "flux" used prevents oxidation in part, and also floats away from between the two surfaces whatever scale or oxide does form, thus insuring intimate contact between them.

It is cohesion that enables the molecules of a liquid to cling together and form drops. This will be further considered under "Surface Tension" (Sec. 124). If a clean glass rod is dipped into water and then withdrawn, a drop of water adheres to it. Obviously the weight of the drop of water is sustained by the molecular attraction between the glass molecules and the water molecules. This force is called *Adhesion*, whereas the force which holds the drop together is *Cohesion* as already stated. That is, *the force of cohesion is exerted between like molecules, adhesion between unlike molecules.*

Two pieces of wood may be held firmly together by means of glue. One surface of the thin layer of glue adheres to one piece of wood, and the opposite surface adheres to the other. When the two pieces of wood are torn apart, the line of fracture will occur at the weakest place. If the fracture occurs between glue and wood in such a way that no glue adheres to the wood, then the adhesion between glue and wood is weaker than the cohesion of either substance. If the layer of glue is torn apart so that a portion of it adheres to each piece of wood, then cohesion for glue is weaker than adhesion between glue and wood. Finally, if portions of the wood are torn out because of adhering to the glue, which often happens, it shows that the adhesion between glue and wood is stronger than the cohesion of wood (at that point).

As a rule, cohesion is stronger than adhesion. The adhesion between the layer of gelatine and the glass of a photographic plate furnishes a striking exception to this rule. Sometimes, in becoming very dry, this gelatine film shrinks with sufficient force to tear itself loose from the glass at some points, while at other points bits of the glass are torn out, leaving the glass noticeably rough to the touch. A thin layer of fish glue spread upon a carefully cleaned glass plate produces, as it dries, a similar and even more marked effect.

103. Elasticity, General Discussion.—When a force is applied to a solid body it always produces *some change* either in its *length*, its *volume*, or its *shape*. The tendency to resume the original condition upon removal of the applied force is called *Elasticity*. When a metal bar is slightly stretched by a force, it resumes its original length upon removal of the force, by virtue of its *Tensile Elasticity*. If the bar is twisted, its recovery upon removal of the applied torque is due to its *Elasticity* of *Torsion*, *Rigidity*, or *Shearing*, as it is variously termed. If the bar is subjected to enormous hydrostatic pressure on all sides, its volume decreases slightly. Upon removal of the pressure, the tendency to immediately resume its original volume is due to the *Volume Elasticity* of the metal of which the bar is made.

If, upon removal of the distorting force, the body regains immediately and completely its original shape or size, it is said to be *perfectly elastic*. Liquids and gases are perfectly elastic, but no solids are. Ivory, glass, and steel are more nearly perfectly elastic than any other common solid substances. Such substances as putty have practically no tendency to recover from a distortion and are therefore called *inelastic*. They are also called *plastic*, which distinguishes them from brittle inelastic substances such as chalk.

Through wide ranges, most elastic substances are distorted in proportion to the applied or distorting force, *e.g.*, doubling the force produces twice as great stretch, twist, or shrinkage in volume, as the case may be. Such substances are said to obey Hooke's law (Sec. 107).

Any change in the shape of a body must entail a change in the relative positions of its molecules, hence elasticity of shape or rigidity may be considered to be due *primarily* to the tendency of the molecules to resume their former relative positions. The resistance which the molecules offer to being crowded more closely

together, or rather their tendency to again spring apart, gives rise to volume elasticity.

Elasticity is one of the most important properties of substances, and for this reason it has been very much studied. The subject will be taken up more in detail in subsequent chapters, especially under "Properties of Solids." For a more complete study the reader is referred to advanced works on Physics or Mechanics, some of which are mentioned in the preface.

PROBLEMS

1. By the use of the table, find the densities of air, sea-water, mercury, and gold in the British system.
2. A rectangular block of wood 4 in. $\times$2 in. $\times$1/2 in. weighs 44 gm. Find its density.
3. Find the weight of 1/2 mi. of 1/8-in. iron wire.
4. A cylindrical metal bar 1 cm. in diameter and 20 cm. in length weighs 165.3 gm. Of what metal is it composed? What is its density?
5. Find the mass of a cubic yard of each of the following substances: hydrogen, air, water, ice. A cubic foot of water weighs 62.4 lbs.
6. How many cubic feet of ice will 50 gal. of water form upon freezing? Water weighs very closely 62.4 lbs. per cu. ft., or 8.33 lbs. per gal.
7. A hollow iron sphere 10 cm. in diameter weighs 3 kilos. What is the volume of the cavity within it?
8. A piece of brass has a density of 8.4 gm. per cm.^{3} Assuming that the volume of the brass is exactly equal to the sum of the volumes of copper and zinc that compose it, what percentage of the brass, by volume, is zinc? Density of copper is 8.92, zinc 7.2 gm. per cm.^{3} Suggestion: Represent by x the fractional part that is zinc.
9. From the answer to problem 7, find what percentage of the brass by *weight* is zinc.

CHAPTER IX

PROPERTIES OF SOLIDS

104. Properties Enumerated and Defined.—The following properties are obviously peculiar to solids: ***hardness, brittleness, malleability, ductility, tenacity*** or tensile strength, and ***shearing elasticity.***

Hardness and brittleness often go hand in hand. Thus steel when tempered "glass hard" is brittle. Glass is both hard and brittle. Chalk, however, is brittle but not hard. *Brittleness* may be defined as the property of yielding very little before breaking. Thus glass or chalk cannot be bent, twisted, or elongated appreciably before breaking, and are therefore brittle.

If a substance may be made to scratch another, but cannot be scratched by it, then the former substance is *Harder* than the latter. Ten substances, with diamond at the head of the list, sapphire next as 9, and talc at the bottom of the list as 1, have been used as a "scale of hardness." If a certain substance may be scratched by diamond as readily as it can be made to scratch sapphire, then the substance is 9.5 in the scale of hardness.

Malleability is that property of a solid by virtue of which it may be hammered into thin sheets. Gold is very malleable, indeed it is the most malleable known substance. By placing a thin sheet of gold between two sheets of "gold beater's skin" it may be hammered into foil about 1/200000 inch thick. Lead is malleable. Iron becomes quite malleable when heated to a white heat. Wrought iron is slightly malleable at ordinary temperatures.

Ductility is that property of a metal which enables it to be drawn out into the form of a fine wire. Brass, copper, iron and platinum are very ductile. Although lead is malleable, it is not strong enough to be ductile.

The *Tenacity* or tensile strength of a metal or other substance, depends, as stated in Sec. 102, upon the cohesive force between its molecules. Iron has a large tensile strength—from 40,000 to 60,000 lbs. per sq. in. Copper and lead have relatively low tensile strengths.

105. Elasticity, Elastic Limit, and Elastic Fatigue of Solids.—If several balls, made of different metals, are successively dropped upon an anvil from a height of a few inches, it will be found that the first rebound carries the steel ball nearly to the height from which it was dropped. The brass ball rebounds less, and the iron one still less than the brass one. The lead ball does not rebound, but merely flattens slightly where it strikes the anvil. Ivory rebounds better than steel. The *sudden* stopping of the ball by the anvil requires a *large* force ($F=Ma$), which flattens the ball in each case. If the material is elastic, however, the flattened portion springs out again into the spherical form as soon as the motion of the ball is stopped, and in so doing throws the ball into the air. If the ball and anvil were both perfectly elastic the first rebound would bring the ball back to the point from which it was dropped. This is a very simple, rough test of elasticity. If the ball were *perfectly elastic,* the average force required to flatten it would be exactly equal to the average force with which it would tend to restore its spherical shape. Obviously, these two forces would each act through the same distance, hence, the work of *flattening* and the work of *restoring* would be equal. But the former work is equal to—is in fact due to—the potential energy of the ball in its original position, and the latter work is used in throwing the ball back to the height of the first rebound. Accordingly, this height should be equal to the distance of fall. Because of molecular friction, the above restoring force is smaller than the flattening force even in the case of the ivory ball, which accounts for its failure to rebound to the original height.

If a straight spring is moderately bent for a short time and is then slowly released (to prevent vibration), it returns to its original straight condition. If, however, it is moderately bent and left for years in this bent condition and is then slowly released, it will immediately become nearly straight, and then *very slowly recover* until it becomes practically straight. It might be said that the steel becomes "fatigued" from being bent for so long a time. Accordingly, it is said to be *Elastic Fatigue* of the steel (see also Sec. 108) which in this case prevents the *immediate* return of the spring to its straight condition. Again, if the spring is very much bent and then released it will remain slightly bent, *i.e.*, it will have a slight permanent "set." In such case, the steel is said to be "strained" beyond the *Elastic Limit.* All solids are more or less elastic. Even a lead bar if very slightly bent will

recover; but the elastic limit for lead is very quickly reached, so that if the bar is appreciably bent, it *remains* bent upon removal of the applied force.

106. Tensile Stress and Tensile Strain.—In Sec. 103 a brief discussion of elasticity was given, in which it was shown that solids possess three kinds of elasticity. We shall now discuss more in detail the simplest of these, namely, *Tensile Elasticity*, and consider the other two in subsequent sections. Before a systematic study of the elastic properties of a substance can be made, it is necessary to understand clearly the meaning of each of the terms, *Stress*, *Strain*, and *Modulus*.

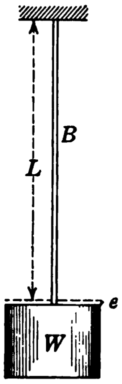

FIG. 63.

Whenever an elastic body is acted upon by a force tending to stretch it, there arises an equal internal force tending to shorten it. See Principle of d'Alembert (Sec. 43). Thus, in Fig. 63, let B be a steel bar of length L, say 10 ft., and of cross section A, say 2 sq. in. When an external force F of 20,000 lbs. is applied, the bar stretches a distance e (elongation), say 0.04 in. It is at once evident that in this stretched condition, which is also an equilibrium condition, the internal forces due to which the bar tends to resume its normal length must just equal the 20,000 lbs. which tends to make the bar lengthen; otherwise the weight W would move downward causing the stretch of the bar to be still further increased. This *internal* force divided by the cross section of the bar, in other words, the force per unit cross section, is called the *Tensile Stress*. But, since the internal force that arises is always equal to the applied force, we have

$$\text{Stress} = \frac{\text{applied force}}{\text{cross section}} = \frac{F}{A} \tag{62}$$

which here is $\frac{20000}{2}$ or 10,000 lbs. per sq. in.

The increase in length, or the elongation e of the bar, divided by its original length, in other words, the stretch per unit length, is called the *Tensile Strain*. Accordingly, we here have

$$\text{Tensile strain} = \frac{\text{elongation}}{\text{orig. length}} = \frac{e}{L} = \frac{0.04 \text{ in.}}{120 \text{ in.}} = 0.000333 \tag{63}$$

A column, in supporting a load, is subjected to a stress and suffers a strain, both of which are defined essentially as above.

The stress is the load divided by the cross section of the column, and the strain is the *decrease* in length divided by the original length of the column. It is an observed fact that a column, in supporting a load in the usual way, is decreased in length by an amount exactly equal to the stretch that it would experience if its upper end were fastened to a support and the same load were suspended from its lower end. In other words, within certain limits, the elasticities of extension and compression are alike. It appears, then, that within certain limits, the molecules of an elastic solid resist having their *normal spacing decreased* with the same force that they resist having it *increased* a like amount.

107. Hooke's Law and Young's Modulus.—If the bar B (Fig. 63) supports twice as large a load it will stretch twice as much, and so on for still larger loads, so long as it is not strained *beyond the elastic limit.* A glance at the above equations shows that both the stress and the strain must, then, increase directly as does the load. This being true, it follows that

$$\frac{\text{Stress}}{\text{Strain}}=\text{a } constant,$$

which is known as *Hooke's law.* If a substance is strained beyond the elastic limit it does not obey Hooke's law; conversely, if an elastic body does not obey Hooke's law, it must be strained beyond the elastic limit.

A spiral spring of steel obeys Hooke's law, *i.e.*, the elongation is proportional to the load it supports. This property is utilized in the ordinary spring balance used in weighing. If a certain torque twists a rod or shaft through an angle of 20°, and if doubling the torque twists it 40°, then the rod or shaft follows Hooke's Law for that torque. If 5 times as great a torque twists the rod say 130° (instead of 100°), it shows that it is strained beyond the elastic limit, since for this larger torque it *does not* follow Hooke's Law.

The constant of Hooke's Law is called the *Stretch Modulus* or *Young's Modulus* for the substance, when applied to *tensile stress* and *tensile strain*, or

$$\text{Young's Modulus } E=\frac{\text{tensile stress}}{\text{tensile strain}}=\frac{F/A}{e/L}=\frac{FL}{Ae} \qquad (64)$$

Substituting the values used in Eqs. 62 and 63, we have

$$E=\frac{FL}{Ae}=\frac{20000\times 120}{2\times 0.04}=30{,}000{,}000 \text{ lbs. per sq. in.}$$

The above assumed stretch is about what would be found by experiment if the bar were very good steel. Hence Young's modulus for good steel is 30,000,000 lbs. per sq. in. In the metric system, the force would usually be expressed in dynes (sometimes in kilograms), the distance in centimeters, and the cross section in square centimeters. Young's modulus for steel as expressed in this system is 1.9×10^{12} dynes per $cm.^2$ For most substances Young's modulus is very much smaller than for steel; in other words, most substances offer less resistance to stretching than steel does.

If, in Eq. 64, A were unity, and if e were equal to L, *i.e.*, if B had unit cross section and were stretched to double its original length (assuming that to be possible), then the equation would reduce to $E=F$. Hence Young's modulus E is *numerically* equal to the force that would be required to stretch a bar of unit cross section to twice its original length, provided it continued to follow Hooke's law. Although a bar of steel, or almost any other substance except rubber, would break long before reaching twice its original length, still this concept is useful. For, by its use in connection with the above data, we see at once, since a force of 30,000,000 lbs. would double the length of a bar of 1 sq. in. cross section (*assuming* Hooke's law to hold), that a force of 30,000 lbs., for which force Hooke's law *would* hold, would increase its length 1/1000 as much, or 1 part in 1000.

108. Yield Point, Tensile Strength, Breaking Stress.—If the bar B (Fig. 63) is made of steel, it will be found that as the load is increased the bar will stretch more and more, in accordance with Hooke's law, until the stress is about 60,000 lbs. per sq. in. Upon further increasing the load, it will be found that the bar begins to stretch very much more—perhaps 50 times more—than for previous increases of like magnitude. This change in the behavior of the steel, this very great increase in the strain produced by a slight increase in the stress, is due to a yielding of the molecular forces, which yielding permits the molecules to slide slightly with reference to each other. We may say for this specimen of steel, that a stress of 60,000 lbs. per sq. in. strains it to the *elastic limit*, and that a slightly greater stress brings it to the *Yield Point*.

As soon as the yield point is reached, further increase of load causes the bar to stretch until the elongation is 25 or 30 per cent. of the original length, in the case of *soft* steel. The maximum elongation for *hard* steel may be as small as 1 per cent. If the

load is removed after the yield point has been passed, the bar remains permanently elongated, *i.e.*, it has a *Permanent Set.* This elongation is accompanied by a decrease in cross section. The maximum load required to cause breaking, divided by the *original* cross section, gives the *Breaking Stress* or *Tensile Strength of the Steel.*

A slight difference in the amount of carbon in steel, changes its elastic behavior very much. Thus, a certain specimen of steel containing 0.17 per cent. carbon had an elastic limit of 51,000 lbs. per sq. in. and a breaking stress of 68,000 lbs. per sq. in. For another specimen, containing 0.82 per cent. carbon, the elastic limit was 68,000 lbs. per sq. in., and the breaking stress was 142,000 lbs. per sq. in. The annealing or tempering of steel is also an important factor in determining its elastic properties.

In addition to iron and carbon, steel may contain various other substances, important among which are nickel, silicon, and manganese, which greatly influence its elastic properties and its hardness, even though present in very small quantities (1 to 5 per cent. more or less). A piano wire, having the enormous tensile strength of 340,000 lbs. per sq. in., or 170 tons per sq. in., was found upon analysis to contain 0.01 per cent. sulphur, 0.018 per cent. phosphorous, 0.09 per cent. silicon, 0.4 per cent. manganese, and 0.57 per cent. carbon. Because of the great commercial importance of steel, this brief statement concerning its composition and elastic properties is made here. For further discussion consult some special engineering work on the subject, or an encyclopedia, such as "Americana" or "Britannica."

Factor of Safety.—If steel is subjected to a great many repetitions of stresses which are well below its tensile strength, or even below its elastic limit, it is greatly weakened thereby, and it may finally break with a load which it would have easily carried at first. This weakening of material by a great number (several millions) of repetitions of a stress is said to be due to *Elastic Fatigue.* (See also Sec. 105.) Of course in any structure the stress should always be well below the elastic limit for the material used. Thus steel whose elastic limit is 50,000 lbs. per sq. in. would rarely be subjected to stresses greater than 25,000 lbs. per sq. in. In such case the *Factor of Safety* is 2. Structures or machine parts which are exposed to vibrations and sudden stresses or shocks, especially if constructed of very hard steel or other relatively brittle material, require a much higher factor of safety. The factor of safety also guards against breakage from flaws in the material.

109. Strength of Horziontal Beams.—If a straight beam of wood or metal (Fig. 64) of length L, having a rectangular cross section of depth h and width a, is supported at each end and loaded in the middle as shown, it will bend slightly. Obviously, in the process of bending, the material near the upper portion of the beam is compressed, while that below is stretched. The horizontal layer of particles through the middle of the beam, that is, through the line, BCD, is called the *Neutral Plane*, because this portion is neither compressed nor stretched. The material at G is stretched only 1/2 as much as that at H, because it is only 1/2 as far from the neutral plane. Hence if the load is made too

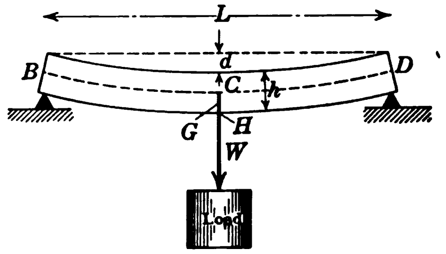

FIG. 64.

great the material at H, called the "outer fiber," is the first to be strained to the *yield point*, and when fracture occurs, it starts at this point.

It can be shown by means of advanced mathematics that

$$d = \frac{WL^3}{4Eah^3} \qquad (65)$$

in which d is the deflection of the middle of the beam produced by the load W, and E is Young's modulus for the material of the beam. Eq. 65 shows that the beam will deflect less, and hence be stronger if placed on edge than if flatwise.

As an illustration, consider a 2-in. by 6-in. joist such as is sometimes used to support floors. In changing the joist from the flat to the edgewise position, we treble h and make a 1/3 as large. Trebling h makes h^3 27 times as large, consequently ah^3 is 1/3 times 27, or 9 times as large as before. This makes d 1/9 as large. In other words, the beam would require 9 times as large a load to give the same amount of bend, which means that the *Stiffness* of the beam is made 9 times as great by turning it on edge.

In the edgewise position, however, the distance of the "outer fiber" (Fig. 64) from the *neutral plane* is three times as large as before, and consequently a given bend or deflection produces 3 times as great a strain on this fiber as before, so that the *Strength* of the beam is not 9 times as great, but only 3 times as great on edge as flatwise.

Next consider the effect on d of variation in length, all other quantities remaining the same. If the beam is made 3 times as long, L^3 and hence also d become 27 times as great as before. If the beam is three times as long, it must bend 9 times as much (*i.e.*, d must be 9 times as great) to produce the same strain in the material. For to produce the same strain in the longer beam, it must bend to an arc of the same radius of curvature as the shorter beam. But, for *small* arcs, the distance d from the middle point of the chord to the middle point of its arc varies approximately as the square of the length of the chord. Consequently, the strain is 3 (not 27) times as great as before, and the beam will therefore support only 1/3 as great a load as before. This relation will be clearly seen from an application. Suppose that a pine beam 4 ft. long and 2 in. by 4 in. in cross section, will support 1000 lbs. at its center. Then if twice as long it will support 1/2 as much, or 500 lbs. If 3 times as long it will support 1/3 as much, and so on.

To summarize, we may state that for rectangular beams supported at the end and loaded in the middle (or supported in the middle and loaded at the ends, which amounts to the same thing), the *strength varies directly as the first power of the width and as the second power of the depth; while it varies inversely as the first power of the length. For such beams, the stiffness varies directly as the first power of the width, and as the cube of the depth* (other things not being varied); *while it varies inversely as the cube of the length.*

110. Three Kinds of Elasticity, of Stress, and of Strain; and the Three Moduli.—In Sec. 103 it was stated that a solid, for example a metal bar, may be acted upon by forces in three distinct ways bringing into play its three elasticities. Thus the metal bar B (Fig. 65) of length L and cross section A, is acted upon by a force F which produces an elongation e. Upon removal of this force it returns to its original length due to *tensile elasticity*. B_1 illustrates the same bar acted upon by forces from all sides, *i.e.*, over its entire surface of area A_1. Let us suppose these forces to be due to hydrostatic pressure, which pressure causes a decrease V' in the original volume (V) of the bar. As soon as the pressure is removed, the bar returns to its original volume by virtue of its *volume elasticity*. B_2 illustrates the same bar again, this time with its *lower* surface fixed. Consequently the force F applied to its *upper* surface of area A_2 makes it slide or *shear* a distance s with respect to the lower surface. The distance between the two surfaces we shall call d. Upon removal of the force F the shear disappears due to *shearing elasticity*. In all three cases, recovery upon removal of the force is *practically* immediate and complete, provided the bar has not been strained beyond the elastic limit.

The Three Moduli.—The stress to which a certain material is subjected, divided by the resulting strain, is constant (Hooke's Law), and this constant is called the Modulus of Elasticity. Since there are three kinds of stress and three kinds of strain, it follows that there must be *three moduli.*

Stress is *always* the *total* applied force F divided by the area to which it is applied. Thus in the first case (B), *tensile* stress is F/A, in the second case (B_1), the hydrostatic stress or *volume* stress is F/A_1; while in the third case (B_2), the *shearing* stress is F/A_2. In the first case, the *tensile* strain is the change in length divided by the original length or e/L; in the second case, the *volume* strain is the change in volume divided by the original volume, or V'/V; while in the third case, the *shearing* strain is the distance sheared divided by the distance between the two shearing surfaces, or s/d.

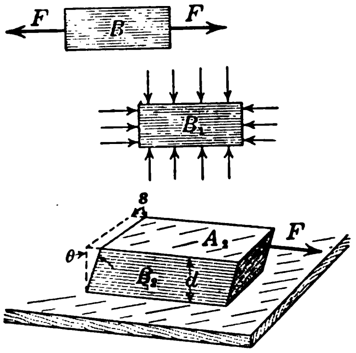

FIG. 65.

Summarizing, then, we have:

$$\text{The mod. of Tension (Young's modulus)} = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{e/L} \qquad (64 \text{ bis})$$

$$\text{The Volume modulus (bulk mod.)} = \frac{\text{hydrostatic pressure}}{\text{volume strain}} = \frac{F/A_1}{V'/V} \qquad (66)$$

$$\text{The Shearing modulus (mod. of rigidity)} = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{F/A_2}{s/d} \qquad (67)$$

Observe that if s is very small with respect to d, then $s/d = \theta$. The angle θ is called the angle of shear. For this reason the shearing strain is usually called the *angle of shear.* To illustrate shearing, the bar B_2 may be considered to be made up of a great number of horizontal layers of molecules, a few of which layers are indicated in the sketch. Evidently, when the force F is applied, and the bar is changed from the rectangular form to the sheared position, each layer is shifted to the right a slight distance,—s for

the top layer, ½ s for the middle layer, and so on. Furthermore, each layer is shifted or displaced very slightly with respect to the next layer below it, thereby causing a slight change in the relative positions of the molecules of successive layers. If F is *decreased*, the tendency of the molecules to resume their original relative positions reduces the relative shift between successive layers, and hence reduces the angle of shear. If F is *removed* the angle of shear becomes zero, *i.e.*, the molecules completely return to their normal relative positions, and the bar again becomes rectangular, provided it has not been strained beyond the elastic limit.

111. The Rigidity of a Shaft and the Power Transmitted.—If one end A of a shaft is clamped and the other end B is turned through one revolution by some applied torque, the shaft is said to be twisted through an angle of 360°. Evidently the layer of molecules on the end B has been displaced or sheared through 1 revolution with respect to the layer at end A, through 1/2 revolution with respect to the transverse layer through the middle of the shaft, through 1/4 revolution with respect to the layer 1/4 way from B to A, and so on. Indeed every transverse (circular) layer in the shaft is sheared slightly with respect to its neighbor. Obviously this shear is greatest for the particles farthest from the axis of the shaft. Accordingly it is the "outer fibers" (on the surface of the shaft) which first give way when it is twisted in two. Observe that when a bolt is twisted in two, the central fibers are the last to break. Observe also that fracture in this case consists in a shearing apart of adjacent layers.

By knowing the values of the shearing modulus and the shearing strength for the steel used, and with the aid of certain formulas, the derivation of which requires a knowledge of advanced mathematics, the engineer can readily compute the proper size of shaft for a specified purpose. The shaft must be of such size that the maximum torque to which it is to be subjected shall not strain the outer fibers beyond the "safe" limit. Although the mathematical treatment of this topic is too complicated for an elementary work, it may be stated that the strength of a shaft, that is the maximum torque which it can safely transmit, varies as the cube of its radius, while the "stiffness" varies as the 4th power of the radius. Thus a 2-inch shaft can transmit 8 times as great a torque as a 1-in. shaft; while, if the length of the shaft and the applied torque are the same for

both, the smaller shaft will be twisted through 16 times as great an angle as the larger.

Since power is torque multiplied by the angular velocity ($P = T\omega$, Sec. 83), it follows that a given amount of power can be transmitted by 1/4 as great a torque, and hence by 1/4 as strong a shaft by making the angular velocity 4 times as great. We may also add that the power which a belt of given strength can transmit varies directly as the speed of the belt. For, in this case, $P = Fv$, in which v is the belt speed, and F is the difference in tension between the tight and the slack belt.

PROBLEMS

1. A certain steel bar 10 ft. in length and 2 sq. in. in cross section is elongated 0.22 in. by a 50-ton pull. What is Young's modulus E for this specimen?

2. A steel wire 3 meters in length and 2 mm. in diameter supports a load of 10 kilos. How much will the wire elongate under this load, if Young's modulus for the wire is 1.9×10^{12} dynes per cm.^{2}?

3. How much will a copper wire 10 meters in length and 2 sq. mm. in cross section stretch under a load of 3 kilos? Young's modulus for copper is 1.2×10^{12} dynes per cm.^{2}

4. A certain shaft A can safely transmit 50 H.P. What power can be transmitted by a shaft of the same material having twice as great a diameter and 3 times as great an angular velocity as A?

5. An oak timber 3 in. by 12 in. rests edgewise upon two supports which are 8 ft. apart. How much will the beam bend (deflect at the middle) under a load of 1000 lbs. applied midway between the supports? Young's Modulus for oak is 1,500,000 lbs. per sq. in.

6. How much would the 1000-lb. load bend the timber (Prob. 5) if the timber rested flatwise upon the supports?

CHAPTER X

PROPERTIES OF LIQUIDS AT REST

112. Brief Mention of Properties.—Some of the properties of liquids in addition to the general properties of matter (Sec. 101), are *Viscosity, Solvent Action, Diffusion, Osmosis, Pressure Production, Pressure Transmission, and Surface Tension.*

Elasticity.—The only kind of elasticity that liquids or gases can have is of course *volume elasticity* (Sec. 110). Liquids (also gases) are perfectly elastic, that is, however much a liquid is compressed, upon removing the pressure the liquid expands to exactly its former volume. There is no such thing as elastic fatigue or elastic limit for liquids. It requires very high pressure to produce appreciable compression of a liquid. Thus a pressure of 100 lbs. per sq. in. applied to a volume of water causes a shrinkage of only 1 part in 3000.

Viscosity.—If a vessel filled with syrup has a small hole made near the bottom, the syrup will flow slowly through the hole. If the vessel were filled with water instead, it would be found that the water, having less viscosity, would flow much more quickly through the hole. Syrup is said to be *viscous*, and water *mobile.* Water, however, has some viscosity. Glycerine has greater viscosity than water but less than molasses. Viscosity arises from internal friction, that is, friction between the molecules of the liquid. The greater viscosity of glycerine as compared with that of water is then due to the fact that glycerine molecules do not glide over each other so readily as do water molecules.

It may easily be observed that the water on the surface of a river moves more rapidly than that near the bottom, and also that the water near the center of the stream moves more rapidly than that near the shore. This difference in velocity is due to friction upon the bed of the river (and upon its shores), which causes the layers very near the bottom to move very slowly. These slowly moving layers of water, due to friction of *water on water, i.e.*, due to the viscosity of water, tend to retard the motion of the layers above. The greatest retarding effect is

exerted upon the nearest layers, and the least upon the surface layer. Hence the velocity of flow gradually increases from the bottom up.

Solvent Action.—Some solids when placed in certain liquids slowly disappear. Thus salt readily "dissolves" in water, forming a *solution.* Paraffine dissolves in kerosene, but not in water; while salt dissolves in water but not in kerosene. When water has dissolved all of the salt it is possible for it to hold in solution, the brine thus formed is said to be a *saturated solution* of salt. Solution is usually attended by either evolution or absorption of heat; *i.e.*, by either heating or chilling action.

Gold, zinc, and some other metals dissolve to a certain extent in mercury, forming gold amalgam, zinc amalgam, etc. These *amalgams* are really *solutions* of the metals in mercury.

Some liquids dissolve in other liquids. Thus, if some ether and water are thoroughly stirred together in a vessel and then allowed to stand a moment, the water, being the heavier, settles to the bottom and the layer of ether rests upon it. Upon examination it will be found that there is about 10 per cent. ether in the water, and about 3 per cent. water in the ether, which shows that a saturated solution of ether in water is about 10 per cent. ether, while a saturated solution of water in ether is about 3 per cent. water.

Some liquids dissolve certain gases. Thus water dissolves air to a slight extent, and at room temperature and atmospheric pressure, water dissolves 450 times its volume of hydrochloric acid gas (HCl), or 600 times its volume of ammonia gas (NH_3). What is known *commercially* as ammonia or as hydrochloric acid is simply an aqueous solution of the one or the other of these gases. *Pure* liquid ammonia is used in ice manufacture (Sec. 200). Hydrochloric acid gas can be condensed to a liquid, thus forming *pure* liquid hydrochloric acid, by subjecting it to very high pressure and low temperature. A given volume of water will dissolve about an equal volume of carbon dioxide (CO_2) at ordinary pressure and temperature. Under greater pressure it dissolves considerably more, and is then called *soda water.* When drawn from the fountain, the pressure upon it is reduced, and the escaping CO_2 produces effervescence.

Diffusion.—Many liquids if placed in the same vessel, mix even though of quite different densities. Thus, if some ether is very carefully introduced onto the surface of some water in such

a way as to prevent mixing when introducing it, it will be found after a time that the *heavier* liquid (water) has diffused *upward* into the ether until the latter contains about 3 per cent. water, while the ether, although lighter, has diffused *downward* into the water.

Osmosis.—Osmosis is the mixing or diffusing of two different liquids or gases through a membrane that separates them. Membranes of animal or plant tissue readily permit such diffusion of certain substances through them. Thus a bladder filled with water does not leak, but if lowered into a vessel of alcohol it slowly collapses. This shows that the water passes readily through the bladder; the alcohol less readily, or not at all. On the other hand, if a rubber bag is filled with water and is then lowered into a vessel of alcohol, it becomes more and more *distended*, and may finally burst. In this case it is the *alcohol* which passes most readily through the separating membrane.

If a piece of parchment or other such membrane is tied tightly across the mouth of an inverted funnel filled with sugar solution, and the funnel is placed in water, it will be observed that the solution slowly rises in the stem. By prolonging the stem a rise of several feet may be obtained. Obviously the pure water passes more readily through the membrane than does the *sweetened* water, or sugar solution. If the solution is 1.5 per cent. sugar (by weight), it will finally rise in the stem about 34 ft. above the level of the water outside.

Since a column of water 34 ft. in height exerts a pressure of about one atmosphere (Sec. 136), which pressure in this case would tend to force the *solution* through the membrane *into the water*, it follows, when equilibrium is reached, *i.e.*, when no further rise of the column occurs, that the *Osmotic Pressure* developed by the tendency of the *water* to pass through the membrane *into the solution*, must be one atmosphere for a 1.5 per cent. sugar solution.

With weak solutions, the osmotic pressure varies approximately as the strength of the solution. Thus a 3 per cent. sugar solution would develop an osmotic pressure of about 2 atmospheres. The osmotic pressure also differs greatly for different solutions. Thus, for example, if a solution of common salt is used the osmotic pressure developed will be much more than for the same strength (in per cent.) of sugar solution.

In accordance with the kinetic theory of matter (Sec. 99) we may explain osmotic pressure by assuming, in the case cited

above, that the water molecules in their vibratory motion, pass more readily through the animal membrane (the bladder) than do the more complicated and presumably larger alcohol molecules. This is the commonly accepted explanation. The fact, however, that substituting a rubber membrane reverses the action, makes it seem probable that something akin to chemical affinity between the membrane and the liquids plays an important rôlé. From this standpoint, we would explain this reversal in osmotic action by stating that the rubber membrane has greater affinity for alcohol than for water; while in the case of animal tissue the reverse is true. Osmosis plays an important part in the physiological processes of nutrition, secretion by glands, etc., and in the analogous processes in plant life. Gases also pass in the same way through membranes. In this way the blood is purified in the capillary blood-vessels of the lungs by the oxygen in the adjacent air cells of the lungs.

In chemistry, *Dialysis*, the process by which crystalloids, such as sugar and salt are separated from the colloids—starch, gum, albumin, etc., depends upon osmosis. Crystalloids pass readily through certain membranes; colloids, very slowly, or not at all. In case of suspected poisoning by arsenic or any other crystalloid, the contents of the stomach may be placed on parchment paper floating on water. In a short time the crystalloids (only) will have entered the water, which may then be analyzed.

Pressure and its Transmission.—Liquids exert and also transmit pressure. In deep-sea diving the pressure sustained by the divers is enormous. By means of our city water mains, pressure is transmitted from the pumping station or supply tank to all parts of the system. (This property will be fully discussed in Secs. 113 and 114. *Surface Tension* will be considered in Sec. 124.)

113. Hydrostatic Pressure.—The study of fluids at rest is known as *Hydrostatics*, and that of fluids in motion, as *Hydraulics*. From their connection with these subjects we have the terms *hydrostatic pressure* and *hydraulic machinery* such as hydraulic presses, hydraulic elevators, etc.

A liquid, because of its weight, exerts a force upon any body immersed in it. This force, divided by the area upon which it acts, is called the *Hydrostatic Pressure*, or

$$\text{Hydrostatic pressure (average)} = \frac{\text{total force}}{\text{area acted upon}}$$

Note that pressure, like all *stresses* (Sec. 110), is the *total force* applied divided by the *area* to which it is applied. The unit in which to express pressure will therefore depend upon the units in which the *force* and the *area* are expressed. Some units of pressure are the poundal per square inch, the pound per square inch, the pound per square foot, and the dyne per square centimeter.

Let it be required to find the pressure at a depth h below the surface of the liquid of density d in the cylindrical vessel of radius r, Fig. 66. The formula for the pressure on the bottom of the vessel is, by definition,

$$\text{Pressure} = \frac{\text{total force on the bottom}}{\text{area of bottom}}$$

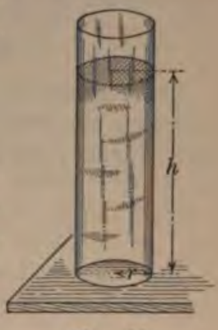

FIG. 66.

The force on the bottom is obviously the weight W of the liquid, and the area A is πr^2; so that the pressure is $W/\pi r^2$. We may express W in dynes, poundals, or pounds force, and πr^2 in square centimeters, square inches, etc. The weight in dynes is Mg, but the mass M in grams is the product of $\pi r^2 h$, the volume of the liquid in cubic centimeters, and d its density in grams per cubic centimeter. Hence

$$\text{Pressure } p = \frac{\text{force}}{\text{area}} = \frac{W}{A} = \frac{Mg}{\pi r^2} = \frac{\pi r^2 hdg}{\pi r^2} = hdg \text{ dynes per cm.}^2 \quad (68)$$

In the British system, $\pi r^2 h$ would be the volume of the liquid column in cubic feet, and d the density in pounds per cubic foot; so that $\pi r^2 hd$ would be the weight in pounds, and $\pi r^2 hdg$ would be the weight in poundals. Note that 1 lb. = g poundals, *i.e.*, 32.17 poundals (Sec. 32). Accordingly, the pressure produced by a column of liquid whose height is h feet is hdg *poundals* per square foot, or hd *pounds* per square foot.

114. Transmission of Pressure.—If a tube A (Fig. 67) with side branches B, C, D and E, is filled with water, it will be found that the water stands at the same level in each branch as shown. Further, if A contains four small holes, a, b, c, and d, all of the same size and at the same level, and covered by valves a', b', c', and d', respectively, it will be found that it requires the same amount of force to hold the valve a' closed against the water pressure as to hold b', c', or d' closed.

If the branch tube B were removed, everything else being left

just as before, it is evident from symmetry that a small valve at e in order to prevent water from coming out would have to resist an upward pressure at e (say p_2) equal to the upward pressure at c, d, etc. With B in place, however, the water does not come out of e, but is at rest; hence the downward pressure at e (say p_1) due to the column of water in B must just balance the above-mentioned pressure p_2. The pressure, p_1, however, is equal to hdg (Eq. 68). If the pressure at a, b, c, and d, is represented by p_a, p_b, p_c, and p_d respectively, we have

$$p_2 = p_a = p_b = p_c = p_d = p_1 = hdg.$$

The experiment shows, then, that in liquids the *pressure* (*a*) is *exactly equal in all directions at a given point* (see also experiment below); (*b*) *is transmitted undiminished to all points at the*

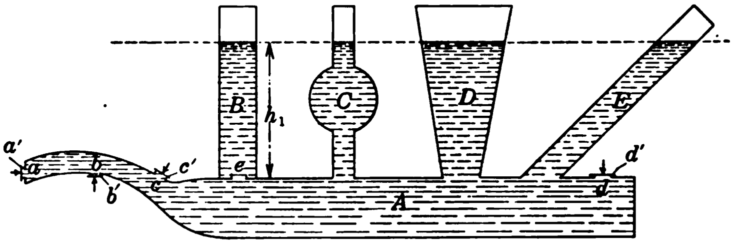

FIG. 67.

same level; and (*c*) *is numerically hdg*, in which h is the *vertical distance* from the point in question to the upper *free* surface of the liquid causing the pressure.

The above three facts or principles (*a*), (*b*), and (*c*) are fundamental to the subjects of hydrostatics and hydraulics. They are utilized in our city water systems, in hydraulic mining, and in all hydraulic machinery. They must be reckoned with in deep-sea diving and in the construction of mill dams and coffer-dams. In these and hundreds of other ways these principles find application.

The greater pressure in the water mains in the low-lying portions of the city as compared with the hill sections, is at once explained by (*c*), noting that the vertical distance from these points to the level of the water in the supply tank is greater for these places than it is on the hills.

An exceedingly simple experimental proof of the principle (*a*) may be arranged as follows: A glass jar containing water

has placed in it several glass tubes which are open at both ends. Some of these tubes are bent more or less at the lower end, so that the lower opening in some cases faces upward, in others downward, and still others horizontally or at various angles of inclination. If these openings are all at the same depth, the fact that the water stands at the same height in all of the tubes, that is, at the general level of the water in the vessel, shows that the outward pressure at each lower opening must be the same. Consequently, since no flow takes place, the inward pressure at each opening, which is due to the general pressure of the main body of water, and which is exerted in *various directions* for the different tubes, must be the same for all.

Pressure Perpendicular to Walls.—The pressure exerted by a liquid, against the wall of the containing vessel at any point is always perpendicular to the wall at that point. For if the pressure were aslant with reference to the wall at any point, it would have a component parallel to the wall which would tend to move the liquid along the wall. We know, however, that the liquid is at rest; hence the pressure can have no component parallel to the wall, and is therefore perpendicular to the wall at all points.

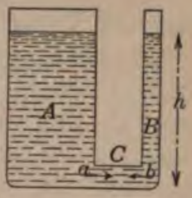

FIG. 68.

115. The Hydrostatic Paradox.—A small body of liquid, for example the column in tube B (Fig. 68), may balance a large body of liquid, such as the column in tube A. This is known as the *Hydrostatic Paradox*. From the preceding sections, we see that the pressure tending to force the liquid through C in the direction of arrow b, is hdg, due to the column of liquid B, while the pressure tending to force it in the direction of arrow a is likewise hdg due to the column of liquid A. Evidently the liquid in C will be in equilibrium and will not tend to move either to the right or left when these two pressures are equal, *i.e.*, when h is the same for both columns. Thus, viewed from the pressure standpoint, we see that there is nothing paradoxical in the behavior of the liquid. If A contained water and B contained brine, then the liquid level in A would be higher than in B (Sec. 116).

116. Relative Densities of Lqiuids by Balanced Columns.—A very convenient method of comparing the densities of two liquids, is that of balanced columns, illustrated in Fig. 69. A U-shaped

glass tube, with arms A and B, contains a small quantity of, mercury C, as shown. If water is poured into the arm A and at the same time enough of some other liquid, *e.g.*, kerosene, is poured into the arm B to just balance the pressure of the water column A, as shown by the fact that the mercury stands at the same level in both arms; then it is evident that the pressure p_2 due to the kerosene, which tends to force C to the left, must be equal to the pressure p_1 due to the water, which tends to force C to the right. But the former pressure is h_2d_2g while the latter pressure is h_1d_1g, in which h_1 and h_2 are the heights of the water and the kerosene columns respectively, and d_1 and d_2 the respective densities of the two liquids. Hence

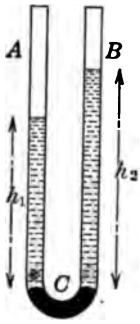

FIG. 69.

$$h_1d_1g = h_2d_2g$$

$$\text{or } \frac{d_2}{d_1} = \frac{h_1}{h_2}, \text{ or } d_2 = \frac{h_1}{h_2}d_1$$

The density d_1 of water is almost exactly 1 gm. per cm.^{3}; therefore if h_1 is found to be 40 cm., and h_2 is found to be 50 cm., then the density of kerosene is 4/5 that of water or *practically* 0.8 gm. per cm.^{3}

117. Buoyant Force.—Any body immersed in a liquid experiences a certain buoyant force. This force, if the body is of small density compared with the liquid, causes the body to rise rapidly to the surface. Thus cork floats on water, and iron on mercury. This buoyant force is due to the fact that the upward pressure on the body is greater than the downward pressure on it.

Let B, Fig. 70, be a cylindrical body immersed in a vessel of water. Let A_1 and A_2 be the areas of the lower and upper ends respectively, and let p_1 and p_2 be the corresponding pressures. If A_1 is 3 times as far below the surface as A_2, then p_1 will equal $3p_2$. The forces on the sides of B will of course neutralize each other and produce neither buoyant nor sinking effect. The entire *Buoyant Force* of the water upon B is, then, $F_1 - F_2$, in which F_1 is the upward push or force on A_1, and F_2 the much smaller downward push on A_2. Force, however, is the pressure multiplied by the area; *i.e.*,

$$F_1 = p_1 A_1, \text{ and } F_2 = p_2 A_2, \text{ or, since } A_1 = A_2,$$

$$\text{Buoyant force} = F_1 - F_2 = p_1A_1 - p_2A_2$$
$$= (p_1 - p_2)A_1$$

If this buoyant force, which tends to make the body rise, is (*a*) greater than the weight W of B, which of course tends to make it sink, the body will move upward—*rapidly* if much greater, and *slowly* if but little greater. (*b*) If the buoyant force is equal to W, then B will remain in equilibrium and float about in the liquid. Finally (*c*), if W is greater than the buoyant force, then B will sink to the bottom, and the rapidity with which it *sinks* depends upon how much its *weight exceeds* the *buoyant force*.

If the body were of irregular shape such as C, it would be very difficult to find its area, and also difficult to find the average vertical components of pressure on the upper and lower surfaces. It is, nevertheless, obvious that the average downward pressure on the body would be less than the average upward pressure, and it is just this *difference* in pressure that gives rise to the buoyant force *whatever shape the body may have* (see Sec. 118). The horizontal components of pressure would, of course, have no tendency to make the body either float or sink.

Fig. 70.

118. The Principle of Archimedes.—If any body, whatever be its shape, *e.g.*, A (Fig. 71), is immersed in a vessel of water, it will be found to be lighter in weight than if it were weighed in air. This difference in weight is referred to as the "Loss of Weight" in water, and is found to be equal to the weight of the water that would occupy the space now occupied by A. In other words, the *loss of weight in water is equal to the weight of the water displaced*. This principle, of course, holds for any other liquid, and also for any gas (Sec. 134), and is known as the *Principle of Archimedes*, so called in honor of the Grecian mathematician and physicist Archimedes (B. C. 287–212) who discovered it.

Theoretical Proof of Archimedes' Principle.—Imagine the body A (Fig. 71) to be replaced by a body of water A' of exactly the same size and shape as A and enclosed in a membranous sack of negligible weight. It is evident that A' would have no tendency either to rise or to sink. It then appears that this particular portion of water loses its *entire weight*, hence it must be true that the buoyant force exerted upon A' is exactly equal to its weight. Since this buoyant force is the direct result of the greater average pressure upon the lower side than upon the upper side of the

body, it can in no wise depend upon the material of which the body is composed. Consequently, the body A must experience this *same* amount of buoyant force, and therefore must lose this same amount of weight, namely, the *weight of the water displaced.*

Experimental Proof of Archimedes' Principle.—A small cylindrical bucket B is hung from the beam of an ordinary beam balance, and a solid metal cylinder C (Fig. 72) which accurately *fits* and completely *fills* the bucket is suspended from it. Sufficient mass is now placed in the pan at the other end of the beam to secure a "balance." Next a large beaker of water is so placed that the solid cylinder is immersed. This, of course, buoys it up somewhat and destroys the "balance." Finally the bucket is filled with water, whereupon it will be found that exact "bal-

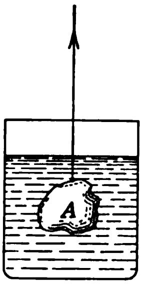

Fig. 71.

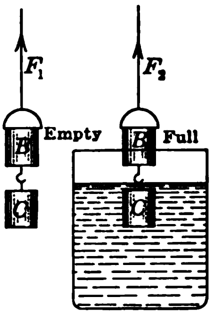

Fig. 72.

ance" is restored, *i.e.*, $F_1 = F_2$. This fact shows that the weight of the water in the bucket just compensates for the buoyant force that arises from the immersion of the cylinder. In other words the *loss of weight* experienced by the cylinder is equal to the weight of the water which fills the bucket, and is therefore equal to the *weight of the water displaced* by the cylinder.

119. Immersed Floating Bodies.—In case the body A (Fig. 71) is denser than water, it will weigh more than the water which it displaces and will therefore tend to sink. If, however, it has the same density as water, the buoyant force will be just equal to its weight, and it will therefore lose its entire weight and float about *in* the liquid.

If a tall glass jar is about one-third filled with strong brine and is then carefully filled with water, the two liquids will mix

slightly, so that the jar will contain a brine varying in strength, and hence in density, from that which is almost pure water at the top, to a strong dense brine at the bottom. If pieces of resin, wax, or other substances which sink in water but float in brine are introduced, they will sink to various depths, depending upon their densities. Each piece, however, sinks until the buoyant force exerted upon it is equal to its weight, that is, until the *weight of the liquid displaced is equal to its own weight.*

Occasionally the query arises as to whether heavy bodies such as metals will sink to the bottom of the ocean. They certainly do, regardless of the depth. To be sure, the enormous pressure at a great depth compresses the water slightly, making it more dense, and hence more buoyant. The increase in density due to this cause, however, even at a depth of one mile amounts to less than 1 per cent. (closely 3/4 per cent). Since the compressibility of metals is about 1/100 as great as that of water, its effect in this connection may be ignored. Substances, however, which are more readily compressed than water, *e.g.*, porous substances containing air, actually become less buoyant at great depths.

120. Application of Archimedes' Principle to Bodies Floating Upon the Surface.—If a piece of wood that is lighter than water is placed in water, it sinks until the weight of the water displaced is equal to its own weight. If placed in brine it will likewise sink until the weight of the liquid displaced is equal to its own weight; but it will not then sink so deep. A boat, which with its cargo weighs 1000 tons, is said to have 1000 tons "displacement," because it sinks until it displaces 1000 tons of water. As boats pass from the fresh water into the open sea they float slightly higher.

FIG. 73.

If a wooden block B (Fig. 73) is placed in water and comes to equilibrium with the portion *mnop* immersed, then the volume *mnop* is the volume of water displaced, and the weight of this volume of water is equal to the entire weight of the block. Further, if d is 9/10 c, we know that the block of wood displaces 9/10 of its volume of water, hence its density is 0.9 gm. per cm.^{3} (since a cm.^{3} of water weighs almost exactly 1 gm.).

Ice is about 9/10 as dense as sea water; consequently icebergs float with approximately 9/10 of their volume immersed and 1/10 above the surface. If some projecting points are 100 ft. above

the sea, it does not follow, of course, that the iceberg extends 900 ft. below the surface.

121. Center of Buoyancy.—If a rectangular piece of wood is placed in water in the position shown at the left in Fig. 74, the center of gravity of the displaced water *mnop* is at *C*. This point *C* is called the *Center of Buoyancy*. It is the point at which the entire upward lift or buoyant force *F*, due to the water, may be considered as concentrated. The center of gravity, marked *G*, is the point at which the entire weight *W* of the block of wood may be considered as concentrated. The block in this position is unstable, since the least tipping brings into play a torque (as shown at the right in Fig. 74) tending to tip it still farther. Consequently the block tips over and floats lengthwise on the water. For the same reason logs do not float on end, but lie lengthwise on the water.

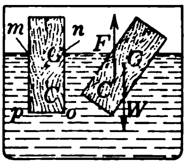

Fig. 74.

If a sufficiently large piece of lead were fastened to the bottom of the block of wood so as to bring its center of gravity below its center of buoyancy, the block would then be stable when floating on end. Ballast is placed deep in the hold of a vessel in order to lower the center of gravity. It does not necessarily follow, however, that the center of gravity of ship and cargo must be below the center of buoyancy of the ship. For, as the ship rolls to the right, say, the form of the hull is such that the center of bouyancy shifts to the right, and therefore gives rise to a righting or restoring torque.

122. Specific Gravity.—The *Specific Gravity* (*S*) of a substance is the *ratio* of the density of the substance to the density of water at the same temperature. Representing the density of water by d', and the density of the substance referred to by d, we have

$$S = d/d' \tag{69}$$

Since the value of d' is very nearly one (*i.e.*, one gm. per cm.^{3}) at ordinary temperatures, it follows that the *Specific Gravity* of a substance and its *density* have almost the same value, but they must *not* be considered as identical.

Density, however, is mass divided by volume, so that if we consider equal volumes of the substance and of water, and represent the mass of the former by M and that of the latter by M', Eq. 69 may be written

$$S = d/d' = \frac{M/V}{M'/V} = M/M' = Mg/M'g = W/W' \tag{70}$$

in which W is the weight of a certain volume of the substance and W' the weight of the same volume of water. Hence the specfic gravity of a substance might be defined as the ratio of the weight of a certain volume of that substance to the weight of an equal volume of water.

Specific Gravity of a Liquid.—If a bottle full of liquid, say kerosene, weighs W_1, and the same bottle full of water weighs W_2, while the empty bottle weighs W_3, then W_1-W_3 is the weight W of the kerosene in the bottle, and W_2-W_3 is the weight W' of an equal volume of water; hence from Eq. 70 we have for the specific gravity of kerosene

$$S=\frac{W}{W'}=\frac{W_1-W_3}{W_2-W_3}$$

If a piece of metal which has first been weighed in air, is then immersed in water and again weighed, it will be found to be lighter. This "loss of weight" in water, *i.e.*, its weight in air minus its weight in water, is of course due to the buoyant force and is equal to the weight of the *water* displaced. If the piece of metal is again weighed while immersed in brine, the loss of weight will be equal to the weight of the *brine* displaced. This loss of weight will be greater than the former loss. Dividing it by the former loss we obtain the specific gravity of the brine.

Specific Gravity of a Solid.—Evidently the volume of any body immersed in water is exactly equal to the volume of water which it displaces. Consequently its specific gravity is the ratio of the weights of these two volumes, or the weight of the body in air divided by its loss of weight in water. This is a convenient method for determining the specific gravity of irregular solids, such as pieces of ore.

If a stone weighs 30 gm. in air and 20 gm. in water, then the weight of the water it displaces must be 10 gm.; so that the stone weighs 3 times as much as the same volume of water and its specific gravity is, therefore, 3. Since the density of water d' (Eq. 69) is very slightly less than 1.0 at room temperature, the density d of the stone would be very slightly less than its specific gravity.

123. The Hydrometer.—The hydrometer, of which there are several kinds, affords a very rapid means of finding the specific gravity of a liquid. It is also sufficiently accurate for most purposes. The most common kind of hydrometer consists of a

glass tube *A* (Fig. 75), having at its lower end a bulb *B* containing just enough mercury or fine shot to properly ballast it when floating. From Sec. 120 we see that such an instrument will sink until it displaces an amount of water equal to its own weight. To do this it will need to sink deeper in a light liquid than in a heavy liquid; hence the depth to which it sinks indicates the specific gravity of the liquid in which it is placed. From a scale properly engraved upon the stem of the hydrometer, the specific gravity of the liquid in which it is floating may be read by observing the mark that is just at the surface. Thus, if the hydrometer sinks to the point *a* in a given liquid, we know that the specific gravity of the liquid is 1.12, *i.e.*, it is 1.12 times as dense as water. The scale shown is called a *Specific Gravity Scale*, because the specific gravity of the liquid is given directly. It will be observed that it is *not* a scale of equal divisions.

Fig. 75.

The Beaumé Scale.—The Beaumé Scale, which is very much used, has on the one hand the advantage of having equal scale divisions; but on the other hand it has the disadvantage that it is entirely arbitrary, and that its readings do not give directly the specific gravity of the liquid. There are two Beaumé scales, one for liquids heavier than water, the other for liquids lighter than water.

To calibrate a hydrometer for heavy liquids it is placed in water, and the point to which it sinks is marked 0°. It is next placed in a 15 per cent. brine (15 parts salt and 85 parts water, by weight) and the point to which it sinks is marked 15°. The space between these two marks is then divided into 15 equal spaces and the graduation is continued *down* the stem. If, when placed in a certain liquid, the hydrometer sinks to mark 20, the specific gravity of the liquid is 20° *Beaumé heavy.*

For use in light liquids, the point to which the instrument sinks in a 10 per cent. brine is marked 0°, and the point to which it sinks in water is marked 10°. The space between these two marks is divided into 10 equal spaces, and the graduation is extended up the stem. If, when placed in a certain liquid, the hydrometer sinks to mark 14, the specific gravity of the liquid is 14° *Beaumé light.*

124. Surface Tension.—Small drops of water on a dusty or oily surface assume a nearly spherical shape. Small drops of

mercury upon most surfaces behave in the same manner. Dew-drops and falling raindrops are likewise spherical. When the broken end of a glass rod having a jagged fracture is heated until soft, it becomes smoothly rounded. These and many other similar phenomena are due to what is called *Surface Tension* (defined in Sec. 126).

Surface tension arises from the intermolecular attraction (or cohesion) between adjacent molecules. Some of the effects of this attraction have already been discussed in Sec. 102. Certain experiments indicate that these molecular forces do not act appreciably through distances greater than about two-millionths of an inch. A sphere, then, of two-millionths inch in radius described about a molecule may be called its sphere of influence, or *sphere of molecular attraction.*

Let A, B, and C (Fig. 76) represent respectively a molecule of water well below the surface, one very near the surface, and one

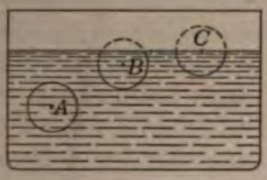

Fig. 76.

Fig. 77.

on the surface; and let the circles represent their respective spheres of molecular action. Evidently A, which is completely surrounded by water molecules, will be urged equally in all directions and hence will have no tendency to move. It will therefore, barring friction, not require any force to move it about in the liquid; but, as we shall presently see, it *will* require a force to move it to the surface. Accordingly, work is done in increasing the amount of surface of a liquid (Sec. 126), *e.g.*, as in inflating a soap bubble. Part of B's sphere of molecular attraction projects above the surface into a region where there are no water molecules, and hence the aggregate downward pull on B exerted by the surrounding molecules is greater than the upward pull upon it. In the case represented by C, there is no upward pull, except the negligible pull due to the adjacent molecules of air. Consequently B and C, and all other molecules *on* or *very near* the surface, are acted upon by downward (inward)

forces. The nearer a molecule approaches to the surface, the greater this force becomes.

In Fig. 77, A represents a small water drop and a, b, c, d, etc., surface molecules. Since every surface molecule *tends* to move inward, the result is quite similar to uniform hydrostatic pressure on the entire surface of the drop. But such pressure would arise if the surface layer of molecules were a stretched membranous sack (*e.g.*, of exceedingly thin rubber) enveloping the drop. This fact, that the surface layer of molecules of any liquid behaves like a stretched membrane, *i.e.*, like a membrane under *tension*, makes the name *Surface Tension* very appropriate. Although there is no stretched film over the drop, the surface molecules differing in no sense from the inner molecules except that they *are* on the surface, it is, nevertheless, very *convenient* to regard the phenomenon of surface tension as *arising* from the action of stretched films, and in the further discussion it will be so regarded. It must be kept in mind, however, that this is merely a *matter of convenience*, and that the true cause of surface tension is the *unbalanced molecular* attraction just discussed.

When certain insects walk upon the water, it is easily observed that this "membrane" or "film" sags beneath their weight. A needle, especially if slightly oily, will float if carefully placed upon water. We may note in passing that the weight of the water displaced by the sagging of the surface film is equal to the weight of the needle (Archimedes' Principle).

125. Surface a Minimum.—Evidently a stretched film enclosing a drop of liquid would cause the drop to assume a form having the least surface, *i.e.*, requiring the least area of film to envelop it. The sphere has less surface for a given volume than any other form of surface. Hence drops of water are spherical. For the same reason soap bubbles, which are merely films of soapy water enclosing air, tend to be spherical. A large drop of water, or mercury, or any other liquid is not spherical if resting upon a surface, but is flattened, due to its weight (see Fig. 78). Quite analogous to this is the fact that a small rubber ball filled with water and resting upon a plane surface will remain almost spherical; while a large ball made of equally thin rubber would flatten quite appreciably, due to its greater weight.

FIG. 78.

If the effect of the weight of the drop is removed, this flatten-

ing does not take place even for very large drops. Thus, if a mixture of alcohol and water having the same density as olive oil is prepared, it will be found that a considerable quantity of this oil retains the spherical form when carefully introduced well below the surface of the mixture.

That a film tends to contract so as to have a *minimum area*, and that in so doing it *exerts* a force, is beautifully illustrated by the following experiment. If the wire loop B (Fig. 79), to which is attached a small loop of thread a, is dipped into a soap solution and withdrawn, it will have stretched across it a film in which the loop a "floats" loosely as indicated. Evidently the film, pulling equally in all directions on a, has no tendency to stretch it. If, however, the film *within* a is broken, the inward pull disappears, whereupon the outward pull causes the loop to assume the circular form shown at the right (Fig. 79). A loop has its *maximum* area when circular; consequently, the annular film between the thread and the wire must have a *minimum* area when the thread loop is circular.

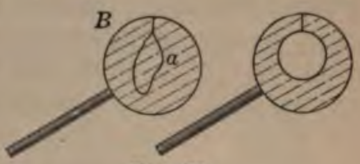

Fig. 79.

If a piece of sealing wax with sharp corners is heated until slightly plastic, the corners are rounded, due to surface tension of the wax; and in this rounding process the amount of surface is reduced. Glass and all metals behave in the same way when sufficiently heated. All metals when melted, indeed all substances when in the liquid state, exhibit surface tension. This property is utilized in making fine shot by dropping molten lead through the air from the shot tower. During the fall, the drops of molten lead cool in the spherical form *produced* by *surface tension*.

126. Numerical Value of Surface Tension.—The *Surface Tension* T of a liquid is numerically the force in dynes with which a surface layer of this liquid one centimeter in width resists being stretched. There are several methods of finding the surface tension, in all of which the force required to stretch a certain width of surface layer is first determined. This force, divided by the width of the surface layer stretched, gives the value of the surface tension.

The simplest method of finding the surface tension is the follow-

ing: An inverted U of fine wire 1/2 cm. in width is immersed in a soap solution (Fig. 80) and then suspended from a sensitive Jolly balance. (The Jolly balance is practically a very sensitive spring balance.) Since the film across the U has two surfaces, one toward and one away from the reader, it is evident that in raising the U, a surface layer 1 cm. in width must be stretched. Hence the reading of the Jolly balance (in dynes) immediately before the film breaks minus the reading after, gives the surface tension for the soap film in dynes per centimeter. For pure water, T is approximately 80 dynes per cm. Its value decreases due to rise in temperature, and also due to the presence of impurities (Sec. 127.) Observe that T is *numerically* the force required to keep *stretched* a surface layer having a width (counting both sides) of 1 cm.

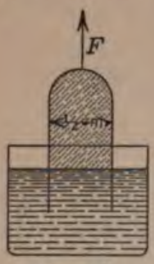

FIG. 80.

In raising the wire (Fig. 80) a distance of 1 cm., a force of 80 dynes (for pure water) must be exerted through a distance of 1 cm., that is, 80 ergs of work must be done. But 1 cm.^{2} of surface has been formed; showing that 80 ergs of work are required to form 1 cm.^{2} of surface. In other words, 80 ergs of work are required to cause enough molecules to move from position A to that of C (Fig. 76) to form 1 cm.^{2} of additional surface.

Observe that a soap bubble has an *outer* and an *inner* surface. Between these two surfaces is an exceedingly thin layer of soapy water. This soapy water, as it flows down between the two surfaces, forms the drop which hangs below the bubble and at the same time causes other portions of the bubble to become thinner and thinner until it finally bursts. The greater viscosity of soapy water, as compared with pure water, causes the downward flowing to be much slower than with pure water, and therefore causes a soap bubble to last much longer than a water bubble.

In blowing a soap bubble, work is done *upon* the film in increasing its area; on the other hand, if the film is permitted to contract by forcing air out through the pipestem, work is evidently done *by* the film. Barring friction, these two amounts of work must be equal.

In another method of determining surface tension, quite similar in principle to the one just given, a wire ring suspended in a horizontal position from a Jolly balance is lowered until it rests flat

upon the water, and is then raised, say 1/16 inch. In this position it would be found that a film *tube* of water, having the diameter of the ring and a length of 1/16 inch, connects the ring with the water and exerts upon the ring a downward pull. The reading of the Jolly balance just before this film breaks, minus the reading after (or F_1, say), gives this downward pull. The width of surface layer that is stretched is *twice* the circumference of the ring, or $4\pi r$. Note that a tube has an outer and an inner surface. Hence

$$4\pi r T = F_1$$

which may be solved for T.

127. Effect of Impurities on Surface Tension of Water.—Most substances when dissolved in water produce a marked decrease in its surface tension. For this reason, parings of camphor move rapidly over the surface of water if dropped upon it. Let A, B, and C, Fig. 81, be three pieces of camphor upon the surface of water. The piece A dissolves *more rapidly* from the point a than elsewhere, so that the surface tension on the end a is reduced more than on the opposite end, and the piece moves in the direction of the stronger pull, as indicated by the arrow. In the case of C, this same effect at c gives rise to a rotary motion, as shown; while B describes a curved path due to the same cause.

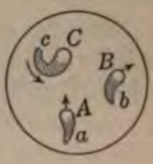

Fig. 81.

128. Capillarity.—If a glass A, Fig. 82, contains water, and another glass B contains mercury, it may easily be observed that most of the surface of each is perfectly flat, as shown, but that near the edge of the glass, the water surface curves upward, while the mercury surface curves downward. If the glass A were made slightly oily, the water would curve downward; while if B were replaced by an amalgamated zinc cup, the downward curvature of the mercury would disappear. Thus the form of the surface depends upon *both* the liquid and the containing vessel.

If a clean glass rod is dipped into water and then withdrawn, it is wet. This shows that the adhesion between glass and water exceeds the cohesion between the water particles. For the water that wets the glass rod must have been more strongly attracted by the glass than by the rest of the water, or it would not have come away with the rod. If the glass rod is slightly oily it will

not be wet after dipping it into the water. If a clean glass rod is dipped into mercury and then withdrawn, the fact that no mercury comes with it shows that the cohesion between mercury molecules exceeds the adhesion between mercury and glass molecules. It is, indeed, the *relative values* of cohesion and adhesion that determine surface curvature at edges. If the cohesion of the liquid molecules for each other just equals their adhesion for the substance of which the containing vessel is made, the surface will be flat from edge to edge. If *greater*, the curvature is *downward* (B, Fig. 82), while if *smaller*, it is *upward* (A, Fig. 82). Thus, in the latter case, the water at the edge rises above the general level, wetting the surface of the glass, simply because glass molecules attract water molecules more strongly than other water molecules do. This phenomenon is most marked in the case of small tubes (capillary tubes) and is therefore called *capillarity*.

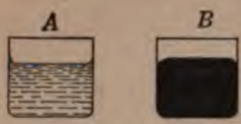

FIG. 82.

129. Capillary Rise in Tubes, Wicks, and Soil.—If clean glass tubes a and b (Fig. 83) are placed in the vessel of water A, and c and d in the vessel of mercury B, it will be found that the capillary *rise* in a and b, and the capillary *depression* in c and d is greater for the tube of smaller bore. Indeed, it will be shown in the next section, and it is easily observed experimentally with tubes of different bore, that a given liquid rises n times as high in a tube of $1/n$ times as large bore.

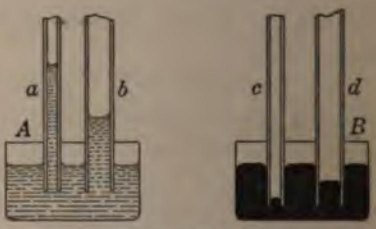

FIG. 83.

Any *porous* material produces a marked capillary rise with any liquid that wets it. There are numerous phenomena due to capillary action, many of which are of the greatest importance. If one corner of a lump of sugar, or clod of earth, touches the water surface, the entire lump or clod becomes moist. Due to capillarity, the wick of a lamp carries the oil to the flame where it is burned. If the substratum soil is moist, this moisture, during a dry time, is continually being carried upward to the roots of plants by the capillary action of the soil. Capillarity is probably an important factor, in connection with osmosis (Sec. 112), in the

transference of liquid plant food from the rootlets to the topmost parts of plants and trees.

Cultivating the soil to the depth of a *few inches* greatly reduces the amount of evaporation, and hence helps retain the moisture for the use of the plants. For, stirring the ground destroys, in a large measure, the continuity, and hence the capillary action, between the surface soil and the moist earth a few inches below. Consequently the surface soil dries more quickly, and the lower soil more slowly, than if the ground had not been stirred.

130. Determination of Surface Tension from Capillary Rise in Tubes.—In Fig. 84, B represents a capillary tube having a bore of radius r cm., and giving, when placed in water, a capillary rise of h cm. It may be *considered* to be the *upward pull* of the surface layer f that holds the column of water in the capillary tube above the level of the water in the vessel. The weight of this column is πr^2hdg (see Eq. 68, Sec. 113). The hemispherical surface layer that sustains this weight, however, is attached to the bore of the tube by its margin abc (as shown at A), so that the "width of surface" (see Sec. 126) that must support this weight is $2\pi r$, consequently

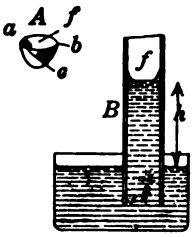

Fig. 84.

$$2\pi rT = \pi r^2hdg, \text{ or}$$

$$T = \tfrac{1}{2}rhdg \text{ dynes per cm.} \tag{71}$$

The above method is the one most frequently used for determining surface tension. It is usually necessary first to clean the tube with nitric acid or caustic soda, or both, and then carefully rinse before making the test.

PROBLEMS

1. What is the pressure at a depth of 2 mi. in the ocean?

2. A water tank has on one side a hole 10 cm. in diameter. What force will be required to hold a stopper in the hole if the upper edge of the hole is 4 meters below the water level?

3. What horizontal force will a lock gate 40 ft. in width exert on its supports if the depth of the water is 18 ft. above the gate, and 6 ft. below it?

4. Express a pressure of 15 lbs. per in.^{2} in dynes per cm.^{2}

5. The right arm of a U-tube, such as shown in Fig. 69, contains mercury only and the left arm some mercury upon which rests a column of brine 60

cm. in height. The mercury stands 5.2 cm. higher in the right arm than in the left. What is the density of the brine? Sketch first.

6. The weight of a stone in air is 60 gm., in water 38 gm., and in a certain oil 42 gm. What is the sp. gr. (*a*) of the stone? (*b*) of the oil?

7. Two tons increase in cargo makes a boat sink 1.2 in. deeper (in fresh water). What is the area of a horizontal section of the boat at the water line?

8. A marble slab (density 2.7 gm. per cm.^{3}) weighs 340 lbs. when immersed in fresh water. What is its volume?

9. How much lead must be attached to 20 gm. of cork to sink it in fresh water? Consult table of densities, Sec. 101.

10. What capillary rise should water give in a tube of (*a*) 1 mm. bore, (*b*) 2 mm. bore?

11. A wire ring of 5 cm. radius is rested flat on a water surface and is then raised. The pull required to raise it is 5 gm. more before the "film" breaks than it is after. What value does this give for the surface tension?

CHAPTER XI

PROPERTIES OF GASES AT REST

131. Brief Mention of Properties.—Gases have all the properties of liquids that are mentioned in Sec. 112 (to which section the reader is referred) except solvent action and surface tension. Gases have also properties not possessed by liquids, one of which is *Expansibility*.

Viscosity.—The viscosity of gases is much smaller than that of liquids, but it is not zero, nor is it even negligible. In order to force water to flow rapidly through a long level pipe, the pressure upon the water as it *enters* the pipe must be considerably greater than the pressure upon it as it flows *from* the pipe. This difference in pressure is known as Friction Head. It requires a pressure difference or pressure drop to force water through a level pipe because of the viscosity of water. To produce the same rate of flow through a given pipe would require a much *greater* pressure drop if the fluid used were molasses instead of water, and very much smaller drop if the fluid used were a gas. This difference is due to the fact that the viscosity of water is less than that of molasses and greater than that of the gas. The slight viscosity of illuminating gas necessitates a certain pressure drop to force the required flow through the city gas mains.

Usually in ascending a high tower there is a noticeable, steady increase in the velocity of the wind; which shows that the higher layers of air are moving more rapidly than those below (compare with the flowing of a river, Sec. 112). Indeed, just as in the case of the layers of water in the river, each layer experiences a forward drag due to the layer above it and a backward drag due to the layer below it, and therefore moves with an intermediate velocity. The lower layers are retarded by trees and other obstructions.

It is probable that the viscosity of gases should not be attributed to molecular friction but rather to molecular vibration (see Kinetic Theory of Matter, Sec. 99). Consider a rapidly moving stratum of air gliding past a slower moving stratum below. As molecules from the upper stratum, due to their vibratory motion,

wander into the lower stratum, they will, in general, accelerate it; whereas molecules passing from the lower stratum to the upper will, in general, retard the latter. Thus, any interchange of molecules between the two strata results in an equalization of the velocities of the portions of the strata near their surface of separation. Of course sliding (molecular) friction would produce this same result, but the fact that a rise in temperature causes the viscosity to *decrease* in liquids and *increase* in gases, points to a difference in its origin in the two cases. As a gas is heated, the vibrations of its molecules, according to the Kinetic Theory of Gases (Sec. 171), become more violent, thus augmenting the above molecular interchange between the two layers and thereby increasing the apparent friction between them.

Diffusion.—Diffusion is very much more rapid in the case of gases than with liquids, probably because of greater freedom of molecular vibration. Thus if some carbon dioxide (CO_2) is placed in the lower part of a vessel and some hydrogen (H) in the upper part, it will be found after leaving them for a moment that they are mixed due to diffusion; *i.e.*, there will be a large percentage of carbon dioxide in the upper portion of the vessel, notwithstanding the fact that it is more than twenty times as dense as hydrogen. Escaping coal gas rapidly diffuses so that it may soon be detected in any part of the room. An example of gas *Osmosis* has already been given (see Sec. 112).

Since gases have weight, they *produce pressure* for the same reason that liquids do (Sec. 113). Thus the air produces what is known as atmospheric pressure, which is about 15 lbs. per sq. in. In the case of illuminating gas, we have an example of *Transmission of Pressure* by gas from the gas plant to the gas jet. Another example is the transmission of pressure from the bicycle pump to the bicycle tire.

Elasticity.—Gases, like liquids, are perfectly elastic, *i.e.*, after being compressed they expand to exactly their original volume upon removal of the added pressure. Gases are very easily compressed as compared with liquids. Indeed, if the pressure upon a given quantity of gas is doubled or trebled, its volume is thereby reduced very closely to 1/2 or 1/3 its original volume, as the case may be. The fact that doubling the pressure on a certain quantity of gas halves the volume, or, in general, increasing the pressure n-fold reduces the volume to $1/n$ the original volume provided the *temperature* is *constant*, is known as *Boyle's*

Law. This very important gas law will be further considered in Sec. 139. It may be stated that Boyle's law does not apply *rigidly* to any gas, but it does apply *closely* to many gases, and through wide ranges of pressure.

Expansibility.—Gases possess a peculiar property not possessed by solids or liquids, namely, that of indefinite expansibility (Sec. 98). A given mass of any gas may have any volume, depending upon the pressure (and also the temperature) to which it is subjected. If the pressure is reduced to 1/10 its original value the volume expands 10-fold, and so on. A mass of gas, however small, always (and instantly) expands until it entirely fills the enclosing vessel.

The expansibility and also the compressibility of a gas may be readily shown by the use of the apparatus sketched in Fig. 85. A is a circular brass plate which is perfectly flat and smooth on its upper surface. B is a glass bell jar turned open end down against A. The lower edge D of B is carefully ground to fit accurately against the upper surface of A, over which some vaseline is spread. A and B so arranged constitute what is called a *receiver*. The receiver forms an air-tight enclosure in which is placed a bottle C, across the mouth of which is secured a thin sheet of rubber a, thus enclosing some air at ordinary atmospheric pressure.

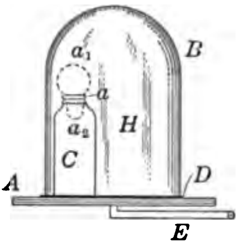

FIG. 85.

By means of the pipe E leading to an air pump, it is possible to withdraw the air from the space H within the receiver, or to force air into the space H. In the former case the air pressure in H is reduced so as to be less than one atmosphere, and the thin membrane of rubber stretches out into a balloon-like form a_1; while in the latter case, that is, when the air in H is compressed, this increased pressure, being greater than the pressure of the air confined in C, causes the membrane to assume the form a_2. The process by which the air pump is able to withdraw from H a portion of the air, also depends upon the property of expansibility. A reduction of pressure is produced in the pump, whereupon the air in H expands and rushes out at E. (This process will be further considered in Secs. 145 and 147.)

Gas Pressure and the Kinetic Theory.—According to the Kinetic Theory of Gases (Sec. 171), the pressure which a gas

exerts against the walls of the enclosing vessel is due to the bombardment of these walls by the gas molecules in their to-and-fro motion. The fact that the *ratio* of the *densities* of any two gases, *e.g.*, carbon dioxide and hydrogen, when subjected to the same pressure and temperature, is the same as the *ratio* of their *molecular weights*, shows that *a certain volume of hydrogen contains the same number of molecules as does the same volume of carbon dioxide or any other gas under like conditions as to pressure and temperature.* This is known as *Avogadro's Law.* It will be recalled that momentum change is equal to the impulse required to produce it (Eq. 19, Sec. 45). Consequently, since the hydrogen molecule is 1/16 as heavy as the oxygen molecule, it will need to have 4 times as great velocity as the oxygen molecule to produce an equal contribution toward the pressure. For each impulse of the hydrogen molecule would then be 1/4 as great as those of the oxygen molecule, but, because of the greater velocity of the former, these impulses would occur 4 times as often.

Knowing the density of the gas, it is comparatively easy to compute the molecular velocity required to produce the observed pressure. The *average velocity* of the hydrogen molecule at 0° C. is, on the basis of this theory, slightly more than 1 mi. per sec., while that of the oxygen molecule is 1/4 as great, as already explained.

The very rapid diffusion of hydrogen as compared with other gases would be a natural consequence of its greater velocity, and therefore substantiates the kinetic theory. The observed increase in pressure resulting from heating confined gases is attributed to an increase in the average velocity of its molecules with temperature rise. The kinetic theory of gas pressure affords a very simple explanation of Boyles' law (close of Sec. 139).

132. The Earth's Atmosphere.—Because of the importance and abundance of the mixture of gases known as *air*, the remainder of the chapter will be devoted largely to the study of it. It may be remarked that most of the gases are very much like air with respect to the properties here discussed.

The term "atmosphere" is applied to the body of air that surrounds the earth. Dry air consists mainly of the gases nitrogen and oxygen—about 76 per cent. of the former and 23 per cent. of the latter, by weight. The remaining 1 per cent. is principally argon. In addition to these gases there are traces of other gases, impor-

tant among which are carbon dioxide (CO_2) and water vapor. The amount of carbon dioxide in the air may vary from 1 part in 3000 outdoors (not in large cities), to 10 or 15 times this amount in crowded rooms. The oxygen of the air in the lungs (see Osmosis, Sec. 112) is partially exchanged for carbon dioxide and other impurities of the blood; as a result the exhaled air contains 4 or 5 per cent. carbon dioxide. If the breath is held for an instant and then carefully and slowly exhaled below the burner of a lamp (the hands being held in such a position as to exclude other air from the burner), the flame is quickly extinguished. The air in this case does not have enough oxygen to support combustion. Through repeated inhalation, the air in crowded, poorly ventilated rooms becomes vitiated by carbon dioxide. Carbon dioxide escapes from fissures in the earth and forms the deadly "choke damp" of mines. It also results from the explosion of "fire damp," or marsh gas (CH_4), as it is known to the chemist. If a candle when carefully lowered into a shaft is extinguished upon reaching the bottom, the presence of choke damp is indicated.

In nature, even in deserts, air never occurs dry. The amount of *water vapor* in the air varies greatly, sometimes running as high as 1/2 oz. per cubic yard (about 1.5 per cent.) in hot, sultry weather. As moist air is chilled, its ability to retain water vapor decreases rapidly and precipitation (Sec. 221) occurs. Consequently during extremely cold weather the air is very dry.

133. Height of the Atmosphere.—As meteors falling toward the earth strike the earth's atmosphere, the heat developed by them through air friction as they rush through the upper strata of rarefied air causes them to become quite hot, so that they shine for an instant. Suppose that one is seen at the same instant by two observers 40 or 50 miles apart. The meteor will appear to be in a *different direction* from one observer than from the other. This makes possible the calculation of the height of the point at which the meteor began to glow. But it could not glow before striking the earth's atmosphere; hence the earth's atmosphere extends to *at least* that height.

The duration of twilight after sunset also enables the calculation of the height of the atmosphere. Fine dust particles floating in the upper regions of the air are, of course, flooded with sunlight for a considerable time after sunset. The general glow from these particles constitutes twilight. If an observer at A (Fig. 85a) looking in the direction AX observes the last trace of

twilight when it is sunset at B, then the intersection X of the tangents at A and B is the highest point at which there are enough dust particles to give appreciable twilight effect.

Knowing the angle θ and the radius of the earth, the height of X above the earth is readily found. Since twilight lasts until the sun is 15 or 20 degrees below the horizon, we see that θ is 15 or 20 degrees. If θ is 18°, X is about 50 miles above the earth. Extremely rare air, almost free from dust particles, doubtless extends far above this height. Estimates of the height of the atmosphere range from 50 to 200 miles.

The upper strata of air are very rare and the lower strata comparatively dense due to compression caused by the weight of the air above; so that upon a mountain 3.5 miles high about half of the weight of the atmosphere is above and half below. The entire region above 7 miles contains only 1/4 of the earth's atmosphere.

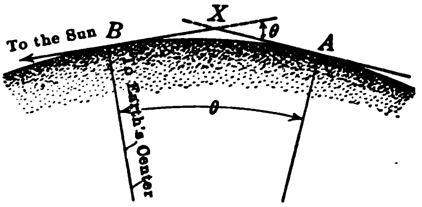

FIG. 85a.

134. Buoyant Effect, Archimedes' Principle, Lifting Capacity of Balloons.—Since air has weight, it produces a certain *buoyant effect* just as liquids do, but since it is about 1/800 as dense as water, the buoyant effect is only 1/800 as great. That air has weight may easily be shown by weighing a vessel, *e.g.*, a brass globe, first with air in it, and then weighing it again after the air has been partially pumped out of it by means of an air pump. The difference in weight is the weight of the air withdrawn. Galileo (1564–1642) weighed a glass globe when filled with air at atmospheric pressure, and again after forcing air into it. The observed increase in weight he rightly attributed to the additional air forced in.

Archimedes' Principle (Sec. 118) applies to gases as well as to liquids; therefore any body weighed in *air* loses weight equal to the weight of the *air* displaced by the body. Thus a cubic yard

of stone, or any other material, weighs about two pounds less in air than it would in a *Vacuum*, *i.e.*, in a space from which all air has been removed. The buoyant force exerted by the air upon a 150-lb. man is about 3/16 lb., *i.e.*, $1/800 \times 150$ lb.; since his body has about the same density as water. Observe that he would lose practically his *entire* weight if immersed in water; hence, since air is about 1/800 as dense as water, he loses 1/800 of his weight by being immersed in air.

The *lifting capacity* of a balloon, if it were not for the weight of the balloon itself and the contained gas, would be the weight of the air displaced, or approximately 2 lbs. for each cubic yard of the balloon's volume. If a balloon is filled with a light gas, *e.g.*, with hydrogen, its lifting capacity is much more than if filled with a heavier gas. The car or basket attached to a balloon contains ballast, which may be thrown overboard when the aeronaut wishes to rise higher. When he wishes to descend he permits some of the gas to escape from the balloon, thereby decreasing the volume and hence the weight of the air displaced.

135. Pressure of the Atmosphere.—Since the air has weight, the atmosphere must inevitably exert pressure upon all bodies with which it comes in contact. This pressure at sea level is closely 14.7 lbs. per sq. in., and at an altitude of 3.5 miles, about half of this value. Ordinarily the atmospheric pressure is not observable. It seems hard to believe that the human body withstands a pressure of about 15 lbs. on *every square inch* of surface, which amounts to several tons of force upon the entire body, without its even being perceptible. It is certain, however, that such is the case. We may note in this connection that the cell walls in the tissues of the body do not have to sustain this pressure, since the cells are filled with material at this same pressure. Thus, the atmospheric pressure of about 15 lbs. per sq. in. has no tendency to crush the lung cells when they are filled with air at this same pressure. Sudden changes in pressure, however, such as accompany rapid ascent or descent in a balloon, or in a diving bell, produce great discomfort.

The pressure exerted by water at a depth of about 34 ft. is one atmosphere (Sec. 136), so that a diver 34 ft. below the surface of a lake experiences a pressure of 2 "atmospheres," one atmosphere due to the air, and one due to the water. Divers can work more than 100 ft. beneath the surface of water, and must then experience a pressure of 4 or 5 atmospheres, *i.e.*, 60 or 75 lb. per sq.

in. The air which the diver breathes must, under these circumstances, be also under this same high pressure.

The pressure of the atmosphere acts in a direction which is at all points perpendicular to the surface of a body immersed in it. Compare the similar behavior of liquids (Sec. 114). That the atmospheric pressure may be exerted vertically upward, and that it may be made to lift a heavy weight, is forcibly shown by the following experiment.

A cylinder A, having a tight-fitting piston P to which is attached the weight W, is supported as shown (Fig. 86). If, by means of an air pump connected to the tube C, the air is partly withdrawn from the space B, it will be found that P will rise even if W is very heavy. If it were possible to remove all of the air from B, producing in the cylinder a perfect vacuum, the pressure within the cylinder, and hence the downward pressure on P would be zero. The upward pressure upon P, being atmospheric pressure or about 14.7 lbs. per sq. in., would enable it to lift 147 lbs., provided it had an area of 10 sq. in.

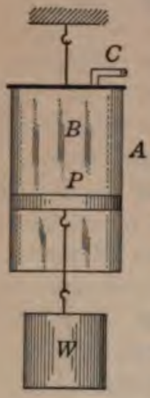

FIG. 86.

If only part of the air is withdrawn from B, so that the pressure within the cylinder is say 5 lbs. per sq. in., P would then exert a lifting force of 14.7 minus 5, or only 9.7 lbs. for each square inch of its surface. The pressure of the atmosphere cannot be computed by use of the formula $p = hdg$; because the height is uncertain, and also because the density d varies, being much less at high altitudes. The pressure is very easily obtained, however, by means of the barometer described in the next section.

136. The Mercury Barometer.—There are several different kinds of barometers. The simplest, and also the most accurate form is shown in Fig. 87. Various devices found in the practical instrument for making adjustments, and for determining very accurately the height of the mercury column (vernier attachment), are omitted in the sketch for the sake of simplicity in showing the essentials and in explaining the principle involved.

A glass tube A, about 1/3 in. in diameter and 3 ft. in length, and closed at the end a, is filled with mercury, and then, a stopper being held against the open end to prevent any mercury from escaping, it is inverted and placed open end down in a vessel

of mercury B, as shown. Upon removing the stopper, it might be expected that the mercury would run out until it stood at the same height inside and outside the tube. Indeed it would do this if there were at a the slightest aperture to admit the air to the upper portion of the tube, for then the pressure inside and outside the tube would be exactly the same, namely, atmospheric pressure. If a is perfectly air-tight, it will be found that some mercury runs out of the tube until the upper surface sinks to a point c. The height h of the mercury column c to b, is called the *Barometric Height*, and is usually about 30 in. near sea level. Evidently the space a to c contains no air nor anything else. Such a space is called a *Vacuum*. The downward pressure on the surface of the mercury at c is then zero.

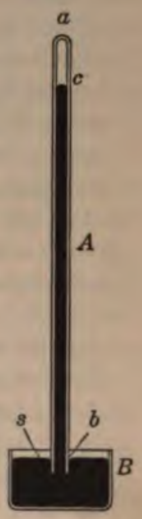

FIG. 87.

This experiment was first performed in 1643 by Torricelli (1608–1647) and is known as *Torricelli's experiment*. A few years later the French physicist Pascal (1623–1662) had the experiment performed on a mountain, and found, as he had anticipated, that the column bc was shorter there than at lower altitudes.

Consider the horizontal layer of mercury particles b within the tube and on the same level as the surface s outside the tube. The *downward* pressure on this layer is hdg in which h is the height of the column bc, and d is the density of mercury (13.596 gm. per cm.^{3}). But the *upward* pressure on this layer b must have this same value, since the layer is in equilibrium. The only cause for this upward pressure, however, is the pressure of the atmosphere upon the surfaces of the mercury, which pressure is transmitted by the mercury to the inside of the tube. Hence the *pressure* of the *atmosphere* is equal to the *pressure* exerted by the *mercury* column, or hdg. The barometric height varies greatly with change of altitude and also considerably with change of weather. Standard atmospheric pressure supports a column of mercury 76 cm. in height, at latitude 45° and at sea level ($g=980.6$); hence

Standard Atmos. Pr. $= hdg = 76 \times 13.596 \times 980.6 = 1,000,000$ dynes per sq. cm. (approx.).

This is approximately 14.7 lbs. per sq. in.

Quite commonly the pressure of the atmosphere is expressed

simply in terms of the height of the barometric column which it will support, as "29.8 in. of mercury," "74 cm. of mercury." At sea level the pressure of the atmosphere is usually about 30 in. of mercury; at an altitude of 3.5 mi., about 15 in. of mercury; while aeronauts at still higher altitudes have observed as low a barometric height as 9 in.

Unless great care is taken in filling the tube (Fig. 87), it will be found that some air will be mixed with the mercury, and that therefore the space from *a* to *c*, instead of containing a vacuum, will contain *some air* at a slight pressure. This counter pressure will cause the mercury column to be somewhat shorter than it otherwise would be, and the barometer will accordingly indicate too low a pressure. If the mercury is boiled in the tube before inverting, the air will be largely driven out and the error from this source will be greatly reduced. It will be evident that this slight counter pressure of the entrapped air, in case a trace of air is left in the space *ac*, plus *hdg* for the column of mercury *bc*, gives the total downward pressure at *b*. But this total pressure must equal the upward pressure at *b*, due to the atmosphere as shown. Hence *hdg* will give a value for the atmospheric pressure, which is too small by exactly the amount of pressure on *c*, due to the entrapped air.

Since water is only 1/13.6 times as dense as mercury, it follows that atmospheric pressure will support 13.6 times as long a column of water as of mercury, or about 13.6×30 in., which is approximately 34 ft. Accordingly, the pressure required to force water through pipes a vertical height of 340 ft. is approximately 10 atmospheres, or 150 lbs. per sq. in., in addition to the pressure required to overcome friction in the pipes.

137. The Aneroid Barometer.—The *Aneroid Barometer* consists of an air-tight metal box of circular form having a corrugated top and containing rarefied air. As the pressure of the atmosphere increases, the center of this top is forced inward, and when the pressure decreases the center moves outward, due to the elasticity of the metal. This motion of the center is very slight but is magnified by a system of levers connecting it with a pointer that moves over the dial of the instrument. The position of this pointer upon the dial at a time when the mercury column of a simple barometer is 75 cm. high is marked 75, and so on for other points. This type of barometer is light, portable, and easily read.

138. Uses of the Barometer.—Near a storm center the atmospheric pressure is low (Sec. 225), consequently a falling barometer indicates an approaching storm. Knowing the barometric readings at a great number of stations, the *Weather Bureau* can locate the storm centers and predict their probable positions a few days in advance. Thus this Bureau is able to furnish information which is especially valuable to those engaged in shipping.

Due to the capricious character of the weather, these predictions are not always fulfilled. Although the forcasting of the weather a year in advance is absolute nonsense, there are many who have more or less faith in such forcasts. Of course one is fairly safe in predicting "cold rains" for March, "hot and dry" for August, etc., but to fix a month or a year in advance the date of a storm from the study of the stars (which certainly have nothing to do with the weather), is surely out of place in this century.

As stated in Sec. 136, the barometric height decreases as the altitude increases. Near sea level the rate of this decrease is about 0.1 in. for each 80 ft. of ascent. At higher altitudes this decrease is not so rapid because of the lesser density of air in those regions. A formula has been developed, by the use of which the mountain climber can *determine his altitude* fairly well from the readings of his barometer. An "altitude scale" is engraved on many aneroid barometers, by means of which the altitude may be roughly approximated.

139. Boyle's Law.—The volume of a *given mass* of gas, multiplied by the pressure to which it is subjected, is found to be nearly constant if the temperature remains unchanged. This is known as Boyle's Law and may be written

$$pV \text{ (temp. constant)} = K \tag{72}$$

This important law was discovered by Robert Boyle (1627–1691) and published in England in 1662. Fourteen years later it was *rediscovered* by the French physicist Marriotte. This illustrates the slow spread of scientific knowledge in those days. In France it is called *Marriotte's Law.*

From the equation it may be seen that to cause a certain volume of gas to shrink to $1/n$ its original volume will require the pressure to be increased n-fold, provided that the temperature remains constant. The equation also shows that if we permit a certain mass of confined gas to expand to, say, 10 times its

original volume, then the new pressure will be 1/10 as great as the original pressure. By *original pressure* and *volume* we mean the pressure and volume before expansion occurred. As already stated, Boyle's Law applies closely to many gases, rigidly to none.

To illustrate Boyle's Law by a problem, let P (Fig. 88) be an air-tight, frictionless piston of, say, 10 sq. in. surface and of negligible weight, enclosing in vessel A a quantity of air at atmospheric pressure, say 15 lbs. per sq. in. Let it be required to find how heavy a weight must be placed upon P to force it down to position P_1, thereby compressing the entrapped air to 1/3 its original volume.

From Eq. 72, we see that the pressure of the entrapped air in the latter case will be increased 3-fold and hence will exert upon P when at P_1, an upward pressure of 45 lbs. per sq. in. The outside atmosphere exerts a pressure of 15 lbs. per sq. in. on P; consequently the remaining 30 lbs. pressure required to hold P down must be furnished by the added weight. A pressure of 30 lbs. per sq. in. over a piston of 10 sq. in. surface amounts to 300 lbs. force; hence the added weight required is 300 lbs.

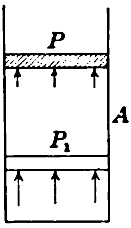

FIG. 88.

We may explain Boyle's Law in full accord with the Kinetic Theory of gas pressure (Sec. 131). For when the volume of the air in the vessel represented in Fig. 88 is reduced to 1/3 its original volume, the molecules, if they continue to travel at the same velocity, would strike the piston three times as frequently, and experience each time the same amount of momentum change, as in the original condition. They would therefore produce three times as great pressure against the piston as they did in the original condition, which, it will be noted, accords with experimental results.

140. Boyle's Law Tube, Isothermals of a Gas.—A bent glass tube A (Fig. 89), having the short arm closed at e, and the long arm open and terminating in a small funnel at b, is very convenient to use in the verification of Boyle's Law. The method of performing the experiment is given below.

A few drops of mercury are introduced into the tube and adjusted until the mercury level c in the long arm is at the same height as the mercury level d in the short arm. As more mercury is poured into the tube at b, the pressure on the air enclosed

in *de* is increased, which causes a proportional decrease in its volume.

If now we plot these values of the pressure as ordinates (Sec. 41) and the corresponding values of the volume as abscissæ, we obtain, provided the room temperature is 20°, the curve marked 20° in Fig. 90. This is called the *Isothermal* for air at 20° C.

Method in Detail.—If the barometer reads 75 cm., that is, if the atmospheric pressure is 75 cm. of mercury, then, since c and d are at the same level, it follows that the pressure on the entrapped air is 75 cm. of mercury. If the tube has 1 sq. cm. cross

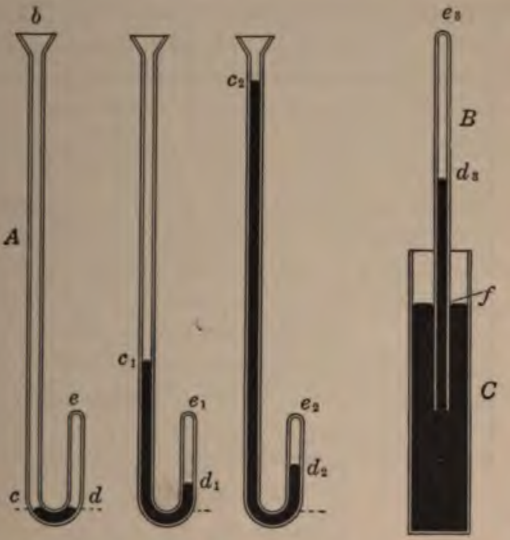

Fig. 89.

section and *de* is 20 cm., then the corresponding volume of the air is 20 cm.³ Accordingly the point marked A on the curve (ordinate 75, abscissa 20) represents the *initial* state of the entrapped air. Next, mercury is poured into b until it stands at c_1 and d_1 in the tubes. If the *vertical* distance from c_1 to d_1 is 25 cm., the pressure upon the air in d_1e_1 will be 25 cm. more than atmospheric pressure, or a total of 100 cm. Since this is 4/3 of the initial pressure, the corresponding volume *should be* 3/4 of the initial volume, or 15 cm.³ Measurement will show that d_1e_1 *is* 15 cm.³ Hence point B (ordinate 100 and abscissa 15)

represents the new state of the entrapped air as regards its pressure and volume. When still more mercury is poured in, the mercury stands at, say, c_2 and d_2, the vertical distance c_2d_2 being 75 cm. The pressure upon the entrapped air (d_2e_2) is now this 75 cm. plus atmospheric pressure, or a total of 150 cm. Since this pressure is *twice* the initial pressure, the corresponding volume is, as we should expect, *one-half* the original volume, or 10 cm.³ Hence the point on the curve marked C (ordinate 150, abscissa 10) represents this, the third state of the entrapped air.

To obtain smaller pressures than one atmosphere, a different form of apparatus shown at the right in Fig. 89 is more conven-

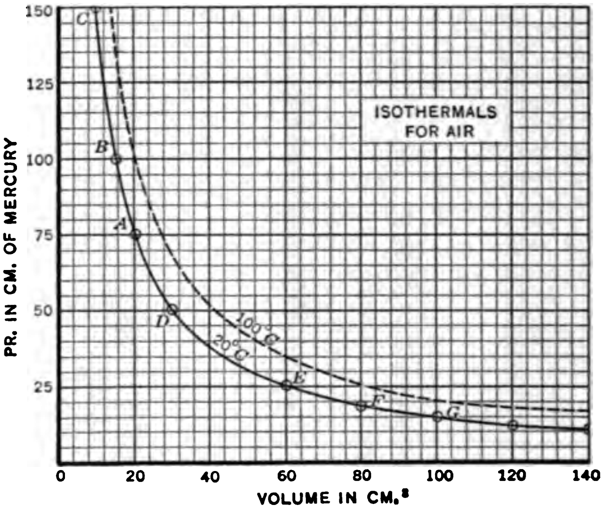

Fig. 90.

ient. A small tube B of, say 1 sq. cm. cross section, is filled with mercury to within 20 cm. of the top and then stoppered and inverted in a large tube C which is nearly filled with mercury. Upon removing the stopper and pressing the tube down until the mercury in both tubes stands at the same height, it will be seen that the volume of the entrapped air (which is now at atmospheric pressure) is 20 cm.³ If, now, tube B is raised until the mercury within it stands at d_3, and if d_3f is 25 cm., then the pressure upon the entrapped air is 50 cm.; for this pressure plus the pressure of the column of mercury d_3f must balance the atmospheric pressure of 75 cm. Since this pressure (50 cm.) is 2/3 of the initial pres-

sure, the corresponding volume in accordance with Boyle's Law must be 3/2 of the original volume, or 30 cm.^{3} Measurement will show that d_3e_3 is 30 cm. Hence the point on the curve marked D (ordinate 50, abscissa 30) represents this state of the air. If tube B is raised still farther until the mercury within it stands 50 cm. higher than in C, then the pressure of the entrapped air is 25 cm., or 1/3 of the initial pressure, and its volume will be found to be three times the initial, or 60 cm.^{3} Hence point E (ordinate 25, abscissa 60) represents this, the fifth state of the entrapped air. In the same way points F, G, etc., are determined. Drawing a smooth curve through these points A, B, C, etc., gives the *isothermal* for air at 20° C. When we take up the study of heat we will readily see that the 100° isothermal would be drawn about as shown (see dotted curve).

Observe that the three rectangles, A–75–0–20, D–50–0–30, and E–25–0–60 all have the same area and that this area represents the product of the pressure—75, 50, or 25 as the case may be, and the corresponding volumes of the entrapped air for the three different states which are represented respectively by the points A, D, and E on the curve. Thus the curve verifies Boyle's Law as expressed in Eq. 72, and shows that the constant K in this equation is, *for this particular amount of gas*, 1500; for 75×20, 50×30, and 25×60, each gives 1500.

141. The Manometers and the Bourdon Gage.—Manometers are of two kinds, the *Open Tube Manometer*, usually used

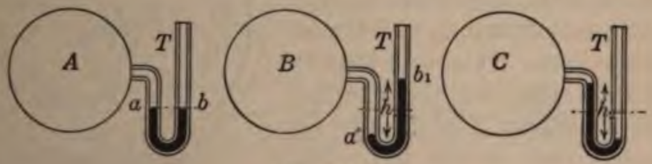

FIG. 91.

for measuring small differences in pressure, and the *Closed Tube Manometer* which may be used to measure the total pressure to which a gas or a liquid is subjected.

The *Open Tube Manometer* (Fig. 91) consists of a U-shaped glass tube T, open at both ends and containing some liquid, frequently mercury. If, when the manometer is connected with the vessel A containing some gas, it is found that the mercury stands at the same height in both arms, namely, at a and b, then

the pressure of this *gas* which acts upon a, must be equal to the pressure of the *atmosphere* which acts upon b. If the mercury meniscus b_1 is higher than a_1 by a distance h_1 cm., then the pressure in B is 1 atmosphere + h_1dg dynes per cm.^{2}, in which d is the density of the mercury. The pressure of the gas in C is evidently less than one atmosphere by the amount h_2dg. If very small differences in pressure are to be measured it is best to employ a light liquid for the manometer.

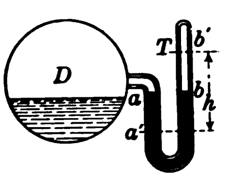

FIG. 92.

The Closed Tube Manometer (Fig. 92) may be used for measuring high pressures, such as the pressure of steam in steam boilers, city water pressure, etc. Let D represent a steam boiler containing some water, and T, an attached closed tube manometer. If the mercury stands at the same height in both arms a and b when valves leading from D to the outside air are open, it shows that the entrapped air in the manometer is at *one atmosphere* pressure. If, upon closing these valves and heating the water in D, the pressure of the steam developed forces the mercury down to a' in the left arm and up to b' in the right arm, thereby reducing the volume of the entrapped air to 1/3 *its original volume*, it follows from Boyle's Law that the pressure on it is increased 3-fold and is therefore 3 *atmospheres*. The steam in D is then at 3 atmospheres pressure. It is really slightly more than this, for the mercury stands a distance h higher in the right arm than in the left. The correction is clearly hdg. That is, the pressure upon the enclosed air above b' would be, under these circumstances, *exactly* 3 atmospheres while the steam pressure in the boiler would be 3 atmospheres plus the pressure hdg due to the mercury column of height h.

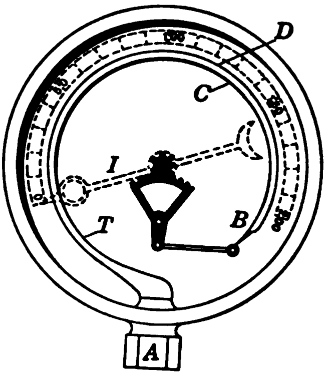

FIG. 93.

The Bourdon Gage.—The essentials of the Bourdon gage, which is widely used for the measurement of steam pressure and water pressure, are shown in Fig. 93. The metal tube T, which is closed at B, is of oval cross section, CD being the smaller diameter.

If A is connected to a steam boiler, the pressure of the steam causes the cross section of the tube to become more nearly circular, *i.e.*, it causes the smaller diameter CD to increase. Obviously, pushing the sides C and D of the tube farther apart will cause the tube to straighten slightly, thereby moving B to the right and causing the index I to move over the scale as indicated. By properly calibrating the gage, it will read directly the steam pressure in pounds per square inch. Most steam gages are of this type. The same device may be used to measure the pressure of water, or the pressure of any gas.

The Vacuum Manometer or *Vacuum Gage.*—If the space above b in tube T (Fig. 92) were a perfect vacuum (*e.g.*, if that arm of the tube were first entirely filled with mercury), and if nearly all of the air were pumped out of D, then T would be a "vacuum" gage. If, under these circumstances, meniscus b stood 0.05 mm. higher than a, it would show that the pressure of the remnant of the air in D was only equal to that produced by a column of mercury 0.05 mm. in height. If the "vacuum" in D were perfect, then a and b would stand at the same height.

PROBLEMS

1. What is the pressure of the atmosphere (in dynes per cm.^{2}) when the mercury barometer reads 74.2 cm.?

2. What is the pressure of the atmosphere (in lbs. per in.^{2}) when the barometer reads 28.2 in.?

3. If, in Fig. 89, $d_1f = 30$ cm. and the barometer reads 74 cm., what is the pressure on the entrapped air in centimeters of mercury? In atmospheres?

4. An aneroid barometer, at a certain time, reads 29.9 in. at sea level and 29.35 in. on a nearby hill. What is the approximate altitude of the hill? (Sec. 138.)

5. The liquid (oil of density 0.9 gm. per cm.^{3}) in an open tube manometer stands 4 cm. higher in the arm which is exposed to the confined gas than it does in the other arm. What is the pressure exerted by the gas? The barometric reading is 29 in.

6. A closed tube manometer contains an entrapped air column 8 cm. in length when exposed to atmospheric pressure, and 3.2 cm. in length when connected to an air pressure system. What is the pressure of the system? The mercury stood at the same level in both arms in the *first* test.

7. If a 1000-lb. weight is rested upon P (Fig. 88), what will be the new volume of the enclosed air in terms of the old?

8. A certain balloon has a volume equal to that of a sphere of 15-ft. radius. What weight, including its own, will it lift when the density of the air is (a) 2 lbs. per cubic yard? (b) 0.0011 gm. per cm.^{3}? Express the weight in pounds in both cases.

9. Plot a curve similar to that shown in Fig. 90 and explain how it is obtained.

CHAPTER XII

PROPERTIES OF FLUIDS IN MOTION

142. General Discussion.—The *steady* flow of a fluid, either a liquid or a gas, at a uniform velocity through a level pipe from one point to another, is always due to a difference in pressure maintained between the two points (friction head, see footnote). This difference in pressure multiplied by the cross section of the pipe gives the total force which pushes the column of fluid through the pipe. Since the velocity of this column is neither increasing nor decreasing, there is no accelerating force, and the above *pushing* force must be just equal to the friction force exerted upon the column by the pipe. If at any point the fluid is increasing in velocity, an accelerating force F must be present, and part of the pressure difference (velocity head)[1] is used in producing this accelerating force. F is equal to the mass M of the liquid being accelerated, multiplied by its acceleration a (Sec. 25, $F = Ma$).

Just as the canal boat (Sec. 43), by virtue of its inertia, develops a forward driving inertia *force* ($F = Ma$) which pushes it onto the

[1] *Head of Water.*—In hydraulics, the pressure at a point, or the difference in pressure between two points, is called *pressure head,* and is measured in terms of the height (in feet) of the column of water required to produce such pressure, or pressure difference. To illustrate, suppose that in certain hydraulic mining operations, the supply reservoir is 600 ft. above the hose nozzle, and that the velocity of the water as it leaves the nozzle is 100 ft. per sec. Since a body must fall about 150 ft. to acquire a velocity of 100 ft. per sec., the head required to impart this velocity to the water would be 150 ft. (see Sec. 143). Consequently the *Velocity Head* required is 150 ft. The remainder of the 600-ft. head, namely 450 ft., is used in overcoming friction in the conveying pipes and hose, and is called *Friction Head.* As the water from the reservoir enters the conveying pipes it must acquire velocity. As the water passes from the pipe into the much smaller hose, and again as it passes from the hose into the tapering nozzle, it must acquire additional velocity. Thus the total head of 600 ft. is equal to the sum of the velocity heads of the pipe, the hose, and the nozzle, in addition to the friction head for all three. If the size of the conveying pipe or hose changes abruptly (either increases or decreases) eddies will be formed which cause considerable friction and consequent loss of head. To reduce this loss, the pipe should flare as it enters the reservoir.

sand bar; so also a moving fluid (*e.g.*, water, steam, or air) exerts a driving *inertia force* ($F = Ma$) against any body that changes its velocity. It is this inertia force which drives the wind mill, the steam turbine, and the turbine water wheel, or any other water wheel which utilizes the velocity of the water.

A thorough understanding of the above principles and their applications gives one a fair elementary knowledge of the subject of *Hydraulics*. A discussion of Fig. 94 will aid in securing such an understanding.

Let B be a level water pipe communicating with the vertical pipes C, E, and F, and with the tank A. If B is closed at G, so that no water flows through it, the water will stand at the same level, say at a–d–e–f, in the tank and in the vertical pipes. If G is removed the water will at first flow out slowly, for it will

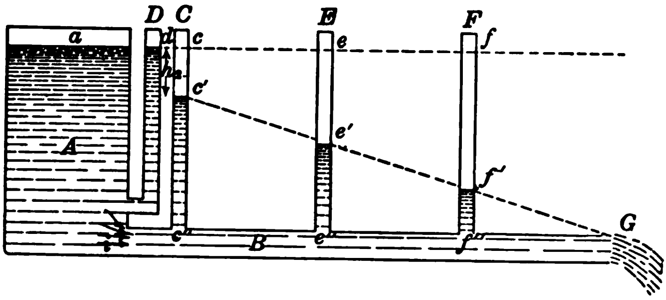

FIG. 94.

take the force at i, due to the tank pressure, a short interval of time to impart to all of the water in pipe B a high velocity. After a few seconds the water will be flowing rapidly and steadily at G, and the water in the vertical pipes will stand at the different levels c', d, e', and f'. Observe that c', e', f', and G all lie in the same straight line. This uniform pressure drop or friction head loss is due to the fact that the friction is the same in all parts of B. If the pipe B between c'' and e'' were rusty and rough, or smaller than elsewhere, the friction head between these two points would be greater than elsewhere, causing e' to be lower than shown. In such case c', e', and f' would not lie in a straight line.

Observe that removing G has not lowered the level in pipe D, but has produced a decided drop in C. This difference in level (h_2), corresponding to a difference in pressure of h_2dg, cannot be

due to friction head in the short distance ic''. This difference in pressure is *mainly due* to the pressure head (velocity head) required to accelerate the water as it passes from the tank, where it is almost without motion, to the pipe B, where it moves rapidly.

If the pipe B were nearly closed at G, the flow would be slow, and the friction head small, so that the water would stand nearly as high in C, E, and F as in the tank A. Just as the heavy flow from G lowers the water pressure at f'' and hence in pipe F, so, during a fire, when many streams of water are thrown from the same main, the heavy flow lowers the available pressure at the hose.

143. Gravity Flow of Liquids.—In the last section, where the flow of water in level pipes was discussed, it was shown that a pressure difference sufficient to overcome the friction of the water on the pipes is always necessary to maintain such flow. In the case of pipes which are not level, but have a slight slope, such as tile drains and sewer pipes, friction between the water and the pipe is overcome, not by difference in pressure, but by a component of the weight of the water itself. The weight of a car on a grade may be resolved into two components, one of which is parallel to the grade and therefore urges the car down the grade (Sec. 19). Likewise, the weight of the water in the tile drain may be resolved into two components, one of which is parallel to the drain and therefore urges the water along the drain. If the slope of the drain is increased, the component parallel to the drain becomes larger, and the flow becomes more rapid. The other component which is perpendicular to the drain does not interest us in the present discussion.

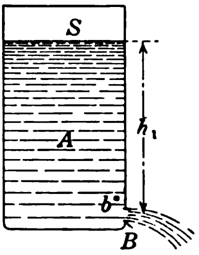

FIG. 95.

The flowing of the water in a river is maintained in the same way as in a tile drain. The bed of the river has a certain average slope down stream. The component of the weight of the water in the river which is parallel to the bed, constitutes the driving force that overcomes the friction on the shores and on the bottom. At points where the slope is great this force is great and "rapids" exist.

Velocity of Efflux, Torricelli's Theorem.—As the water in A (Fig. 95), of depth h_1, flows from the orifice B it acquires a velocity v, given by the equation $v = \sqrt{2gh_1}$.

Proof: As M pounds of water pass through orifice B, the water level in A is lowered slightly, and the potential energy of A is reduced by Mh_1 foot-pounds or Mgh_1 foot-poundals. The kinetic energy of the M lbs. of flowing water is $\frac{1}{2}Mv^2$ (Eqs. 50 and 51, Sec. 75). From the conservation of energy it follows that this kinetic energy must be equal to the potential energy lost by the tank; *i.e.*, $\frac{1}{2}Mv^2 = Mgh_1$, from which we have $v^2 = 2gh$ or $v = \sqrt{2gh_1}$. From Eq. 14, Sec. 34, we see that $\sqrt{2gh}$ is the velocity acquired by a body in falling from rest through a height h. By this proof, known as *Torricelli's theorem*, we have shown that the velocity of *free* efflux produced by a given head h is equal to the velocity of *free* fall through this same height h. If a pipe were connected to B of a length such as to require a friction head of $\frac{1}{3}h_1$ to maintain the flow in it, then the velocity head would be $\frac{2}{3}h_1$, and the velocity of flow in the pipe would be $\sqrt{2g \times \frac{2}{3}h_1}$ or that acquired by a body in falling a distance $\frac{2}{3}h_1$.

144. The Siphon.—The siphon, which is a U-shaped tube T (Fig. 96), may be used to withdraw water or other liquids from tanks, etc. If a siphon is filled with water and stoppered and then inverted and placed in a vessel of water A, as shown, it will be found that the water flows from A through T to B. There must be an unbalanced pressure that forces this water through T. This pressure may be readily found.

Imagine, for a moment, a thin film to be stretched across the bore of the tube at C. The pressure tending to force this film to the right, minus the pressure tending to force it to the left is evidently the unbalanced pressure which causes the flow in the actual case. The former pressure is the atmospheric pressure, frequently called B (from barometer) minus h_1dg, or $B - h_1dg$, while the latter is $B - h_2dg$. The unbalanced pressure is, therefore,

$$\text{Unbalanced pressure} = (B - h_1dg) - (B - h_2dg) = (h_2 - h_1)\ dg = h_3dg$$

From this equation we see that the difference of pressure is proportional to the difference in level (h_3) of surfaces S_1 and S_2. In addition to this difference in level, the factors that determine the rate of flow are the length of the tube, the smoothness and size of its bore, and the viscosity of the liquid.

Since atmospheric pressure cannot support a column of water which is more than 34 ft. in height (Sec. 136), it follows that h_1 (Fig. 96) must not exceed this height or the atmospheric pressur on S_1 will not force the water up to C, and the siphon will fail t

operate. In case mercury is the liquid used, h_1 must not be more than 29 or 30 inches. If made greater than this, a vacuum will be formed at C and no flow will take place. Since a partial vacuum is formed at C, the siphon walls must not easily collapse.

Observe that the water flows from point a to point b, both points being at the same pressure, namely, atmospheric pressure. In Sec. 143 it was shown that pressure difference is not the only thing which may maintain a steady flow, but that in sloping pipes a *component* of the weight of the liquid overcomes the friction resistance. In vertical pipes the *full weight* of the liquid maintains the flow. Hence, in the case of the siphon we may consider that it is the excess weight of the right column over that of the left which provides the force that overcomes the friction between the flowing column and the tube T.

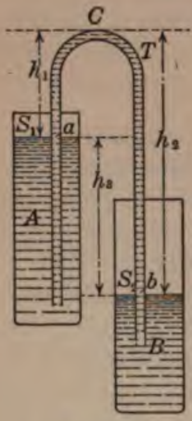

Fig. 96.

145. The Suction Pump.—The common "suction pump" used for cisterns and shallow wells, is shown in Fig. 97 in three stages of operation. The "*cylinder*" C is open at the top and closed at the bottom, except for a valve a which opens upward. Within C is a snug-fitting piston P, containing a valve b also opening upward. D is a pipe extending below the surface of the water. As P is forced downward by means of the piston rod R attached to he pump handle, valve a closes, and as soon as the air in E is sufficiently compressed it lifts the valve b and escapes (left sketch). As the piston rises again, b closes, and the remnant of air in E expands to fill the greater volume, thereby having its pressure reduced (according to Boyle's Law) below one atmosphere. The pressure of the air in D is, of course, one atmosphere. Hence the pressure above the valve a is less than the pressure below it, causing it to rise and admit some air into E from D. As air is thus withdrawn from D the pressure of the remaining air is reduced to below atmospheric pressure, consequently the water in the cistern, which is exposed to full atmospheric pressure, is forced up into the tube D (middle sketch, Fig. 97). Another stroke of the piston still further reduces the air pressure in E and D, and the water is forced higher, until it finally passes through valve a into the cylinder. As P descends, valve a closes, and the

water in the cylinder is forced through *b*, and finally, as *P* again rises (right sketch), it is forced out through the spout *d*.

Atmospheric pressure will support a column of water about 34 ft. in height (Sec. 136), provided the space above the column is a vacuum. Hence we see that the theoretical limiting vertical distance from the cylinder to the water in the cistern, or well, is

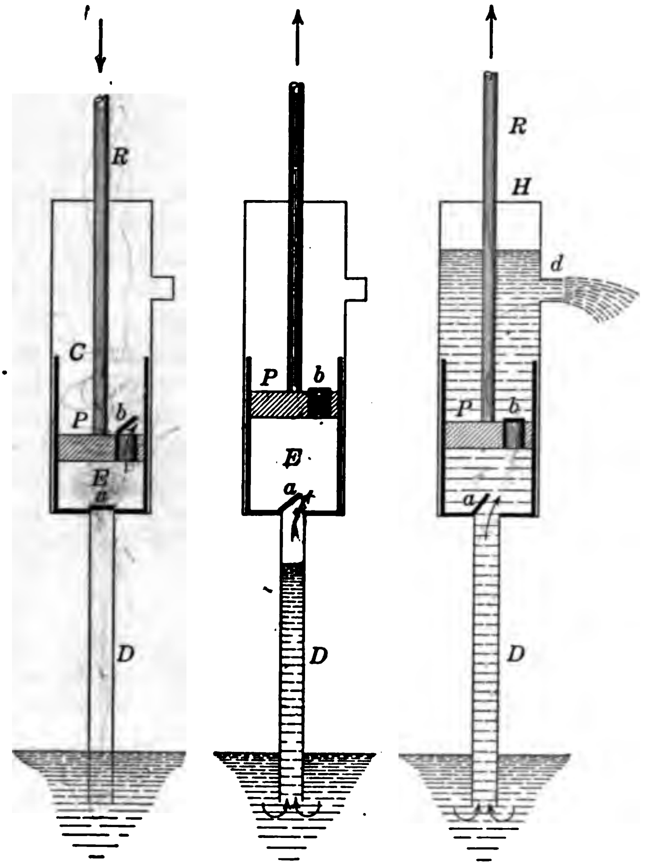

FIG. 97.

34 ft. Suppose this distance to be 40 ft. Then, even if a perfect vacuum could be produced in *E*, the water would still be 6 ft. below the cylinder. In practice, the cylinder should not be more than 20 or 25 ft. above the water. For this reason, pumps for deep wells have the cylinder near the bottom, the piston rod in some cases being several hundred feet in length.

146. The Force Pump.—The force pump is used when it is desired to pump water into a tank which is at a higher level than the pump. The pump described in the last section is sometimes provided with a tight-fitting top at *H* (right sketch, Fig. 97) having a hole just large enough to permit the piston rod *R* to pass through it. By connecting spout *d* with a hose, the pump may then be operated as a force pump.

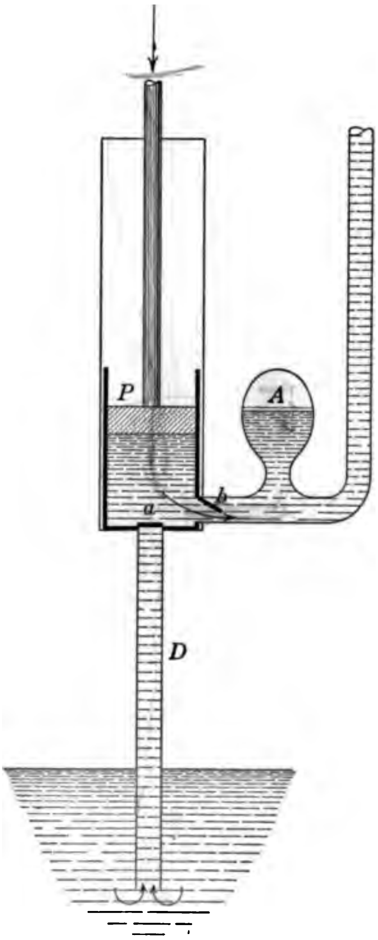

FIG. 98.

The other type of force pump (Fig. 98) "lifts" the water from the well on the upstroke, and forces it up to the tank on the downstroke, thus making it run more evenly, since both strokes are *working* strokes. In this type the piston has no valve. As the piston *P* rises, valve *b* closes and valve *a* opens, permitting water to enter the cylinder. As *P* descends, *a* closes and *b* opens, and the water is forced up into the tank. During the downstroke of *P*, some of the water rushes into the air chamber *A* and further compresses the enclosed air. During the upstroke (valve *b* being then closed) this air expands slightly and expels some water. Thus, by the use of the air chamber, the flow of water from the discharge pipe is made more nearly uniform. The fact that the descending piston may force some water into *A* instead of suddenly setting into motion the entire column of water in the vertical pipe, causes the pump to run more smoothly.

147. The Mechanical Air Pump.—The mechanical air pump operates in exactly the same way as the suction pump (Fig. 97).

In fact, when first started, the suction pump withdraws *air* from D, that is, it acts as an air pump. To withdraw the air from an inclosure (*e.g.*, from an incandescent lamp bulb), the tube D would be connected to the bulb instead of to the cistern. The process by which the air is withdrawn from the bulb is the same as that by which it is withdrawn from D (Sec. 145), and need not be redescribed here. As exhaustion proceeds, the air pressure in the bulb and in D becomes too feeble to raise the valves. Hence the practical air pump must differ from the suction water pump in that its valves are operated *mechanically*. The valves and piston must also fit much more accurately for the air pump than is required for the water pump.

The upper end of the cylinder of an air pump has a top in which is a small hole covered by the outlet valve. If a pipe leads from this valve to an enclosed vessel the air will be forced into the vessel. In such case tube D would simply be opened to the air, and the pump would then be called an *Air Compressor*. Such air compressors are used to furnish the compressed air for operating pneumatic drills, the air-brakes on trains, and for many other purposes. It will be observed that such an air pump, like practically all pumps (see Fig. 98), produces suction at the entrance and pressure at the exit.

Let us further consider the process of pumping air from a bulb connected to the tube D. Assuming *perfect* action of the piston and valve, and assuming that the volume of the cylinder is equal to the combined volume of the bulb and D, we see that the first stroke would reduce the pressure in bulb and D to 1/2 atmosphere. For as P rises to the top of the cylinder, the air in bulb and D expands to double its former volume, and hence the pressure, in accordance with Boyle's Law, decreases to 1/2 its former value. A second upstroke reduces the pressure in bulb and D to 1/4 atmosphere, a third to 1/8 atmosphere, a fourth to 1/16 atmosphere, etc. Observe that each stroke removes only 1/2 of the air then remaining in the bulb.

The Geryk Pump.—In the ordinary mechanical air pump there is a certain amount of unavoidable clearance between the piston and the end of the cylinder. The air which always remains in this clearance space at the end of the stroke, expands as the piston moves away, and produces a back pressure which finally prevents the further removal of air from the intake tube, and therefore lowers the efficiency of the pump. In the Geryk pump, air is eliminated from the clearance space by the use of a thin layer of oil both above and below the piston.

148. The Sprengel Mercury Pump.—The Sprengel pump exhausts

very slowly, but by its use a very much better vacuum may be obtained than with the ordinary mechanical air pump. It consists essentially of a vertical glass tube A (Fig. 99) about one meter in length and of rather small bore, terminating above in a funnel B into which mercury may be poured. A short distance below the funnel a side tube leads from the vertical tube to the vessel C to be exhausted. As the mercury drops, one after another, pass down through the vertical tube into the open dish below, each drop acts as a little piston and pushes ahead of it a small portion of air that has entered from the side tube. Thus any vessel connected with this side tube is exhausted.

Obviously, to obtain a good vacuum, the aggregate length of the little mercury pistons below the side tube must be greater than the barometric height, or the atmospheric pressure would prevent their descent. The funnel must always contain some mercury, or air will enter and destroy the vacuum. A valve at a is adjusted to permit but a slow flow of mercury, thereby causing the column to break into pistons.

149. The Windmill and the Electric Fan.— The common *Windmill* consists of a wheel whose axis lies in the direction of the wind and

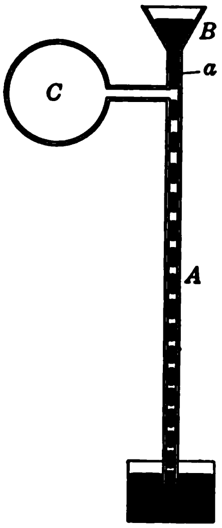

Fig. 99.

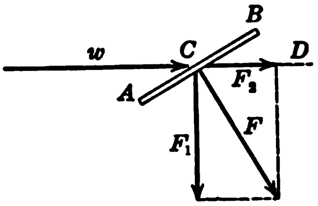

Fig. 100.

is therefore free to rotate at right angles to the direction of the wind. This wheel carries radial vanes which are set obliquely to the wind and hence to the axis of the wheel. In Fig. 100, AB is an end view of a vane which extends toward the reader from the axis (CD) of the windmill wheel. From analogy to the problem of the sailboat (Sec. 20), we see at once that the reaction of the wind w against the vane AB gives rise to a thrust F *normal* to the vane. This force may be resolved into the two components F_1 and F_2. F_2 gives only a useless end thrust on the wheel axle; while F_1 gives the useful force which drives the vane in the direction F_1. When the vane comes to a position directly below the axis of the wheel, F_1 is directed away from

the reader. Thus in these two positions, and indeed in all other positions of the vane, F_1 gives rise to a clockwise torque as viewed from a point from which the wind is coming. Every vane gives rise to a similar, constant, clockwise torque.

The ordinary electric fan is very similar to the windmill in its operation, except that the process is reversed. In the case of the windmill, the wind drives the wheel and generates the power; while with the fan, the electric motor furnishes the power to drive the fan and produce the "wind." In the former, the reaction between the *vane* and the *air* pushes the *vane;* while in the latter it pushes the *air*.

150. Rotary Blowers and Rotary Pumps.—Blowers are used for a great variety of purposes. Important among these are the ventilation of mines; the production of the forced draft for forges and smelting furnaces; the production of the "wind" for fanning mills and the wind-stackers on threshing mchines; and for the production of "suction," as in the case of tubes that suck up shavings from wood-working machinery, foul gases from chemical operations, and dust as in the *Vacuum Cleaner*. All blowers may be considered to be pumps, and like all pumps they are capable of exhausting on one side and compressing on the other, as pointed out in Sec. 147. Hence the above blowers that produce the "wind" do not differ essentially from those that produce "suction." Indeed many of the large ventilating fans used in mines may be quickly changed so as to force the air into the shaft, instead of "drawing" it from the shaft.

Blowers commonly produce a pressure of one pound per square inch or less (*i.e.*, a difference from atmospheric pressure of 1 lb. per sq. in.), although the so-called "positive" blowers may produce eight or ten pounds per square inch. For the production of highly compressed air, such as used in the air-brakes on trains, the piston air pump is used (see Air Compressor, Sec. 147).

Rotary Blowers.—Rotary blowers are of two kinds, *disc blowers* and *centrifugal blowers*. A *disc fan* has blades which are radial and set obliquely to its axis of rotation; while the fan proper has its blades parallel to its axis and usually about radial (like the blades of a steamboat paddle wheel). The common electric fan is of the former type, and the fan used in the fanning mill is of the latter type. If a disc fan is placed at the center of a tube with its axis parallel to the tube, it will, when revolved, force a

stream of air through the tube. The diameter of the fan should be merely enough less than that of the tube to insure "clearance." Such a blower will develop at the intake end of the tube a slight suction and at the other end a slight pressure. This type is widely used for ventilation purposes.

The essential difference between the *Turbine Pump* and the blower just described is that the fan is stronger and propels a stream of water instead of a stream of air. The turbine pump is useful in forcing a large quantity of water up a slight grade for a short distance. It is not a high pressure pump.

The Screw Propeller, universally used on ocean steamships and also used on gasoline launches, is essentially a turbine pump. The propeller forces a stream of water backward and the reacting thrust forces the ship forward.

The Centrifugal Blower is similar in its action to the centrifugal pump described below.

The Centrifugal Pump.—One type of centrifugal pump, shown in section in Fig. 101, consists of a wheel W, an *intake* pipe A

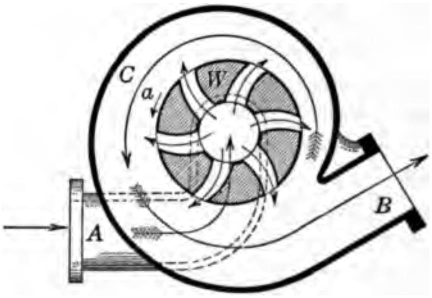

FIG. 101.

which brings water to the center of the wheel, and an *outflow* pipe B, which conveys the water from the periphery of the wheel. The direction of flow of the water at various points is indicated by the arrows. By means of an electric motor or other source of power, the wheel is rotated in the direction of arrow a, and the centrifugal force thereby developed causes the water to flow radially outward through the curved passages in the wheel as indicated by the arrows. In this way, it is feasible to produce a pressure of 20 or 30 lbs. per sq. in. in the space C, which is sufficient pressure to force water to a vertical height of 50 or 60 ft. in a pipe connected with B.

If it is desired to raise water to a greater height than this, several pumps can be used in "series." In such a series arrangement, the lowest pump would force water through *outlet* pipe B to the *intake* of a similar pump, say 50 ft. above. This second pump would force the water on to the next above, and so on.

151. The Turbine Water Wheel.—*The Turbine Water Wheel* operates on the same general principle as the windmill; a stream of water driving the former, a stream of air the latter. Since water is much more dense than air, turbine water wheels develop a great deal more power than windmills of the same size. At the Niagara Falls power plant, water under about 150 ft. vertical head rushes into the great turbines, each of which develops 5000 H.P. Turbines of 10,000 H.P. each are to be used in the power plant at Keokuk, Iowa.

There are several kinds of water turbines. In the "*Axial Flow Turbines*" in which the water flows parallel to the axis, the action of the windmill is practically duplicated; so that Fig. 100 and the accompanying discussion would apply to a vane of such a turbine, provided w were to represent moving water instead of moving air. In the "*Radial Flow Turbines*" the water flows in a general radial direction either toward or away from the axis.

If water under considerable pressure is forced in at pipe A (Fig. 101) through wheel W, and out at pipe B, it will drive W in the direction of arrow a. For, as the water flows outward through the curved radial passages, it would, by virtue of its inertia, produce a thrust against the concave wall of the passage. This thrust would clearly produce a positive (left-handed) torque. Under these circumstances, the wheel would develop power, and would be called a *radial flow turbine water wheel.*

The Steam Turbine, used to obtain power from steam, is similar to the water turbine in principle, but greatly differs from it in detail. The development of the light, high power, high efficiency steam turbine is among the comparatively recent achievements of steam engineering. The steam turbine is further considered in Sec. 235.

152. Pascal's Law.—The fact that liquids confined in tubes, etc., transmit pressure applied at one point to all points, has already been pointed out (Sec. 114). This is known as Pascal's Law. Pascal's Law holds with regard to gases as well as liquids.

This law has many important applications, among which are the transmission of pressure by means of the water mains to all

parts of the city, and the operation of the hydraulic press and the hydraulic elevator.

153. The Hydraulic Press.—The hydraulic press (Fig. 102) is a convenient device for securing a very great force, such as required for example in the process of baling cotton. It consists of a large piston or plunger P, fitting accurately into a hole in the top of a strong cylindrical vessel B. As water is forced into B by means of a force pump connected with pipe D, the plunger P rises. As P rises, the platform C compresses the cotton which occupies the space A. By opening a valve E, the water is permitted to escape and P descends.

In accordance with Pascal's Law, the pressure developed by the force pump is transmitted through D to the plunger P. It will be observed that since the pressure on the curved surface of the plunger is perpendicular to that surface (Sec. 114), it will have no tendency to lift the plunger. The lifting force is pA_1, in which A_1 is the area of the bottom end of the plunger, and p, the water pressure.

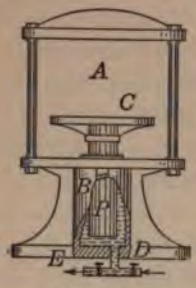

Fig. 102.

If the area of the piston of the force pump is A_2, then, since the pressure below this piston is practically the same as that acting upon plunger P, it follows that the lift exerted by P will be greater than the downward push upon the piston of the force pump in the ratio of A_1 to A_2. In other words, the theoretical mechanical advantage is $A_1 \div A_2$.

Instead of using a force pump, the pipe D may be connected to the city water system. If this pressure is 100 lbs. per in.^{2}, and if $A_1 = 100$ in.^{2}, then P will exert a force of 10,000 lbs. or 5 tons. With some steel forging presses a force of several thousand tons is obtained.

154. The Hydraulic Elevator.—The simplest form of hydraulic elevator, known as the direct-connected or direct-lift type, is the same in construction and operation as the hydraulic press (Fig. 102), except that the plunger is longer. If the elevator, built on platform C, is to have a vertical travel of 30 ft., then the plunger P must be at least 30 ft. in length.

In another type of hydraulic elevator, the plunger and containing cylinder lie in a horizontal position in the basement of the building. The plunger is then connected with the elevator

by means of a system of gears or pulleys and cables in such a way that the elevator travels much farther, and hence also much faster than the plunger. This type is much better than the direct-connected type for operating elevators in high buildings. In both types the valves that regulate the flow of water to and from the cylinder are controlled from the elevator.

155. The Hydraulic Ram.—The hydraulic ram (shown in Fig. 103) depends for its action upon the high pressure developed when a moving stream of water confined in a tube is suddenly stopped. It is used to raise a small percentage of the water from a spring or other source to a considerable height.

The valve C is heavy enough so that the water pressure hdg (see figure) is not quite sufficient to keep it closed. As it sinks slightly, the water flows rapidly past above it; while at the same time the water below it is practically still. In the next

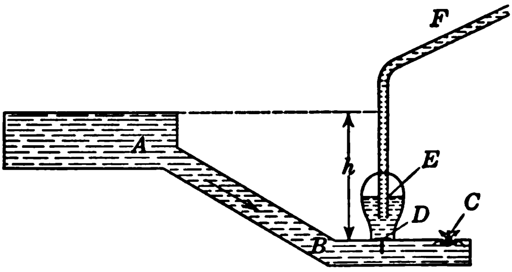

FIG. 103.

section it will be shown that the pressure on a fluid becomes *lower* the *faster* it moves; accordingly, the pressure above the valve is less than the pressure below, and the valve rises and closes. The closing of this valve suddenly checks the motion of the water in pipe B. But suddenly stopping any body in motion requires a relatively large accelerating force (negative), hence here a considerable pressure is developed in pipe B. This "instantaneous," or better *brief*, pressure opens the valve D and forces some water into the air chamber E and also into pipe F. Valve C now sinks, and the operation is repeated, forcing still more water in E, until finally the water is forced through F into a supply tank which is on higher ground than the source A. The action of the air chamber is explained in Sec. 146.

If the hydraulic ram had an efficiency of 100 per cent., from the conservation of energy, we see that it would raise

of the total amount of water to a height nh. Its efficiency, however, is only about 60 per cent.; hence it will force $1/n$ of the total water used to a height 0.6 nh.

156. Diminution of Pressure in Regions of High Velocity.—If a stream of air is forced rapidly through the tube A (Fig. 104), it will be found that the pressure at the restricted portion B is less than elsewhere, as at C or D. If the end D is short and open to the air, manometer F will indicate that D is practically at atmospheric pressure. The pressure at C will be slightly above atmospheric pressure, as indicated by manometer E. That the pressure at B is less than one atmosphere, and hence less than at either C or D, is evidenced by the fact that the liquid stands higher in tube G than in the vessel H.

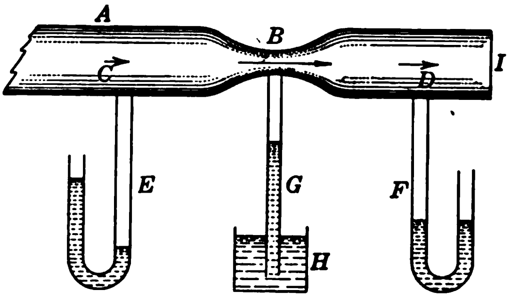

FIG. 104.

That the pressure at B should be less than at C or D is explained as follows: Since the tube has a smaller cross section at B than at C or D, it is evident that the air must have a higher velocity at B than at the other two points, as indicated in the figure by the difference in the length of the arrows. As a particle of air moves from C to B its velocity, then, increases. To cause this increase in velocity requires an accelerating force. Consequently the pressure behind this particle tending to force it to the right must be greater than the pressure in front of it, tending to force it to the left. As the particle moves from B to D it slows down, showing that the backward pressure upon it must be greater than the forward pressure. Thus B is a region of *lower pressure* than either C or D simply because it is a region of *higher velocity*. The reduction in pressure at B is explained

by means of Bernoulli's theorem under "Venturi Water Meter" (see below).

The Atomizer.—If the air rushes through B still more rapidly, the pressure will be sufficiently reduced so that the liquid will be "drawn" up from vessel H and thrown out at I as a fine spray. The tube then becomes an atomizer.

The Aspirator or Filter Pump.—A similar reduction in pressure occurs at B if *water* flows rapidly through the tube. Thus, if the tube is attached in a vertical position to a faucet, the water rushing through B produces a low pressure and consequently "suction," so that if a vessel is connected with G the air is withdrawn from it, producing a partial vacuum. Under these circumstances the tube acts as a *filter pump* or aspirator.

The "forced draft" of locomotives is produced by a jet of steam directed upward in the smoke stack.

The Jet Pump.—If a stream of water from a hydrant is directed through B, a tube connected with G may be employed to "draw"

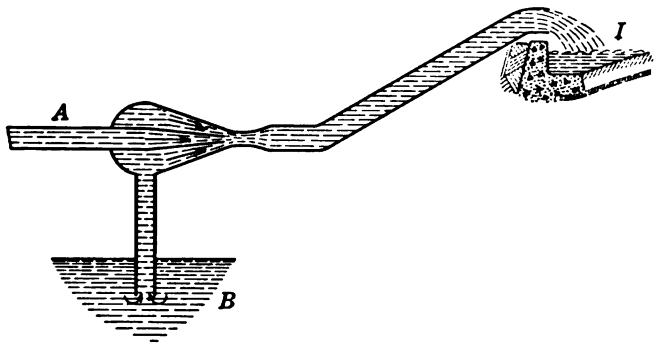

FIG. 105.

water from a cistern or flooded basement. Such an arrangement is a *jet pump*. In Fig. 105 a jet pump is shown pumping water from a basement B into the street gutter I. Pipe A is connected to the hydrant.

Bernoulli's Theorem.—Bernoulli's Theorem, first enunciated in 1738 by John Bernoulli, is of fundamental importance to some phases of the study of hydraulics. We shall develop this theorem from a discussion of Fig. 95.

Let water flow into A at the top as rapidly as it flows out at B, thus maintaining a constant water level. Let us nea sider the energy possessed by a given volume V of water

14

different stages of its passage from the surface S to the outflowing stream at B. Its energy (potential energy E_p) when at S is Mgh_1 (Eq. 50, Sec. 75), and, since $M = Vd$ (volume times density, Sec. 101), we have

$$E_p = Vdgh_1$$

As this given volume reaches point b at a slight distance h above the orifice, it has potential energy Mgh, or $Vdgh$, and, since it now has appreciable velocity v, it has kinetic energy $\frac{1}{2}Mv^2$, or $\frac{1}{2}Vdv^2$. In addition to this it has potential energy, because of the pressure (p) exerted upon it by the water above, which energy, we shall presently prove, is pV. Consequently, its *total energy when at b* is

$$E = Vdgh + pV + \tfrac{1}{2}Vdv^2 \tag{73}$$

Eq. 73 is the mathmetical statement of Bernoulli's theorem. If C.G.S. units are used throughout (*i.e.*, if V is given in cm.^{3}, p in dynes per cm.^{2}, v in cm. per sec., etc.), then E will be the energy in ergs. If the volume chosen is unity, the equation reduces to $E = dgh + p + \frac{1}{2}dv^2$, a form frequently given.

Observe that when the volume V is at S, p and v are zero, hence $E = Vdgh_1$ as already shown; while when this volume reaches the flowing stream, p and h are zero, hence $E = \frac{1}{2}Vdv^2$ (*i.e.*, $\frac{1}{2}Mv^2$). From the law of the conservation of energy we know that these two amounts of energy must be equal, *i.e.*, $Vdgh_1 = \frac{1}{2}Vdv^2$, which reduces to $v = \sqrt{2gh_1}$, an equation already deduced (Sec. 143) from slightly different considerations. When the volume is half way down in vessel A, $Vdgh = pV$, and the third term $\frac{1}{2}Vdv^2$ is practically zero, since v at this point has a small value. It should be observed that when the volume under consideration is below the surface, then the height measured from the volume *up* to the surface determines the *pressure;* whereas the height measured from the volume *down* to the orifice, determines the *potential energy* due to the elevated position. Obviously the energy due to elevation decreases by the same amount that the energy due to pressure increases, and *vice versa*, and the sum of these two amounts of energy is constant so long as the velocity v (last term Eq. 73) is practically zero.

We shall now prove that the potential energy of the above volume V, when subjected to a pressure p, is pV. Let the volume V, as it passes out at B, slowly push a snug-fitting piston in B a

distance d_1 such that $d_1A_1 = V$, in which A_1 is the cross section of the orifice. The work done by the volume V on the piston is $pA_1 \times d_1$ (force times distance), which shows that the potential energy of V immediately before exit was $p \times A_1d_1$ or pV.

The Venturi Water Meter.—The Venturi water meter, used for measuring rate of flow, differs from the apparatus sketched in Fig. 104 in that the medium is water instead of air, and the pressure is measured by ordinary pressure gages instead of as shown. If pipe A were 6 ft. in diameter at C, it would taper in a distance of 100 ft. or so to a diameter of about 2 ft. at B.

Let the pressure, area of cross section, and velocity of flow at C and B, respectively, be p_c, A_c, v_c, and p_b, A_b, v_b. Now the energy of a given volume V when at C must be equal to its energy when at B; hence, from Eq. 73, we have

$$Vdgh + p_cV + \tfrac{1}{2}Vdv_c^2 = Vdgh + p_bV + \tfrac{1}{2}Vdv_b^2$$

from which we get

$$p_c - p_b = \tfrac{1}{2}d(v_b^2 - v_c^2) \tag{74}$$

Since in unit time equal volumes must pass B and C, we have

$$v_bA_b = v_cA_c, \text{ or } v_b = \frac{A_c}{A_b}v_c \tag{74a}$$

Substituting in Eq. 74 this value of v_b gives

$$p_c - p_b = \tfrac{1}{2}d\frac{(A_c^2 - A_b^2)}{A_b^2}v_c, \text{ or } v_c = 2A_b^2\frac{(p_c - p_b)}{d(A_c^2 - A_b^2)} \tag{74b}$$

If the pressure is reduced to *poundals* per square foot, the cross section to square feet, and if the density of the water is also expressed in the British system (*i.e.*, 62.4 lbs. per cu. ft.), then v_c will be expressed in feet per second. Multiplying v_c by A_c (in square feet) gives, for the rate of flow, v_cA_c, in cu. ft. per sec.

157. The Injector.—Injectors are used for forcing water into boilers while the steam pressure is on. Their operation depends upon the decrease of pressure produced by the high velocity of a jet of steam, coupled with the condensation of the steam in the jet by contact with the water spray brought into the jet by the atomizer action (Sec. 156). Some of the commer[illegible] of the injector are quite complicated.

The injector shown diagrammatically in Fig. 106 is

tively simple. If valves a and d are opened, b being closed, the steam from the boiler B rushes through D, E and e and out at a into the outside air. The steam, especially at the restricted portion E of the tube, has a very high velocity, and hence, from Sec. 156, we see that a low pressure exists at E. The pressure at E being less than one atmosphere, the atmospheric pressure upon the water in the tank forces water up through the pipe P into E, where it passes to the right with the steam which quickly condenses. This stream of water, due to its momentum, raises check valve b and passes into the boiler against the boiler pressure. As soon as the flow through b is established, valve a should be closed. In many injectors, the suction due to the partial vacuum at e automatically closes a check valve opening downward at a.

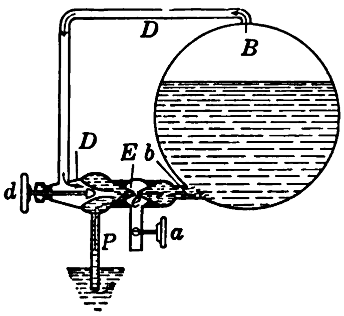

FIG. 106.

It should be pointed out that in the action of the injector, by which steam under a pressure p forces supply water (and also the condensed steam) into the boiler against this same steam pressure plus a slight water pressure (see figure), there is no violation of the law of the conservation of energy. The energy involved is pressure times volume in both cases, but the *volume of water* forced into the boiler in a given time is much *less than* the *volume of steam* used by the injector.

158. The Ball and Jet.—If a stream of air B, Fig. 107, is directed as shown against a light ball A, *e.g.*, a ping pong ball or tennis ball, the ball will remain in the air and rapidly revolve in the direction indicated.

The explanation is simple. There are three forces acting upon the ball, namely, W, F_1, and F_2, as shown. The force F_1 arises from the impact of the stream of air B. The force F_2 is due to the fact that the air pressure at a is less than at b. The pressure at b is one atmosphere, while at a it is slightly less because a is a region of high velocity. W represents the weight of A. If it is desired to determine the magnitude of F_1 and F_2, the magnitude of W may be found by weighing A, and then, since the ball is in equilibrium, these three forces W, F_1, and

F_2, acting upon it must form a closed triangle, as explained in Sec. 18.

Card and Spool.—If a circular card, having a pin inserted through the center, is placed below a spool through the center of which a rapid stream of air is blown, it will be found that the card will be supported in spite of the downward rush of air upon it which might be expected to blow it away. The air above the card is moving rapidly in all directions away from the center; consequently the region between the spool and card, being a region of high velocity, is also a region of low pressure—lower, in fact, than the pressure below the card. This difference in pressure will not only support the weight of the card, but also additional weight.

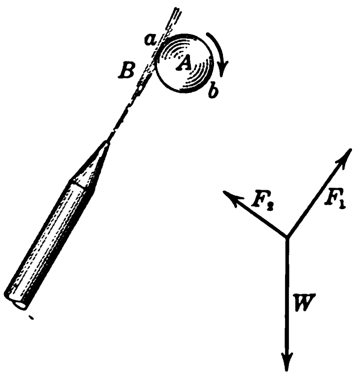

Fig. 107.

159. The Curving of a Baseball.—The principle involved in the pitching of "in curves," "out curves," etc., will be understood from a discussion of Fig. 108. Let A represent a baseball rotating as indicated, and moving to the right with a velocity v. If A were perfectly frictionless, the air would rush past it equally fast above and below, *i.e.*, v_1 and v_2 would be equal. (We are familiar with the fact that a person running 10 mi. per hr. east through still air, faces a 10 mi. per hr. breeze apparently going west.) If the surface of the ball is rough, however, it will be evident that where this surface is moving in the direction of the rush of air past it, as on the upper side, it will not retard that rush so much as if it were moving in the direction opposite to the rush of air, as it clearly is on the lower side of the ball. The air, then, rushes more readily, and hence more rapidly past the upper surface than past the lower surface of the ball; hence, as the ball moves to the right, the air pressure above it is less than it is below, and an "up curve" results.

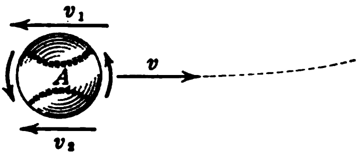

Fig. 108.

The "drop curve" is produced by causing the ball to rotate in a direction opposite to that shown; while the "in curve" and "out curve" require rotation about a vertical axis.

A lath may be made to produce a very pronounced curve by throwing it in such a way as to cause it to rotate rapidly about its longitudinal axis, the length of the lath being perpendicular to its path.

PROBLEMS

1. A force pump, having a 3-ft. handle with the piston rod operated by a 6-in. "arm," (*i.e.*, with the pivot bolt 6 in. from one end of the handle), and having a piston head 2 in. in diameter, is used to pump water into an hydraulic press whose plunger is 1.5 ft. in diameter. What force will a 100-lb. pull on the end of the pump handle exert upon the plunger of the press?

2. An hydraulic press whose plunger is 2 ft. in diameter is operated by water at a pressure of 600 lbs. per sq. in. How much force does it exert? Express in tons.

3. An hydraulic elevator operated by water under a pressure of 100 lbs. per in.^{2} has a plunger 10 in. in diameter and weighs 2.5 tons. How much freight can it carry?

4. If $h_1 = 10$ ft., and $h_2 = 18$ ft. (Fig. 96), what will be the pressure at C (*a*) if the left end of the siphon is stoppered? (*b*) If the right end is stoppered? Assume the barometric pressure to be equal to that due to 34 ft. depth of water.

5. What pressure will be required to pump water from a river into a tank on a hill 300 ft. above the river, if 20 per cent. of the total pressure is needed to overcome friction in the conveying pipes?

6. How long will it take a 10-H.P. pump (output 10 H.P.) to pump 1000 cu. ft. of water into the tank (Prob. 5)?

7. If the water in pipe B (Fig. 94) flows with a velocity of 4 ft. per sec., what will be the value of h_2? Neglect friction head in the portion i to c'' (Secs. 142 and 143).

8. What would be the limiting (maximum) distance from the piston to the water level in the cistern (Fig. 97) at such an altitude that the barometric height is 20 in.?

PART III
HEAT

CHAPTER XIII

THERMOMETRY AND EXPANSION

160. The Nature of Heat.—As was pointed out in the study of Mechanics, a portion of the power applied to any machine is used in overcoming friction. It is a matter of everyday observation that friction develops heat. It follows, then, that mechanical energy may be changed to heat. In the case of the steam engine or the gas engine the ability to do work, that is to run the machinery, ceases when the heat supply is withdrawn. Therefore heat is transformed into mechanical energy by these engines, which on this account are sometimes called heat engines.

Heat, then, is a form of energy, a body when hot possessing more energy than when cold. Cold, it may be remarked, is not a physical quantity but merely the comparative absence of heat, just as darkness is absence of light. The heat energy of a body is supposed to be due to a very rapid vibration of the molecules of the body. As a body is heated to a higher temperature, these vibrations become more violent.

It has been proved experimentally, practically beyond question, that both radiant heat and light consist in waves in the transmitting medium (ether). To produce a wave motion in any medium requires a vibrating body. As a body, for example a piece of iron, becomes hotter and hotter it radiates more heat and light. Hence, since the iron does not vibrate as a whole, the logical inference is that the radiant heat and light are produced by the vibrations of its molecular or atomic particles.

Until about one hundred years ago heat was supposed to be a substance, devoid of weight or mass, called *Caloric*, which, when added to a body caused it to become hotter, and when withdrawn from a body left the body colder. In 1798, Count Rumford showed that an almost unlimited amount of heat could be taken from a cannon by boring it with a dull drill. The heat was produced, of course, by friction. In the process a very small amount of metal was removed. As the drilling proceeded and more "caloric" was taken from the cannon, it actually became

hotter instead of colder as the *caloric theory* required. Furthermore, the amount of heat developed seemed to depend upon the amount of work done in turning the drill. The result was the complete overthrow of the caloric theory.

In 1843, Joule showed by experiment that if 772 ft.-lbs. of work were used in stirring 1 lb. of water, its temperature would be raised 1° F. This experiment showed beyond question that heat is a *form of energy*, and that it can be measured in terms of work units. Later determinations have given 778 ft.-lbs. as the work necessary to raise the temperature of 1 lb. of water 1° F. The amount of heat required to warm 1 lb. of water 1° F. is called the *British Thermal Unit* (B.T.U.); so that 1 B.T.U. = 778 ft.-lbs.

161. Sources of Heat.—As already stated, *Friction* is one source of heat. Rubbing the hands together produces noticeable warmth. Shafts become quite hot if not properly oiled. Primitive man lighted his fires by vigorously rubbing two pieces of wood together. The shower of sparks from a steel tool held against a rapidly revolving emery wheel, and the train of sparks left by a meteor or shooting star, show that high temperatures may be produced by friction. In the latter case, the friction between the small piece of rock forming the meteor, and the air through which it rushes at a tremendous velocity, develops, as a rule, sufficient heat to burn it up in less than a second.

Chemical Energy.—Chemical energy is an important source of heat. The chemical energy of combination of the oxygen of the air with the carbon and hydrocarbons (compounds of carbon and hydrogen) of coal or wood, is the source of heat when these substances are "burned," that is, oxidized. In almost every chemical reaction in which new compounds are formed, heat is produced.

The *Main Source* of heat is the *Sun*. The rate of flow of heat energy in the sun's rays amounts to about 1/4 H.P. for every square foot of surface at right angles to the rays. Upon a high mountain this amount is greater, since the strata of the air below the mountain peak absorb from 10 to 20 per cent. of the energy of the sun's rays before they reach the earth. On the basis of 1/4 H.P. per sq. ft., the total power received by the earth from the sun is easily shown to be about 350 million million H.P. This enormous amount of power is only about 1/2,000,000,000 part of the total power given out by the sun in all directions.

Obviously a surface receives more heat if the sun's rays strike it normally (position AB, Fig. 109) than if aslant (position AB_1), for in the latter case fewer rays strike it. Largely for this reason, the ground is hotter under the noonday sun than it is earlier. The higher temperature in summer than in winter is due to the fact that the sun is, on an average, more nearly overhead in summer than in winter. The hottest part of the day is not at noon as we might at first expect, but an hour or two later. This lagging occurs because of the time required to warm up the ground and the air. A similar lagging occurs in the seasons, so that the hottest and the coldest weather do not fall respectively on the longest day (June 21) and the shortest (Dec. 22), but a month or so later as a rule.

The above-mentioned sources are the three main sources of heat. There are other minor sources. An *electric current* heats a wire or any other substance—solid, liquid, or gas—through

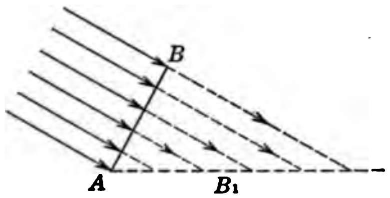

FIG. 109.

which it passes. This source is of great commercial importance. The condensation of water vapor produces a large amount of heat, and this heat is one of the greatest factors in producing wind storms as explained in Sec. 223.

162. The Effects of Heat.—The principal effects of heat are:

(*a*) Rise in temperature.
(*b*) Increase in size.
(*c*) Change of state.
(*d*) Chemical change.
(*e*) Physiological effect.
(*f*) Electrical effect.

(*a*) With but very few exceptions a body becomes hotter, *i.e.*, its temperature rises, when heat is applied to it. Exceptions: If water containing crushed ice is placed in a vessel on a hot stove, the water will not become perceptibly hotter until practically all of the ice is melted. Further application of heat

causes the water to become hotter until the boiling point is reached, when it will be found that the temperature again ceases to rise until all of the water boils away, whereupon the containing vessel becomes exceedingly hot. In this case, the heat energy supplied, instead of causing a *temperature rise* (*a*), has been used in producing a *change of state* (*c*), *i.e.*, it has been used in changing ice to water, or water to steam.

(*b*) As heat is supplied to a body, it almost invariably produces an increase in its size. It might readily be inferred that the more violent molecular vibrations which occur as the body becomes hotter, would cause it to occupy more space, just as a crowd takes more room if the individuals are running to and fro than if they are standing still or moving *less*. Exception to (*b*): If a vessel filled with ice is heated until the ice is melted, the vessel will be only about 9/10 full. In this case heat has caused a decrease in size. This case is decidedly exceptional, however, in that a *change of state* (*c*) is involved. It is also true that most substances expand upon melting instead of contracting as ice does.

(*d*) To ignite wood, coal, or any other substance, it is necessary to heat it to its "kindling" or ignition temperature, before the chemical change called "burning" will take place. In the limekiln, the excessive heat separates carbon dioxide (CO_2) from the limestone, or crude calcium carbonate ($CaCO_3$), leaving calcium oxide (CaO), called *lime*. There are other chemical reactions besides oxidation which take place appreciably only at high temperatures. *Slow* oxidation of many substances occurs at ordinary temperatures. All chemical reactions are much less active at extremely low temperatures such as the temperature produced by liquid air.

(*e*) Heat is essential to all forms of life. Either insufficient heat or excessive heat is exceedingly painful.

(*f*) The production of electrical effect by heat will be discussed under the head of the *Thermocouple* (Sec. 174).

163. Temperature.—The temperature of a body *specifies its state with respect to its ability to impart heat to other bodies.* Thus, if a body A is at a higher temperature than another body B, it will always be found that heat will flow from A to B if they are brought into contact, or even if brought near together. The greater the temperature difference between A and B, other things being equal, the more rapid will be the heat transfer. The tem-

perature of a body rises as the heat vibrations of its molecules become more violent.

The *temperature of a body cannot be measured directly*, but it may be measured by some of the other effects of heat, as (*b*) and (*f*) (Sec. 162), or it may be roughly estimated by the physiological effect or temperature sense. Heat of *itself* always passes from a body of higher temperature to one of lower temperature. The temperature sense serves *usually* as a rough guide in determining temperature, but it is sometimes very unreliable and even *misleading*, as may be seen from the following examples.

If the right hand is placed in hot water and the left hand in cold water for a moment, and then both are placed in tepid water, this tepid water will feel *cold* to the right hand and *warm* to the left hand. Under these conditions heat flows or passes from the right hand to the tepid water. The tepid water being warmer than the left hand, the flow is in the opposite direction. Hence if heat flows *from the hand* to a body, we consider the body to be cold, while if the reverse is true, we consider it to be *warm*. If A shakes B's hand and observes that it feels cold we may be sure that B notices that A's hand is warm.

If the hand is touched to several articles which have been lying in a cool room for some time, and which are therefore at the *same temperature*, it will be found that the articles made of wool do not feel noticeably cool to the touch. The cotton articles, however, feel perceptibly cool, the wooden articles cold, and the metal articles still colder. The metal feels colder than wood or wool, because it takes heat from the hand more rapidly, due to its power (called conductivity) of transmitting heat from the layer of molecules in contact with the hand to those farther away. Wood is a poor conductor of heat and wool is a *very poor* conductor; so that in touching the latter, practically only the particles *touching* the hand are warmed, and hence very little heat is withdrawn from the hand and no sensation of cold results.

One of the most accurate methods of comparing and measuring temperatures, and the one almost universally used, makes use of the fact that as heat is supplied to a body, its *temperature rise*, and its expansion, or *increase in size*, go hand in hand. Thus if 10° rise in temperature causes a certain metal rod to become 1 mm. longer, then an increase of 5 mm. in length will

show that the temperature rise is almost exactly 5 times as great, or practically 50°. This principle is employed in the use of thermometers.

164. Thermometers.—From the preceding section it will be seen that any substance which expands uniformly with temperature rise can be used for constructing a thermometer. Air or almost any gas, mercury, and the other metals meet this requirement and are so used. Alcohol is fairly good for this purpose and has the advantage of not freezing in the far north as mercury does. Water is entirely unsuitable, because its expansion, as its temperature rises, varies so greatly. When ice cold water is slightly heated it actually *decreases* in volume (see Maximum Density, Sec. 185); whereas further heating causes it to expand, but not uniformly.

The fact that in the case of alcohol, the expansion per degree becomes slightly greater as the temperature rises, makes it necessary to gradually increase the length of the degree divisions toward the top of the scale. In the case of mercury, the expansion is so nearly uniform that the degree divisions are made of equal length throughout the scale.

Mercury is the most widely used thermometric substance. It is well adapted to this use because it *expands almost uniformly* with temperature rise; has a fairly *large coefficient of expansion;* does *not stick* to the glass; has a *low freezing point* (−38°.8 C.) and a *high boiling point* (357° C.); and, being opaque, a thin thread of it is *easily seen.*

165. The Mercury Thermometer.—The mercury thermometer consists of a glass tube T (Fig. 110) of very small bore, terminating in a bulb B filled with mercury. As the bulb is heated, the mercury expands and rises in the tube (called the stem), thereby indicating the temperature rise of the bulb. In filling the bulb, great care must be taken to exclude air.

Briefly, the method of introducing the mercury into the bulb is as follows: The bulb is first heated to cause the air contained in it to expand, in order that a portion of it may be driven out of the open upper end of the stem. This end is then quickly placed in mercury, so that when the bulb cools, and consequently the air pressure within it falls below one atmosphere, some mercury is forced up into the bulb. If, now, the bulb is again heated until the mercury in it boils, the mercury vapor formed drives out all of the air; so that upon again placing the end of the stem in the

mercury and allowing the bulb to cool, thereby condensing the vapor, the bulb and stem are completely filled with mercury.

Let us suppose that the highest temperature which the above thermometer is designed to read is 120° C. The bulb is heated to about 125°, expelling some of the mercury from the open end of the tube which is then sealed off. Upon cooling, the mercury contracts, so that a vacuum is formed in the stem above the mercury. It will be evident that as the mercury in B is heated and expands, its upper surface, called its *meniscus* m, will rise; while if it is cooled its contraction will cause the meniscus to fall. Attention is called to the fact that if mercury and glass expanded equally upon being heated, then no motion of m would result. Mercury, however, has a much larger coefficient of expansion than glass (see table, Sec. 171). If heat is *suddenly* applied, for example by plunging the bulb into hot water, the glass becomes heated *first*, and m actually drops slightly, instantly to rise again as the mercury becomes heated.

Fig. 110.

The position of the meniscus m, then, except in the case of very sudden changes in temperature such as just cited, indicates the temperature to which the bulb B is subjected. In order, however, to tell definitely what temperature corresponds to a given position of m, it is necessary to "calibrate" the thermometer. To do this, the thermometer is placed in steam in an enclosed space over boiling water. This heats the mercury in B, thereby causing it to expand, and the meniscus m rises to a point which may be marked a. The thermometer is next placed in moist crushed ice which causes the mercury to contract, thereby lowering the meniscus to the point marked b. We have now determined two *fixed points*, a and b, corresponding respectively to the boiling point of water and the melting point of ice. It now remains to decide what we shall call the temperatures corresponding to a and b, which decision also determines how many divisions of the scale there shall be between these two points. Several different "scales" are used, two of which will be discussed in the next section.

Thermometers should not be calibrated until several years

after filling. If calibrated immediately, it will be found after a short time that because of the gradual contraction that has taken place in the glass, all of the readings are slightly too high.

166. Thermometer Scales.—The two thermometer scales in common use are the *Centigrade* and *Fahrenheit* scales. To calibrate a thermometer, according to the centigrade scale, the point *b* (Fig. 110) is marked 0°, and the point *a* is marked 100°, which makes it necessary to divide *ab* into 100 equal parts in order that each part shall correspond to a degree. Accordingly we see that ice melts at zero degrees centigrade, written 0° C., and that water boils at 100° C. Increasing the pressure, slightly lowers the melting point of ice (Sec. 186) and appreciably raises the boiling point of water (Sec. 194). To be accurate, ice melts at 0° C. and water boils at 100° C. when subjected to standard atmospheric pressure (76 cm. of mercury). If the pressure differs from this, correction must be made, at least in the case of the boiling point.

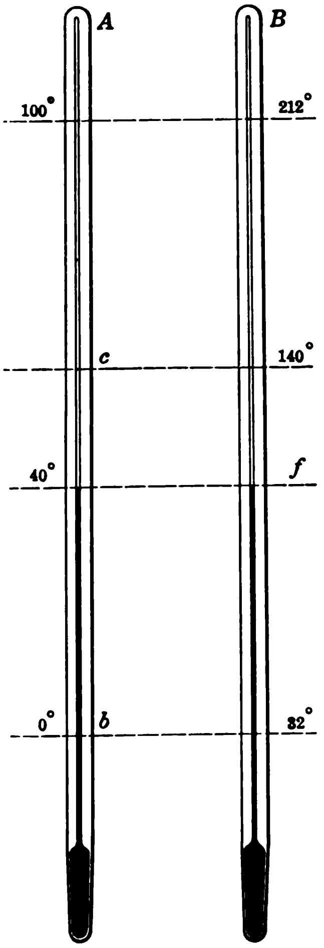

Fig. 111.

The *Fahrenheit* scale is in common use in the United States and Great Britain. To calibrate the thermometer (Fig. 110) according to the Fahrenheit scale, the "ice point" *b* is marked 32°, and the boiling point *a* is marked 212°. The difference between these two points is 180° so that *ab* will have to be divided into 180 equal spaces in order that each space shall correspond to a degree change of temperature. Using the same space for a degree, the scale may be extended above 212 and below 32.

The Fahrenheit scale has the advantage of a low zero point

which makes it seldom necessary to use negative readings, and small enough degree division that it is commonly unnecessary to use fractional parts of a degree in expressing temperatures. The *Reaumer* scale ("ice point" 0°, "boiling point" 80°), used for household purposes in Germany, has nothing to recommend it.

It is frequently necessary to change a temperature reading from the Fahrenheit scale to the centigrade or *vice versa*. For convenience in illustrating the method, let A and B (Fig. 111) represent two thermometers which are exactly alike except that A is calibrated according to the centigrade scale, and B according to the Fahrenheit. If both are placed in crushed ice, A will read 0° C. and B, 32° F.; while if placed in steam, A will read 100° C. and B, 212° F. If both thermometers are placed in warm water in which A reads 40° C., then the temperature f that thermometer B should indicate may be found as follows: The fact that the distance between the ice point and boiling point is 100° on A, and 180° on B, shows that the centigrade degree is 180/100 or 9/5 Fahrenheit degrees. From the figure it is seen that f is 40° C. above ice point or $40 \times 9/5 = 72°$ F. above 32° F., or 104° F. Next, let both thermometers be placed in quite hot water in which B reads 140° F., and let it be required to find the corresponding reading c of A. Since $140 - 32 = 108$, the distance bc corresponds to 108° F., or $108 \times 5/9 = 60°$ C. Hence 140° F. = 60° C. In the same way any temperature reading may be changed from one scale to the other.

167. Other Thermometers.—There are several different kinds of thermometers, each designed for a special purpose, which we shall now briefly consider.

Maximum Thermometer.—In the maximum thermometer of Negretti and Zambra there is, near the bulb, a restriction in the capillary bore of the stem. As the temperature rises, the mercury passes the restriction, but as the temperature falls, and the mercury in the bulb contracts, the mercury thread breaks at the restriction and thus records the maximum temperature. To reset the instrument, the mercury is forced past the restriction down into the bulb by the centrifugal force developed by swinging the thermometer through an arc.

The Clinical Thermometer.—The clinical thermometer, used by physicians, differs from the one just described in that it is calibrated for but a few degrees above and below the normal temperature of the body (98°.4 C.). It also has a large bulb in comparison with the size of the bore of the stem, thus securing long degree divisions and enabling more accurate reading.

Six's Maximum and Minimum Thermometer.—In this thermometer the expansion of the alcohol (or glycerine) in the glass bulb A (Fig. 112), as the temperature rises, forces the mercury down in tube B and up in the tube C. As the mercury rises in C it pushes the small index c (shown enlarged at left) before it. When the temperature again falls, c is held in place by a weak spring and thus records the maximum temperature. The contraction of the alcohol in A as the temperature decreases causes the mercury to sink in C and rise in B. As the mercury rises in B it pushes index b before it and thus records the minimum temperature. This thermometer is convenient for meteorological observations. The instrument is reset by drawing the indexes down to the mercury by means of a magnet held against the glass tube.

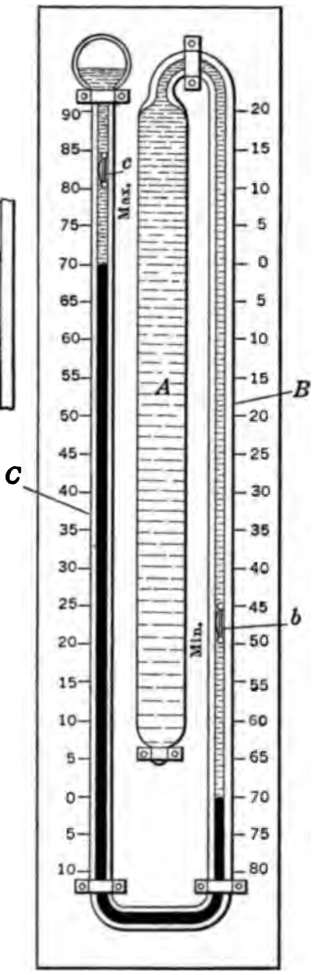

FIG. 112.

The Wet-and-dry-bulb Thermometer, also used in meteorological work, is discussed in Sec. 198 and Sec. 222.

The Gas Thermometer.—There are two kinds of gas thermometers, the constant-pressure and the constant-volume thermometers. A simple form of *Constant-pressure Thermometer* is shown in Fig. 113. As the gas in B is heated or cooled, the accompanying expansion and contraction forces the liquid index I to the right or left. The fact that for each degree of rise or fall in temperature, the volume of a given quantity of gas (under constant pressure) changes by 1/273 of its volume at 0° C. (Sec. 171), makes possible the accurate marking of the degree division on the stem, provided the volume of B and the cross section of the bore of the stem are both known.

A simple form of the *Constant-volume Gas Thermometer* is shown in Fig. 114. The stem A of the bulb B which contains the gas is connected with the glass tube C by the rubber tube T which contains the mercury.

When a quantity of gas is heated and not permitted to increase in volume, its pressure increases 1/273 of its pressure at 0° C. for every degree (centigrade degree) rise in temperature (Sec. 171). If, when B is at 0° C., and meniscus m_1 is at mark a, the meniscus m_2 is at the same level as m_1, then it is known that the pressure of the gas in B is one atmosphere. If, now, the temperature of B rises, m_1 is pushed down; but by

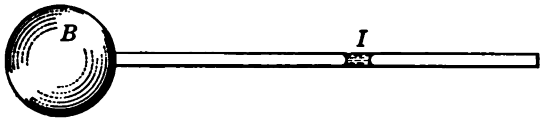

FIG. 113.

raising C until m_2 is at the proper height h above m_1, the mercury is forced back to mark a, thus maintaining the constancy of the volume of air in B and A. Suppose that the required height h is 10 cm. The excess pressure of the gas in B above atmospheric pressure will then be 10/76 or 36/273 atmospheres, and the temperature of B, according to the gas law just stated, must be 36° above zero, that is 36° C.

The *Constant-volume Hydrogen Thermometer* is by international agreement the *standard* instrument for temperature measurements. This instrument differs in detail, but not in principle, from the one shown in Fig. 114.

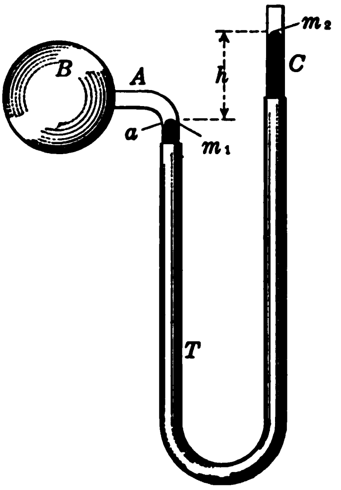

FIG. 114.

The Dial Thermometers.—If the tube of the Bourdon Gage (Sec. 141) is filled with a liquid and then plugged at A, the expansion of the liquid upon being heated will change the curvature of the tube and actuate the index just as explained for the case of steam pressure.

The Metallic Thermometer.—A spiral made of two strips of metal a and b soldered together (Fig. 115) will unwind slightly with temperature rise if the metal b expands more rapidly than a. As the spiral unwinds it causes the index I to move over the scale and indicate the temperature.

Recording Thermometer.—If the scale in Fig. 115 were replaced by a drum revolving about a vertical axis and covered by a suitably ruled sheet of paper, and if, further, the left end of the index I were provided with an inked tracing point resting on the ruled sheet, we would then have represented the essentials of the recording thermometer or *Thermograph.* The drum is driven by a clock mechanism and makes (usually)

one revolution per week. If the temperature remains constant, the tracing point draws a horizontal line on the drum as it rotates under it. As the temperature rises and falls, the tracing point rises and falls and traces on the revolving drum an irregular line which gives a permanent and continuous record of the temperature for the week. Obviously the days of the week, subdivided into hours, would be marked on the sheet around the circumference of the drum; while the temperature lines, properly spaced, would run horizontally around the drum and be numbered in degrees from the bottom upward.

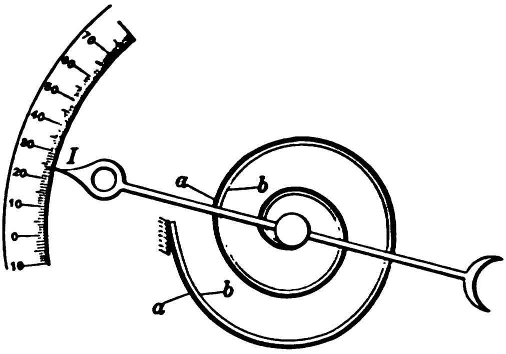

FIG. 115.

168. Linear Expansion.—When a bar of any substance is heated it becomes slightly longer. In some cases, especially with the metals, allowance must be made for this change in length, called *linear expansion*. Thus, a slight space is left between the ends of the rails in railroad construction. If this were not done, the enormous force or end thrust exerted by the rails upon expansion during a hot day would warp the track out of shape. The contraction and expansion of the cables supporting large suspension bridges cause the bridge floor to rise and fall a distance of several inches as the temperature changes. A long iron girder bridge should have one end free to move slightly lengthwise (on rollers) on the supporting pier to permit its expansion and contraction without damage to the pier.

In the familiar process of "shrinking" hot iron tires onto wooden wagon wheels, use is made of the contraction of the tire that takes place when it cools. Cannons are constructed of concentric tubes, of which the outer ones are successively heated and "shrunk" onto the inner ones. This extremely tight fitting

of the outer layers insures that they will sustain part of the stress when the gun is fired.

169. Coefficient of Linear Expansion.—When a bar, whose length at 0° C. is L_o, has its temperature raised to 1° C., its length increases by a certain fraction α of its original length L_o. This fraction α, which is very small for all substances, is called the *coefficient of linear expansion* for the material of which the bar is composed. The actual increase in length of the bar is then $L_o\alpha$. When heated from 0° to 2°, the increase in length is found to be almost exactly twice as great as before, or $L_o 2\alpha$; while if heated from 0° to t°, it is very closely $L_o\alpha t$. Consequently the length of the bar at any temperature t, which length may be represented by L_t, is given by the equation.

$$L_t = L_o + L_o\alpha t = L_o\,(1+\alpha t) \tag{75}$$

whence

$$\alpha = \frac{L_t - L_o}{L_o t} \tag{75a}$$

in which $L_t - L_o$ is the total increase in length for a change of t degrees, and hence $(L_t - L_o)$ divided by t is the total change for one degree. If this total change is divided by the length L_o of the bar (in cms.) we have the *increase in length* per centimeter of length (measured at 0° C.) *per degree* rise of temperature, which by Eq. 75*a* is α. Thus α may also be defined as the increase in length per centimeter (*i.e.*, per cm. of the length of the bar when at 0° C.) produced by 1° C. rise in temperature, or the *increase in length per centimeter per degree*.

To illustrate, suppose that two scratches on a brass bar are 1 cm. apart when the bar is at 0° C. Then, since α for brass is 0.000019 (approx., see table), it follows that at 1° C. the scratches will be 1.000019 cm. apart; at 2° C., 1.000038 cm.; at 10° C., 1.00019 cm. apart, etc. Since the length L_t of a metal bar at a temperature t differs very little from its length at 0°, *i.e.*, L_o, we may for most purposes consider that its increase in length when heated from a temperature t to $t+1$ is $L_t\alpha$ instead of $L_o\alpha$. Consequently, when heated from a temperature t to a still higher temperature t', the increase in length is approximately $L_t\,\alpha\,(t'-t)$. We then have the length L_t' at the higher temperature expressed approximately in terms of L_t by the equation

$$L_t' = L_t + L_t\alpha\,(t'-t) = L_t\,[1+\alpha(t'-t)]$$

This equation is accurate enough for all ordinary work and it is also a very convenient equation to use in all problems involving two temperatures, neither of which is zero. Strictly speaking, α is not constant, but increases very slightly in value with temperature rise.

AVERAGE COEFFICIENT OF LINEAR EXPANSION OF A FEW SUBSTANCES

Substance	Coeff. of Exp. α	Substance	Coeff. of Exp. α
Brass............	0.0000185	Oak, with grain..	0.000005
Copper..........	0.0000168	Platinum........	0.0000088
Glass............	0.0000086	Quartz, fused....	0.000005
Ice..............	0.000050	Silver..........	0.000019
Iron.............	0.000012	Zinc............	0.000029

It is perhaps well for the student to memorize α for platinum and note that for oak it is less than for platinum and for most metals about twice as great. In the case of glass, α varies considerably for the different kinds.

The French Physicist Guillaume recently made the interesting discovery that the coefficient of expansion of a certain nickel-steel alloy (36 per cent. nickel), known as *Invar*, is only about one-tenth as large as that of platinum, or 0.0000009. From these figures we see that the length of a bar of this metal increases less than 1 part in 1,000,000 when its temperature is raised 1° C. Steel tapes and standards of length are quite commonly made of *Invar*.

170. Practical Applications of Equalities and Differences in Coefficient of Linear Expansion.—In the construction of incandescent lamps it is necessary to have a vacuum in the bulb, or the carbon filament that gives off the light will quickly oxidize or "burn out." The electric current must be led through the glass to the filament by means of wires sealed into the glass while hot. If the glass and wire do not expand alike upon being heated, the glass will crack and the bulb will be ruined. Platinum wire is used for this purpose because its coefficient of expansion is almost exactly the same as that of glass.

The *differences between the coefficients* of expansion for any two metals, for example, brass and iron, has many practical applications. Important among the devices which utilize these differences in expansion are the automatic *fire alarm*, the *thermostat*, and the mechanism for operating the "skidoo" lamp used in signs. Another very important application of this difference

in expansion of two metals is in the *temperature compensation* of clock pendulums and the balance wheels of watches. By means of these compensation devices, timepieces are prevented from gaining or losing time with change of temperature.

The Fire Alarm.—The operation of the fire alarm will be understood from a study of Fig. 116. An iron bar I and a brass bar B are riveted together at several points and attached to a fixed support D at one end, the other end C being free. Since the coefficient of expansion for brass is greater than for iron, it will be evident that the above composite bar will curve upward upon being heated, and downward upon being cooled. Consequently the end C will rise when the temperature rises, and fall when the temperature falls. If such a device is placed near the

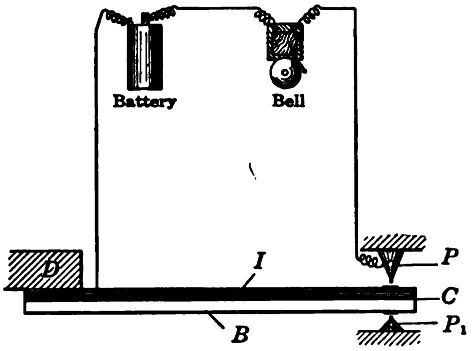

FIG. 116.

ceiling of a room, and if by suitable wiring, electrical connections are made between it and an electric bell, it become a *fire alarm.* For if a fire breaks out in the room, both bars I and B will be equally heated, but B will elongate more than I, thus causing C to rise until it makes contact with P. This contact closes the electrical circuit and causes the electric bell to ring.

The Thermostat.—If the room above considered becomes too cold, C descends and may be caused to touch a suitably placed point P_1, thereby closing another electrical circuit (not shown) connected with the mechanism that turns on more heat. As soon as the temperature of the room rises to its normal value, C again rises enough to break connection with P_1, and the heat supply is either cut off or reduced, depending upon the adjustment and design of the apparatus. When so used, the above bar, with its connections, is called a *thermostat.*

In a common form of thermostat, the motion of *C*, when the room becomes too cold, opens a "needle" valve to a compressed air pipe. This pipe leads to the compressed air apparatus, which is so arranged that when the air escapes from the above-mentioned valve, more heat is turned on.

The "Skidoo Lamp."—This device is very much used in operating several lamps arranged so as to spell out the words of a sign. Such a sign is much more noticeable if the lamps flash up for an instant every few seconds than if they shine steadily. The arrangement (using only one lamp) is shown in Fig. 117. The binding posts *E* and *F* are connected to the lighting circuit. Bars *I* and *B* are arranged just as in Fig. 116, except that the brass bar is above the iron bar instead of below.

When these bars are not touching the point *p*, the electric current passes from *E* to *a*, at which point the wire is soldered

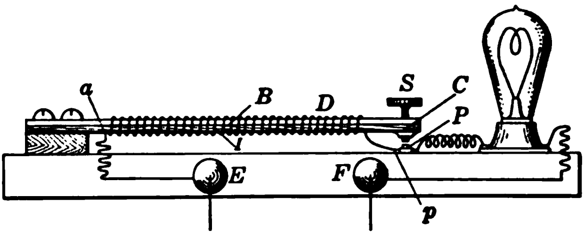

FIG. 117.

to the bars, then on through the coil *D* of very many turns of fine wire wrapped about the bars, to point *P*, where the wire is again soldered, and finally through the lamp, back to the binding post marked *F*.

Since coil *D* offers very great resistance to the passage of current, only a small current flows, and the lamp does not glow. This small current, however, heats coil *D* and therefore bars *B* and *I*; and, since *B* expands more rapidly than *I*, point *C* moves down until it touches point *P* as explained in connection with Fig. 116. The instant that point *C* touches *P*, practically all of the current flows directly from *a* through the heavy bars to *P* and then through the lamp as before. The fact that the current does not have to flow through coil *D* when *C* and *P* are in contact produces two marked changes which are essential to the operation of the lamp. First, since the electrical resistance of the bars is small, the current is much greater than before and

the lamp glows; and second, the coil now having practically no current, cools down slightly, thus permitting the bars to cool down, thereby causing C to rise. The instant C rises, the current is obliged to go through the coil, and is therefore too weak to make the lamp glow, but it heats the coil, causing C to descend again and the cycle is thus repeated indefinitely. If the contact screw S is screwed down closer to P, the lamp "winks" at shorter intervals.

The Balance Wheel of a Watch.—The same principle discussed above is used in the "temperature compensation" of the balance

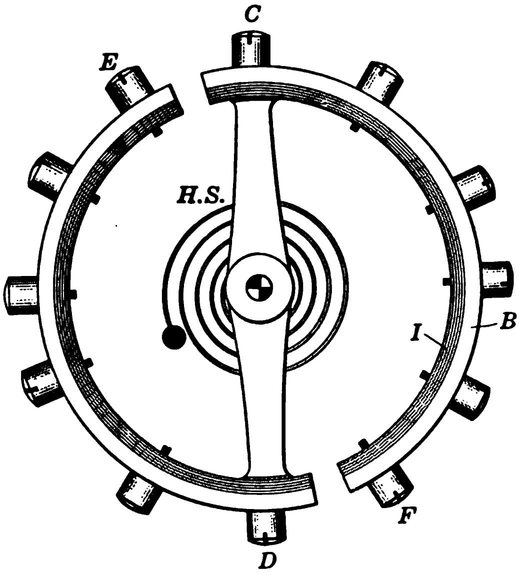

Fig. 118.

wheel of a watch, due to which compensation its period does not change with change of temperature. When an uncompensated wheel is heated the resulting expansion causes its rim to be farther from its axis, thereby increasing its moment of inertia. As its moment of inertia increases, the hairspring (*H.S.*, Fig. 118) does not make it vibrate so quickly and the watch loses time. To make matters worse the hairspring becomes weaker upon being heated.

It will be noticed in the balance wheel, sketched in Fig. 118, that the expansion produced by a rise in temperature causes the masses C and D (small screws) to move *from* the center; while

at the same time it causes E and F to move *toward* the center. For the brass strip B forming the *outside* of the rim expands more than the iron strip I *inside*. If the watch runs faster when warmed it shows that it is overcompensated; whereas if it runs slower when warmed it is undercompensated. Overcompensation would be remedied by replacing screws E and F by lighter ones, at the same time perhaps replacing C and D by heavier ones.

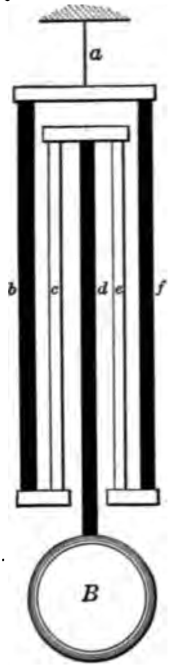

FIG. 119.

The Gridiron Pendulum.—From the sketch of the gridiron pendulum shown in Fig. 119, it will be seen that the expansion of the steel strip a, and the steel rods b, d, and f, causes the pendulum bob B to lower, thereby increasing the period of the pendulum; whereas the expansion of the zinc rods c and e evidently tends to raise B, thereby shortening the pendulum and also its period By having the proper relation between the lengths of the zinc and the iron rods, these two opposing tendencies may be made to exactly counterbalance each other. In this case the period of the pendulum is unaffected by temperature changes, that is, exact temperature compensation is obtained. If rods c and e were brass, their upward expansion would not compensate for the downward expansion of the iron rods. It would then be necessary to have four rods of brass and five of iron.

171. Cubical Expansion and the Law of Charles.—When a given quantity of any substance, say a metal bar, whose volume at 0° C. is V_o, has its temperature raised to 1° C., its volume increases by a certain small fraction β of its original volume V_o. This fraction β is called the *coefficient* of *cubical expansion* of the substance in question. The actual increase in volume is then $V_o\beta$. If the bar is heated from 0° to $t°$, *i.e.*, through t times as great a range, the increase in volume is found to be almost exactly t times as great, or $V_o\beta t$. Accordingly, the volume at $t°$, which may be represented by V_t, is given by the equation

$$V_t = V_o + V_o\beta t = V_o\,(1+\beta t) \tag{76}$$

whence

$$\beta = \frac{V_t - V_o}{V_o t} \tag{77}$$

In Eq. 77, $V_t - V_o$ is the total increase in volume; $(V_t - V_o) \div t$ is the total increase per degree rise in temperature; and dividing the latter expression by V_o gives $(V_t - V_o) \div V_o t$, or the increase per degree per cubic centimeter. But $(V_t - V_o) \div V_o t$ is β from Eq. 77. Hence β is numerically the *increase in volume per cubic centimeter of the "original" volume per degree rise in temperature.* By "original" volume is meant the volume of the bar when at 0° C.

Equations 76 and 77 apply to volumes of solids, liquids, or gases. The values of β however, differ widely for different substances, as shown in the table below. These equations apply to gases only if free to expand against a *constant pressure* when heated.

When a solid, *e.g.*, a metal bar, expands due to temperature rise, it increases in each of its three dimensions—length, breadth, and thickness. For this reason, it may be shown that the coefficient of cubical expansion is 3 times the coefficient of linear expansion for the same substance; *i.e.*, $\beta = 3\alpha$. For, consider a cube of metal, say, each edge of which has a length L_o at 0° C. Then, by Eq. 74, the length of each side at a temperature $t°$ will be $L_o(1+\alpha t)$. The volume at 0°, or V_o, is L_o^3; while the volume V_t at $t°$ is

$$V_t = L_o^3\ (1+\alpha t)^3 \tag{78}$$

Expanding $(1+\alpha t)^3$, we have $1+3\alpha t+3\alpha^2t^2+\alpha^3t^3$. Now, since α is very small, α^2 and α^3 will be negligibly small (observe that $(1/1000)^2 = 1/1{,}000{,}000$), and the terms $3\alpha^2t^2$ and α^3t^3 may be dropped. Eq. 78 then becomes

$$V_t = V_o\ (1+3\alpha t) \tag{79}$$

By comparing Eq. 79 with Eq. 76 we see at once that $\beta = 3\alpha$. which was to be proved. In like manner it may be shown that the *coefficient of area expansion* of a sheet of metal, for example, is 2α.

Accordingly, the fractional parts by which the *length* of a bar of iron, the *area* of a sheet of iron, and the *volume* of a chunk of iron increase per degree, are respectively 0.000012, 0.000024, and 0.000036.

COEFFICIENT OF CUBICAL EXPANSION OF A FEW SUBSTANCES

Substance	β	Substance	β
Alcohol	0.00104	Air, and all gases	0.00367
Ether	0.0017	Iron	0.000036
Mercury	0.00018	Zinc	0.000087
Petroleum	0.00099	Glass	0.000026

The Law of Charles.—If a quantity of gas which is confined in a vessel A (Fig. 120) by a frictionless piston P, at atmospheric pressure and 0° C., is heated to 1° C. it will expand 1/273 (or 0.00367) of its original volume; so that its volume becomes 1.00367 times as great. The fact that this value of β (Eq. 77) is practically the *same* for *all gases* was discovered by Charles and is known as the *Law of Charles.*

If, now, the piston is prevented from moving, then, as the gas is heated it cannot expand, but its *pressure* will *increase* 1/273 for each degree rise in temperature, as might be detected by the attached manometer M; while if *cooled* 1°, its pressure will *decrease* 1/273. If cooled to 10° below zero its pressure will decrease 10/273 of its original value, etc. Hence the *inference,* that if it were possible to cool a gas to −273° C. it would *exert no pressure whatever.*

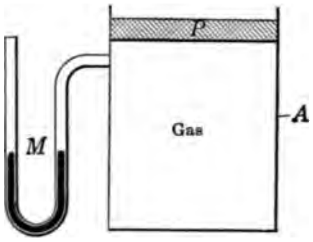

FIG. 120.

Absolute Zero and the Kinetic Theory of Gases.—According to the *Kinetic Theory of Gases,* a gas exerts pressure because of the to-and-fro motion of its molecules (Sec. 131). These molecules are continually colliding with each other, and also bombarding the walls of the enclosing vessel. The impact of the molecules in this bombardment gives rise to the pressure of gases, just as we know that a ball, thrown against the wall and then rebounding from it, reacts by producing a momentary thrust against the wall. Millions of such thrusts per second would, however, give rise to a steady pressure. Under ordinary conditions the average speed of the air molecules required to produce a pressure of 15 lbs. per sq. in. is about 1400 ft. per sec. But a body is supposed to have *heat energy* due to the *motion* of its molecules. It may therefore be said: (*a*) that at −273° C. a gas would *exert no pressure* (see above); hence (*b*) that its *molecular motion must cease;* and therefore (*c*) that it would have *no heat energy* at this temperature. When a body has lost all of its heat energy, it cannot possibly become any colder. This temperature of −273° C. is therefore called the *Absolute Zero* It is interesting to note that extremely low temperatures, within a few degrees of the absolute zero, have been produced artificially. By permitting liquid helium to evaporate in a par-

tial vacuum, Kammerlingh-Onnes (1908) produced a temperature of $-270°$ C., or within 3° of the absolute zero.

172. The Absolute Temperature Scale.—If the above absolute zero is taken as the starting point for a temperature scale, then on this scale, called the *Absolute Centigrade Scale*, ice melts at $+273°$; water boils at 373° (373° A.); a temperature of 20° C. = 293° A., and $-10°$ C. = 263° A., etc. This absolute scale is of great value from a scientific point of view. Its use also greatly simplifies the working of certain problems.

It will now be shown that if the pressure upon a gas is kept constant while its temperature is increased from T_1 to T_2, then its volume will be increased in the ratio of these two temperatures expressed in the *absolute* scale. In other words,

$$\frac{V_2}{V_1}=\frac{T_2}{T_1}$$

in which V_1 and V_2 represent the volume of the gas at the lower and higher temperatures respectively, and T_1 and T_2, the corresponding temperatures in the *absolute* scale.

Proof: Obviously $T_1=t_1+273$, and $T_2=t_2+273$; *i.e.*, the centigrade readings t_1 and t_2 are changed to absolute temperature readings by adding 273, which is the difference between the zeros of the two scales. From Eq. 76, since β is 1/273, we have

$$V_1=V_o\left(1+\frac{t_1}{273}\right),$$

and likewise

$$V_2=V_o\left(1+\frac{t_2}{273}\right)$$

whence

$$\frac{V_2}{V_1}=\frac{V_o\left(1+\frac{t_2}{273}\right)}{V_o\left(1+\frac{t_1}{273}\right)}=\frac{273+t_2}{273+t_1}=\frac{T_2}{T_1}$$

i.e.,

$$\frac{V_2}{V_1}=\frac{T_2}{T_1} \text{ (pressure being kept constant)} \tag{80}$$

Eq. 80 shows that if the *absolute* temperature of a certain quantity of gas is made say 5/4 as great, its volume becomes 5/4 as great; while if the absolute temperature is doubled the volume is doubled, etc. It must be borne in mind that Eq. 80 holds only in case the gas, when heated, is free to expand against a constant pressure. A discussion of Fig. 121 will make clear

the application of Eq. 80. Let A be a quantity of gas of volume V_1 and temperature 27° C. confined in a cylinder by a frictionless piston of negligible weight. Let the upper surface of the piston be exposed to atmospheric pressure. The gas in A will then also be under atmospheric pressure regardless of temperature change. For, as the gas is heated, it will expand and push the piston upward; the pressure, however, will be unchanged thereby, *i.e.*, the pressure will be constant, and therefore Eq. 80 will apply. Next let the gas in A be heated from 27° C. to 127° C., *i.e.*, from 300° A. to 400° A. Since the absolute temperature is 4/3 as great as before, we see from Eq. 80 that P, will be raised to a position P_1 such that the volume of the gas will be 4/3 of its former volume. Experiment will show that the new volume is 4/3 times the old, thus verifying the equation. Let us again emphasize the fact that the two volumes are to each other as the corresponding *absolute temperatures, not centigrade temperatures.*

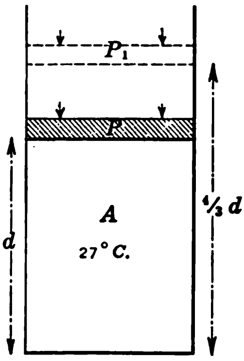

Fig. 121.

Since, as above stated, the pressure of a body of gas that is not permitted to expand increases $t/273$ of its value when the gas is heated from 0° to t° C., it follows that the pressures p_1 and p_2 corresponding to the temperatures t_1 and t_2, are given in terms of p_o (the pressure when the temperature is zero) by the equations

$$p_1 = p_o\left(1 + \frac{t_1}{273}\right) \text{ and } p_2 = p_o\left(1 + \frac{t_2}{273}\right)$$

from which (see derivation of Eq. 80) we have

$$\frac{p_2}{p_1} = \frac{T_2}{T_1} \text{ (volume constant)} \tag{81}$$

This equation shows that if any body of gas, contained in a rigid vessel to keep its volume constant, has its *absolute* temperature increased in a certain *ratio*, then its pressure will be increased in the *same ratio.*

Boyle's Law is expressed in Eq. 72 as $pV = K$. Consequently if the pressure on the gas in question is increased to p_1 the volume will decrease to V_1, but the product will still be K; *i.e.*, $p_1V_1 = K$. Likewise $p_2V_2 = K$, and therefore $p_1V_1 = p_2V_2$ or $V_2/V_1 = p_1/p_2$.

Summarizing, we may write the three important gas laws, namely Boyle's, Charles's, and the one referring to pressure variation with temperature, thus:

$$\left(\frac{V_2}{V_1}=\frac{p_1}{p_2}\right)_T \tag{72 bis}$$

$$\left(\frac{V_2}{V_1}=\frac{T_2}{T_1}\right)_p \tag{80 bis}$$

$$\left(\frac{p_2}{p_1}=\frac{T_2}{T_1}\right)_V \tag{81 bis}$$

Observe that the subscript T indicates that Eq. 72 is true only if the gas whose pressure and volume are varied is maintained at a constant *temperature*. The subscript p of Eq. 80 indicates that the *pressure* to which the gas is subjected must not vary, and V of Eq. 81, that the *volume* must not vary.

Attention is called to the fact that the three important variables of the gas, namely pressure, volume, and temperature, might all change simultaneously. If the *temperature* of the gas is kept constant, Boyle's Law (Eq. 72) states that the volume varies *inversely* as the applied pressure. Eq. 80 states that if the *pressure* upon the gas is kept constant, then the volume varies *directly* as the *absolute temperature;* while Eq. 81 states that if the *volume* of the gas is kept constant, then the pressure varies directly as the *absolute temperature.*

The General Case.—In case both the temperature of a gas and the pressure to which it is subjected change, then the new volume (note that all three variables change) may easily be found by considering the effect of each change *separately; i.e.*, by successively applying Boyle's Law and Charles's Law. To illustrate, let the volume of gas in A (Fig. 121), when at atmospheric pressure and 20° C., be 400 cm.³, and let it be required to find its volume if the pressure is increased to $2\frac{1}{2}$ atmospheres, and its temperature is raised to 110° C. The new pressure is 5/2 times the old; hence, due to *pressure* alone, in accordance with Boyle's Law, the volume will be reduced to $2/5\times400$ cm.³ The original temperature of 20° C. is 293° A., and the new temperature is 383° A.; hence, due to the *temperature change alone*, the volume would be $383/293\times400$ cm.³ Considering *both* effects, the new volume would then be

$$V=\frac{2}{5}\times\frac{383}{293}\times400\text{ cm.}^3$$

We may proceed in a similar manner if both the volume and the temperature are changed, and the new *pressure* that the gas will be under is required in terms of the old pressure.

173. The General Law of Gases.—We shall now develop the equation expressing the relation between the old and the new values of pressure, volume, and temperature of some confined gas when all three of these quantities are changed. Let 1, 2, and 3, respectively, be the initial, second, and final positions of the piston A (Fig. 122). In the initial state, A confines a certain quantity of gas of volume V_o, pressure p_o (say 1 atmosphere), and temperature T_o (say 0° C. or 273° A).

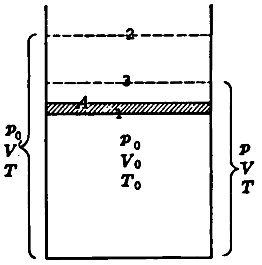

Fig. 122.

The second state is produced by heating the gas from T_o to T, in which T/T_o expressed in the absolute scale is, say, about 3/2. This change in temperature causes the gas to expand against the constant pressure p_o until A is at 2, the new volume V' being about $\frac{3}{2}$ V_o. In this second state of the gas, its condition is represented by p_o, V', and T as indicated in the sketch, and, from Eq. 80, we have

$$\frac{V'}{V_o}=\frac{T}{T_o}, \text{ or } V'=\frac{V_o}{T_o}T$$

The third state of the gas, represented by p, V, and T, is produced by placing a weight on A, thereby increasing the pressure from p_o to p (as sketched $p/p_o=5/4$ approx.), and pushing the piston from position 2 to its final position at 3, and consequently reducing the volume from V' to V (as sketched $V/V'=4/5$ approx.). From Boyle's Law (Eq. 72 bis, just given),

$$\frac{V}{V'}=\frac{p_o}{p}, \text{ or } pV=p_oV'$$

Substituting in this equation the value of V' given above, we have

$$pV=p_oV'=\frac{p_oV_o}{T_o}T=\frac{p_oV_o}{273}T=RT$$

that is,

$$pV=RT \tag{82}$$

in which R is equal to $\frac{p_o V_o}{273}$, and is therefore a known constant if p and V_o are known. Obviously, if twice as great a mass of the same gas, or an equal mass of some other gas half as dense, were placed under the piston, the constant R would then become twice as large.

Eq. 82 expresses the *General Law of Gases*, and is called the *General Gas Equation*. From this general equation, we see (*a*) that for a *given mass* of gas the volume varies *inversely* as the pressure if the temperature is constant (Boyle's Law); (*b*) that the volume varies *directly* as the absolute temperature T if the pressure p is constant (Law of Charles); and (*c*) that the pressure varies *directly* as the absolute temperature if the volume V is constant. The law embodied in (*c*) has not received any name.

Let us now use Eq. 82 to work the problem given under the heading "The General Case" (Sec. 172). Let us represent the first state by $p_1V_1 = RT_1$ and the second state by $p_2V_2 = RT_2$.

Then
$$V_1 = \frac{RT_1}{p_1} \text{ and } V_2 = \frac{RT_2}{p_2}$$

Consequently
$$\frac{V_2}{V_1} = \frac{RT_2}{p_2} \div \frac{RT_1}{p_1}$$

or
$$V_2 = \frac{p_1}{p_2} \times \frac{T_2}{T_1} V_1, \text{ or } \frac{2}{5} \times \frac{383}{293} \times 400$$

as before found. Let us again emphasize the fact that T, T_1, and T_2 represent temperatures on the *absolute scale*.

174. The Thermocouple and the Thermopile.—If a piece of iron wire I (Fig. 123) has a piece of copper wire C fastened to each end of it as shown, it will be found that if one point of contact of these two dissimilar metals, say, B is kept hotter than the other junction A, an electric current will flow in the direction indicated by the arrows. This current might be measured by the instrument D. If B is, say, 60° hotter than A, the electric current will be about 6 times as large as if it is only 10° hotter. Two such junctions so used constitute a *Thermocouple*. Any two different metals may be used for a thermocouple. Antimony and bismuth give the strongest electrical effect for a given difference of temperature between junctions.

One hundred or so thermocouples, made of heavy

properly connected, form a *Thermobattery* of considerable strength. The greatest usefulness of thermocouples, however, is in delicate temperature measurements by means of the thermopile.

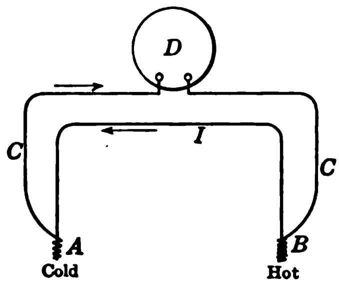

Fig. 123.

The Thermopile.—By observing the readings of D while the temperature difference between A and B is varied through a considerable range (Fig. 123), in other words, by *calibrating* the thermocouple, it becomes a thermometer for measuring temperature differences. A large number of such thermocouples properly connected constitute a *Thermopile*, which will detect exceedingly small differences of temperature. The thermopile readily detects the heat radiated from the hand, or from a lighted match, at a distance of several feet.

PROBLEMS

1. Express 60° C. and −30° C. on the Fahrenheit scale, and also on the absolute scale.

2. Express 200° A. on the Fahrenheit scale, and also on the centigrade scale.

3. An iron rail is 32 ft. long at 0° C. How long is it on a hot day when at 40° C.?

4. A certain metal bar, which is 3 meters in length at 20° C., is 0.30 cm. longer at 100° C. Find α for this metal.

5. If the combined lengths of the iron rods a, b, and d (Fig. 119) is 100 cm., how long must c and e each be to secure exact temperature compensation?

6. How many H.P. does the sun expend upon one acre at noon? Assume the sun to be directly overhead.

7. The cavity of a hollow brass sphere has a volume of 800 cm.^{3} at 20° C. What is the volume of the cavity at 50° C.?

8. If 600 cm.^{3} of gas, at 20° C. and atmospheric pressure, is heated to 40° C., and is free to expand by pushing out a piston against the pressure of the atmosphere, what will be its new volume?

9. If 6 cu. ft. of air, at 20° C. and atmospheric pressure, is compressed until its volume is 2 cu. ft., and is then heated to 300° C., what will be its new pressure?

CHAPTER XIV

HEAT MEASUREMENT, OR CALORIMETRY

175. Heat Units.—Before taking up the discussion of the measurement of quantity of heat, it will be necessary to define the unit in which to express quantity of heat. The unit most commonly used is the *Calorie.* The calorie may be roughly defined as the quantity of heat required to raise the temperature of one gram of water 1° C. To be accurate, the actual temperature of the water should be stated in this definition, since the quantity of heat required varies with the temperature. Thus, the quantity of heat required to raise the temperature of 1 gram of water through a range of 1° is greater at 0° than at any other temperature, and almost 1 per cent. greater than it is at 20°, at which point it is a minimum.

Some authors select this range from 0° C. to 1° C., others 3°.5 C. to 4°.5 C., 4° C. to 5° C., etc., which gives of course slightly different values for the calorie. In selecting 15° C. to 16° C. as the range, we have a calorie of such magnitude that 100 calories are required to raise the temperature of one gram of water from 0° C. to 100° C. Hence the *calorie* is perhaps best defined as the *amount of heat required to raise the temperature of one gram of water from* 15° *C. to* 16° *C.*

In the British system, unit quantity of heat is the quantity required to raise the temperature of 1 lb. of water 1° F., and is called the *British Thermal Unit,* or B.T.U. Since heat is a form of energy, it may be expressed in energy or work units. One B.T.U. = 778 ft.-lbs. This means that 778 ft.-lbs. of work properly applied to 1 lb. of water, for example, in stirring the water, will raise its temperature 1° F. From the above statement, since 1 lb. of water in falling 778 ft. develops 778 ft.-lbs. of energy, we see that if a 1-lb. mass of water strikes the ground after a 778-ft. fall, and if it were possible to have *all of the heat* developed by the impact used in heating the *water,* then this heat would raise its temperature 1° F. In fact this temperature rise is independent of the quantity of water, and depend the height of fall. For, while the heat energy, de

lbs. of water, due to impact after a 778-ft. fall, would be 3 times as much as above given, the amount of water to be heated would also be 3 times as much, and the resulting temperature rise would therefore be 1° F. as before. The calorie is 4.187×10^7 ergs. That is, if 4.187×10^7 ergs of energy are used in stirring one gram of water, its temperature will rise 1° C. This 4.187×10^7 ergs is often called the *Mechanical Equivalent* of heat. The mechanical equivalent in the English system is 778 ft.-lbs.

176. Thermal Capacity.—The thermal capacity of a body is defined as the number of calories of heat required to raise the temperature of the body 1° C., or it is the amount of heat the body gives off in cooling 1° C. It is clear that a large mass would have a greater thermal capacity than a small mass of the same substance. That mass is not the *only* factor involved is shown by the following experiment.

If a kilogram of lead shot at 100° C. is mixed with a kilogram of water at 0° C., the temperature of the mixture will not be 50°, but about 3°. The heat given up by the kilogram of lead in cooling 97° barely suffices to warm the 1 kilogram of water 3°. In fact the thermal capacity of the water is about 33 times as great as that of the lead; consequently, if 33 kilos of lead had been used in the experiment the temperature of the mixture would have been 50°. The very suggestive and convenient term "*water equivalent*" is sometimes used instead of thermal capacity. Multiplying the mass of a calorimeter by its specific heat gives its thermal capacity or the number of calories required to warm it one degree. Suppose that this number is 60. Now 60 calories would also heat 60 grams of water one degree; hence the "*water equivalent*" of the calorimeter is 60; *i.e.*, the calorimeter requires just as much heat to raise its temperature a given amount as would 60 gm. of water if heated through the same range.

177. Specific Heat.—The *Specific Heat* (s) of a substance may be defined as the number of calories required to heat 1 gm. of the substance 1° C. It is therefore the *thermal capacity per gram* of the substance. This, we see from the definition of the calorie, is practically equal to the *ratio* of the heat required to heat a given mass of the substance through a given range of temperature, to the heat required to heat an *equal mass of water* through the *same* range. Thus, the specific heat of lead is 0.031. This means that it would require 0.031 calorie to heat a gram of lead one degree; which is only 0.031 times as much heat as would be re-

quired to heat a gram of water one degree. The specific heat of a substance is sometimes defined as the *ratio* just given. Since the specific heat (calorie per gram per degree) of water varies with the temperature (Sec. 175), this definition lacks definiteness as compared with the one we are here using.

The table below gives the specific heat of a few substances. From the values given, we see that one calorie of heat imparted to a gram of glass would raise its temperature 5° C., while the same amount of heat imparted to a gram of lead would raise its temperature 1/0.031, or about 32°.5. In popular language it might be said that lead heats 6.5 times as easily as glass, and 32.5 times as easily as water.

The specific heat of a substance is usually expressed in calories per gram per degree. Thus, the specific heat of lead is 0.031 cal. per gm. per deg. It may also be written 0.031 B.T.U.'s per lb. per degree, the degree in this case, however, being the Fahrenheit degree. The proof that the numeric (0.031) is the same in both cases may be left as an exercise for the student.

The specific heat of most substances varies considerably with the temperature. In some cases there is a decrease in its value with temperature rise, while in others there is an increase. In the case of water the specific heat decreases up to 20° C. and then increases. The values given in the table for the different substances are average values, taken at ordinary temperatures (excepting in the case of ice and steam).

Substance	Sp. heat in cal. per gm. per deg.	Substance	Sp. heat in cal. per gm. per deg.
Brass	0.088	Ice	0.504
Copper	0.093	Steam	0.4 approx.
Glass	0.200	Water	1.000 (15° to 16°)
Lead	0.031	Alcohol	0.60
Iron	0.11	Petroleum	0.51

To heat a gram of any substance of specific heat s sufficiently to cause a temperature rise of t degrees requires st calories, *i.e.*, t times as much heat as to cause a rise of 1 degree. Further, to heat M grams t degrees requires M times as much heat as to heat one gram t degrees, or Mst calories; hence, the general expression for the heat H required to heat a body of mass M and specific heat s from a temperature t_1 to a temperature t_2, is

$$H = Ms(t_2 - t_1) \quad (8$$

If the substance cools through this *same range*, then H is the heat *given off*.

178. The Two Specific Heats of a Gas.—In general, a body when heated, expands, and in expanding it does work in pushing back the atmosphere. This work makes it require additional heat energy to warm the body, and therefore makes the specific heat of the body larger than it would have been had expansion not occurred. In case a compressed gas is permitted to expand into a space at lower pressure, the above heat energy is taken from the gas *itself* and chills it greatly. This fact is utilized in the manufacture of liquid air (Sec. 205).

In the case of solids and liquids, this expansion upon being heated is inappreciable, but with gases it is very great. Consequently the specifice heat of a gas, *i.e.*, the number of calories required to heat one gram one degree, is less if the gas in confined in a rigid vessel than if it is allowed to expand against constant pressure when heated. The latter is called the specific heat at *constant pressure*, and is 0.237 for air; while the former is called the specific heat at *constant volume*, and is 0.168 for air. The ratio of the two specific heats of air is 0.237/0.168, or 1.41. This ratio differs for the various gases.

179. Law of Dulong and Petit.—Dulong and Petit, in 1819, found by experiment that for thirty of the elements, the product of the atomic weight and the specific heat (in the solid state) is approximately constant. This so-called constant varies from about 6 to 6.6. For a considerable number of the elements it is 6.4. For gases this constant is about 3.4. This law does not hold for liquids, and there are a few solids that do not follow it at all closely.

Let us now utilize this law in finding the specific heat of iron and gold, whose atomic weights are respectively 56 and 196. The mathematical statement of the law of Dulong and Petit is:

$$\text{Sp. heat} \times \text{atomic weight} = 6.4 \text{ (approximately)} \qquad (84)$$

Whence the specific heat of gold is 6.4/196 or 0.0326, and that of iron 6.4/56 or 0 114. These *computed* values of the specific heat are almost exactly the same as those found *experimentally* for iron and gold.

The above law shows that it takes the *same amount of heat to warm an atom one degree* whether it be a gold atom, an iron atom, or an atom of any other substance which follows this law. For, from Eq. 84, it is obvious that if the atomic weight of one element is three times as great as that of another (compare gold with iron), then its specific heat must be 1/3 as great in order to give the same product—6.4. But if the

atomic weight is three times as great for the first metal as for the second, then the number of atoms per gram will be 1/3 as great, which accounts for the first having 1/3 as great specific heat as the second, provided we assume the *same thermal capacity for all atoms.*

180. Specific Heat, Method of Mixtures.—A method which is very commonly used for determining the *specific* heat of substances is that known as the *method of mixtures.* The method can be best explained in connection with the apparatus used, one form of which is shown in section in Fig. 124. H is a heater containing some water and having a tube T passing obliquely through it as shown. This tube contains the substance, *e.g.*, the shot, the specific heat of which is to be determined. D is a calorimeter, usually of brass, containing some water E.

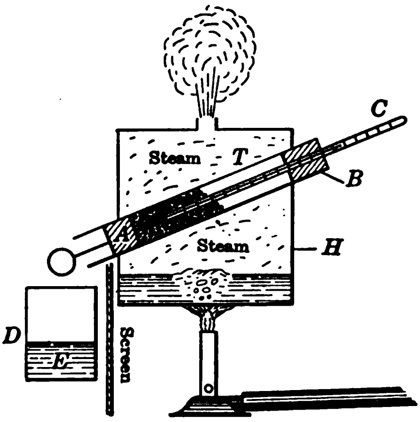

FIG. 124.

First, the shot, the calorimeter D, and the water E, are weighed. Let these masses be M, M_1, and M_2, respectively. Next the water in H is heated to the boiling point and kept boiling for a few minutes. The steam surrounding T soon warms it and the contained shot to 100° C., which may be determined by thermometer C, thrust through cork B. The cork A is now withdrawn, and the hot shot is permitted to fall into the water E to which it rapidly imparts its heat until D, E, and the shot are all at the same temperature. Let this temperature be t', and let the temperature of E before the shot was introduced be t. The heat H_1, which the shot *loses* in cooling from 100° to t', is evidently

equal to the heat H_2 which the calorimeter and water *gain* in rising in temperature from t to t', that is

$$H_1 = H_2 \tag{85}$$

provided no heat passes from the calorimeter to the air or *vice versa* during the mixing process.

This interchange of heat between the calorimeter and the air cannot be totally prevented, but the error arising from this cause is largely eliminated by having D and E a few degrees lower than the room temperature at the *beginning* of the mixing process and a few degrees higher than room temperature at the end; *i.e.*, after D, E, and the shot have come to the same temperature. During the mixing process, the contents of the calorimeter should be stirred to insure a uniform temperature throughout.

Almost always in calorimetric work, it is assumed that the heat given up by the hot body is equal to the heat taken up by the cold body; so that Eq. 85 is the starting point for the derivation of the required equation in all such cases. It is much better to learn how to apply this general equation than to try to memorize special forms of it. One such application will be made here.

If s, s_1, and s_2 represent the specific heats of the shot, calorimeter, and water repectively, then from Eq. 83, the heat *given* up by the shot is $Ms(100-t')$; that *taken* up by the calorimeter is $M_1s_1(t'-t)$; and that *taken* up by the water is $M_2s_2(t'-t)$. Since s_2 is unity it may be omitted, and we have from Eq. 85

$$Ms(100-t') = M_1s_1(t'-t) + M_2(t'-t) = (M_1s_1+M_2)\ (t'-t) \tag{85a}$$

The quantities M, M_1, and M_2 are determined by weighing, and the three temperatures are read from thermometers, so that the one remaining unknown, s, may be solved for.

181. Heat of Combustion.—Chemical changes are, in general, accompanied by the evolution of heat; a few, however, *absorb* heat. Most chemical salts when dissolved in water cool it, in some cases quite markedly. In still other cases solution is attended by the development of heat. A complete study of these subjects is beyond the scope of this volume, but the particular chemical change known as *combustion* is so all-important in connection with commercial heating and power development that a brief discussion of it will be given.

Combustion is usually defined as the violent chemical combination of a substance with oxygen or chlorine, and is accompanied

by heat and light. In a more restricted sense it is what is popularly known as "burning" which practically amounts to the chemical combination of oxygen with hydrogen or carbon.

In scientific work, the *Heat of Combustion* of any substance is the number of calories of heat developed by the complete combustion of 1 gram of that substance. In engineering practice it is the number of B.T.U.'s developed by the complete combustion of 1 pound of a substance. The latter gives 9/5 as large a number as the former for the same substance. Hence it is necessary in consulting tables to determine whether the metric, or the British system is used. Obviously, the burning of one *gram* of coal would heat just as many *grams* of water 1° C. as the burning of a *pound* of coal would heat *pounds* of water 1° C. But to heat a pound of water 1° C. takes 9/5 B.T.U.'s, since 1° C. equals 9/5° F. In the following table, in which the approximate values of the heat of combustion are given in both systems, it will be observed that the numerical values are in the above ratio of 9 to 5.

HEAT OF COMBUSTION WITH OXYGEN

Substance	Product	Calories per gram	B.T.U.'s per pound
Hydrogen(H)	H_2O	34,000	61,000
Carbon (C)	CO_2	7,800	14,000
Marsh gas (CH_4)	CO_2 and H_2O	13,100	23,600
Alcohol (ethyl)	CO_2 and H_2O	7,200	13,000
Petroleum	CO_2 and H_2O	11,000	20,000
Soft coal	Mainly CO_2, H_2O and ash.	7,500 to 8,500	Ave. 14,500
Hard coal		7,800	Ave. 14,000
Wood		4,000 to 4,500	Ave. 7,600
Dynamite		1,300	
Iron	Fe_3O_4	1,600	
Zinc	ZnO	1,300	
Copper	CuO	600	

Hydrogen, it will be seen, produces far more heat per gram than any other substance, indeed over four times as much as its nearest rival, carbon. Coal averages about the same as carbon. Petroleum contains hydrogen combined with carbon (hydrocarbons) and gives, therefore, a higher heat of combustion than pure carbon does. The main gases that are produced in the combustion of all substances known as fuels are water vapor (H_2O) and carbon dioxide (CO_2).

It would be well to memorize the values in the last column for

petroleum, coal, and wood. Observe that dynamite has a surprisingly low heat of combustion. Its effectiveness as an explosive depends upon the *suddenness* of combustion due to the fact that the oxygen is in the dynamite itself, and not taken from the air as in ordinary combustion.

To find how much chemical potential energy in foot-pounds exists in 1 lb. of coal, multiply 14,500 by 778; *i.e.*, multiply the number of B.T.U.'s per pound by the number of foot-pounds in one B.T.U. To reduce this result to H.P.-hours, divide by 550×3600 (1 hr. equals 3600 sec.). Due to various losses of energy in the furnace, boiler, and engine (Chap. XVIII), a steam engine utilizes only about 5 or 10 per cent. of this energy, so that the H.P.-hours above found should be multiplied by 0.05 or 0.10 (depending upon the efficiency of the engine used) to obtain the useful work that may be derived from a pound of coal. With a very good furnace, boiler, and engine, about 1.5 lbs. of coal will do 1 H.P.-hr. of work. Thus it would require about 150 lbs. of coal to run a 100-H.P. engine for an hour.

182. Heat of Fusion and Heat of Vaporization.—As stated in Sec. 162, considerable heat may be applied to a vessel containing ice water and crushed ice without producing perceptible temperature rise until the ice is melted, whereupon further application of heat causes the water to become hotter and hotter until the boiling point is reached, when the temperature again ceases to rise. Other substances behave in much the same way as water. These facts show that heat energy is required to change the substance from the solid to the liquid state, and from the liquid to the vapor state. This heat energy is supposed to be used partly in doing *internal* work against molecular forces. In case the change of state is accompanied by an increase in volume, part of this heat energy is used in doing *external* work in causing the substance to expand against the atmospheric pressure.

The *Heat of Fusion* of a substance is the number of calories required to change a gram of that substance from the solid to the liquid state without causing a rise in temperature. The *Heat of Vaporization* is the number of calories required to change a gram of the substance from the liquid to the vapor state at a definite temperature and at atmospheric pressure. These two changes *absorb* heat while the reverse changes, that is from vapor to liquid and from liquid to solid, *evolve* heat. The amounts of heat *evolved* in these reverse changes are the same respectively

as the amounts *absorbed* in the former changes. This equality should be expected, of course, from the *conservation of energy*.

For water, the heat of fusion is 79.25 calories per gram (also written 79.25 cal./gm.), and the heat of vaporization is 536.5 cal. per gm.; which means that to change one gram of ice at 0° C. to water at 0° C. requires 79.25 calories, and to change 1 gm. of water at 100° to steam at 100° and atmospheric pressure requires 536.5 cal. The value of the latter depends very much upon the temperature. To change a gram of water at 20° to vapor at 20° requires 585 cal., in other words, the heat of vaporization of water at 20° C. is 585 cal. per gm.

From reasoning analogous to that used in changing the heat of combustion from the metric to the British system (Sec. 181), we see that the above heat of vaporization multiplied by 9/5 gives the heat of vaporization in the British system, namely, 966 B.T.U.'s per pound. That is to say, 966 B.T.U.'s are required to change 1 lb. of water at 212° F. to steam at the same temperature. The heat of fusion is rarely expressed in the British system.

HEAT OF FUSION OF VARIOUS SUBSTANCES

Substance	Melting temperature	Calories per gram	Substance	Melting temperature	Calories per gram
Ice	0° C.	79.25	Silver	960° C.	21
Ice	−6	76	Cadmium	315	13.7
Nitrate of soda	306	65	Sulphur	115	9.37
Paraffin	52	35	Lead	325	5.86
Zinc	415	28	Mercury	−38.8	2.82

HEAT OF VAPORIZATION OF VARIOUS SUBSTANCES

Substance	Temperature	Calories per gram	Substance	Temperature	Calories per gram
Water	0° C.	595	Ether	35° C.	90
Water	100	536.5	Mercury	357	62
Ammonia (NH_3)	17	295	Chloroform	61	58.5
Alcohol (ethyl)	78	206	Carbon dioxide	0	56
Sulphur	448	362	Carbon dioxide	30.8	3.72

183. Bunsen's Ice Calorimeter.—A very sensitive form of ice calorimeter is that of Bunsen, in which the amount of ice melted is determined from the accompanying change of volume. It con-

sists of a bulb A (Fig. 125), with a tube B attached, and a test tube C sealed in as shown. The space between A and C is completely filled with water except the lower portion, which contains mercury as does also a portion of B.

By pouring some ether into C and then evaporating it by forcing a stream of air through it (Sec. 197), some ice E is formed about C. As this ice forms, expansion occurs, which forces the mercury farther along in B to, say, point a. Next, removing all traces of ether from C, drop in a known mass of hot substance D at a known temperature t'. The heat from D melts a portion of the ice E, and the resulting contraction causes the mercury to recede, say to a'. The volume of the tube between a and a' is evidently the difference between the volume of the ice melted by D and that of the resulting water formed; and hence, if known, could be used to determine the amount of ice melted. Multiplying this amount by 79.25 would give the number of calories of heat given off by D in cooling to 0° C.

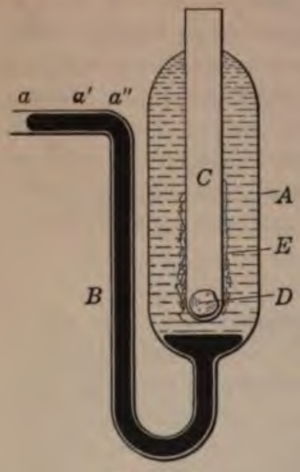

FIG. 125.

A simpler method, however, is to calibrate the instrument by noting the distance, say aa'', that the mercury column recedes when one gram of water at 100° is introduced into C. Suppose it is two inches. Then, since the gram of water in cooling to 0° C. would impart to the ice 100 calories, we see that a motion of one inch corresponds to 50 calories. Accordingly the distance aa' in inches, multiplied by 50, gives the number of calories given off by D in cooling from t' to zero. This enables the calculation of the specific heat of the substance D.

184. The Steam Calorimeter.—Dr. Joly invented a very sensitive calorimeter, known as the Joly Steam Calorimeter, in which the amount of heat imparted to a given specimen in raising its temperature through a known range, is determined from the amount of steam that condenses upon it in heating it. A specimen whose specific heat is sought, *e.g.*, a piece of ore A (Fig. 126), is suspended in an inclosure B by a wire W passing freely through a small hole above, and attached to one end of the beam of a sensi-

tive beam balance. Weights are added to the other end of the beam until a "balance" is secured. As steam is admitted to the inclosure, it condenses upon the ore until the temperature of the ore is 100°, whereupon condensation ceases. The additional weight required to restore equilibrium, multiplied by 536.5, gives the number of calories required to heat the ore and pan from a temperature t (previously noted) to 100°. For it is evident that each gram of steam that condenses upon the ore imparts to it 536.5 calories. If the mass of the ore is known, its specific heat can readily be computed (Eqs. 83 and 85).

The pan in which the ore is placed catches the drip, if any. Obviously the amount of steam that would condense upon the pan in the absence of the ore must be found, either by calculation or by experiment, and be subtracted from the total. By the use

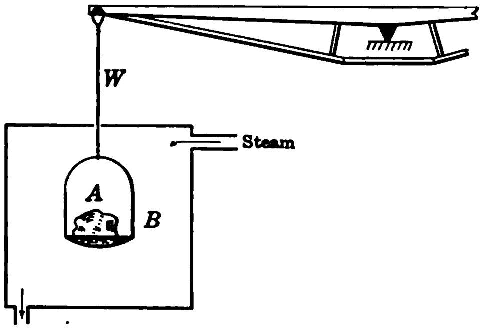

Fig. 126.

of certain refinements and modifications which will not be discussed here, the instrument may be employed for very delicate work, such as the determination of the specific heat of a compressed gas contained in a small metal sphere.

185. Importance of the Peculiar Heat Properties of Water.—The fact that the specific heat, heat of fusion, and heat of vaporization of water are all relatively large is of the utmost importance in influencing the climate. It is also of great importance commercially. From the conservation of energy it follows that if it takes a large amount of heat (heat absorbed) to warm water, to vaporize it, or to melt ice; then an equally large amount of heat will be *given off* (evolved) when these respective changes take place in the reverse sense—that is, when water cools, vapor condenses, or water freezes.

Specific Heat.—In connection with the subject of specific heat, it is seen that the amount of heat a given mass absorbs in being warmed through a given range of temperature depends upon its specific heat. From this fact it is evident that a body of water would change its temperature quickly with change of temperature of the air, if its specific heat were *small.* The specific heat of water is much larger than for most other substances, as may be seen from the table (Sec. 177). Note also that water has about twice as large a specific heat as either ice or steam. Because of the large specific heat of water it warms slowly and cools slowly; so that during the heat of the day a lake *cools* the air that passes over it, while in the cool of the night, it *warms* the air. This same effect causes the temperature on islands in mid-ocean to be much less subject to sudden or large changes than it is in inland countries.

Heat of Fusion.—It requires 79.25 calories to melt 1 gram of ice; hence, according to the conservation of energy, a gram of water must give off approximately 80 calories of heat when it changes to ice. If the heat of fusion were very small, *say 2 calories* per gram, a river would not need to give off nearly so much heat in order to change to ice, so that it might, under those conditions, freeze solid in a night with disastrous consequences to the fish in it, and to the people dependent upon it for water supply. Under these circumstances, it would also be necessary to buy about 40 times as much ice to get the same cooling effect that we now obtain.

Heat of Vaporization.—Since it requires about 600 calories to change a gram of water at *ordinary temperatures* to *vapor,* it follows, from the conservation of energy, that when a gram of vapor condenses to water it gives off about 600 calories of heat. This heat, freed by the condensation of vapor, is one of the main causes of winds. The heat developed causes the air to become lighter, whereupon it rises, and the surrounding air as it rushes in is called a wind (Sec. 223).

If the heat of vaporization of water were much smaller, evaporation and cloud formation would be much more rapid, resulting ultimately in dried rivers and ponds, alternating with disastrous floods.

The *increase in volume* which accompanies the *freezing of water* is of the utmost importance in nature. If ice were more dense than water, it would sink to the bottom when formed, and our

shallow ponds and our rivers would readily freeze solid. As it is, the ice, being less dense, remains at the surface, and thus forms a sheath that protects the water and prevents rapid cooling.

The Maximum Density of water occurs at 4° C. If water at this temperature is either heated or cooled it expands, and consequently becomes less dense. Hence in winter, as the surface water of our lakes becomes cooler and therefore denser, it settles to the bottom, and other water that takes its place is likewise cooled and settles, thus establishing convection currents (Sec. 208). Through this action the temperature of the entire lake tends to become 4° C. At least it cannot become colder than this temperature, for as soon as any surface water becomes *colder* than 4° it becomes less dense, and therefore remains on the surface and finally freezes. As soon as the convection currents cease, the chilling action practically ceases, so far as the deeper strata of water are concerned, for water is a very poor conductor of heat.

186. Fusion and Melting Point.—The *Fusion* of a substance is the act of melting or changing from the solid to the liquid state, and the *Melting Point* is the temperature at which fusion occurs. The melting point of ice is a perfectly definite and sharply defined temperature; for which reason it is universally used as one of the standard temperatures in thermometry. Amorphous or non-crystalline substances, such as glass and resin, upon being heated, change to a soft solid or to a viscous liquid, and finally, when considerably hotter, become perfectly liquid. Such substances have no well-defined melting point.

Solutions of solids in liquids have a *lower freezing point* than the pure solvent, and the amount of lowering of the freezing point is, as a rule, closely proportional to the strength of the solution. It might also be added that the dissolved substance also *raises the boiling point*. For example, a 24 per cent. brine freezes at −22° C. and boils at about 107°. Many other substances dissolved in water produce the same effect, differing in degree only. *Solvents* other than water are affected in the same way.

Alloys, which may be looked upon as a solution of one metal in another, behave like solutions with regard to lowering of the melting point. Thus Rose's metal, consisting by weight of bismuth 4 parts, lead 1, and tin 1, melts at 94° C. and consequently melts readily in boiling water. Wood's metal—bismuth 4, lead 2, tin 1, and cadmium 1—melts at 70°. Solder, consisting of lead

37 per cent., and tin 63 per cent., melts at 180° C. Using either a greater or smaller percentage of lead raises the melting point of the solder. In all these cases, the melting point of the alloy is far lower than that of any of its components, as may be seen by consulting the accompanying table.

TABLE OF MELTING POINTS

Substance	Temperature	Substance	Temperature
Hydrogen	−255° C.	Lead	325° C.
Nitrogen	−210	Zinc	415
Mercury	− 38.8	Salt (NaCl)	800
Ice	0	Silver	960
Phosphorus	44	Gold	1064
Tin	233	Iron	1200 to 1600
Bismuth	267	Platinum	1755
Cadmium	315	Iridium	2300

Supercooling.—It is possible to cool water and other liquids several degrees below the normal freezing point before freezing occurs. Thus water has been cooled ten or twenty degrees below zero, but the instant a tiny crystal of ice is dropped into the water, freezing takes place, and the heat evolved (79.25 cal. per gm. of ice formed) rapidly brings its temperature up to zero. Dufour has shown that small globules of water, immersed in oil, may remain liquid from −20° C. to 178° C. Some other substances, *e.g.*, acetamid and "hypo" (sodium hyposulphite), are not so difficult to supercool as is water.

Pressure.—Some substances when subjected to great pressure have their melting point raised, while others have it lowered. Clearly, if a substance in melting contracts (*e.g.*, ice, Sec. 187), we would expect pressure to aid the melting process, and hence cause the substance to melt at a lower temperature than normal. It has been determined, both by theory and by experiment, that ice melts at 0.0075° C. lower temperature for each additional atmosphere of pressure exerted upon it. This effect is further discussed under *Regelation* (Sec. 188) and *Glaciers* (Sec. 189).

187. Volume Change During Fusion.—Some substances expand during fusion, while others contract. Thus in changing from the liquid to the solid state, water expands 9 per cent., and bismuth 2.3 per cent.; while the following contract, silver (10 per cent.), zinc (10 per cent.), cast iron (1 per cent.). Obviously silver and zinc do not make good, clear-cut castings for the reason that in solidifying *they shrink* away from the mold. Silver

and gold coins have the impressions *stamped* upon them. Iron casts well because it shrinks but slightly. The importance in nature of the expansion of water upon freezing has already been discussed (Sec. 185).

188. Regelation.—If a block of ice *B* (Fig. 127) has resting across it a small steel wire *w*, to each end of which is attached a heavy weight, it will be found that the wire slowly melts its way through the ice. The ice immediately below the wire is subjected to a very high pressure and therefore melts even if slightly below zero (Sec. 186). The water thus formed is very *slightly below* 0° C., and flows around above the wire where it again freezes, due to the fact that it is now at ordinary pressure, and that the surrounding ice is also a trifle below 0° C. Thus the wire passes through the ice and leaves the block as solid as ever. The refreezing of the water as it passes from the region of high pressure is called *Regelation*. Since every gram of ice melted below the wire requires about 80 calories of heat, and since this heat must come from the surrounding ice, we see why the ice above the wire and the water and ice below are cooled *slightly* below 0° C.

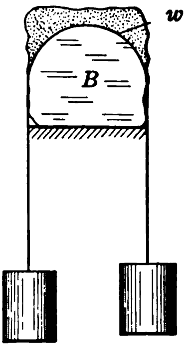

Fig. 127.

If two irregular pieces of ice are pressed together, the surface of contact will be very small and the pressure correspondingly great; as a result of which some of the ice at this point will melt. The water thus formed, being at ordinary atmospheric pressure and slightly below zero as just shown, refreezes and firmly unites the two pieces of ice. A similar phenomenon occurs in the forming of snow balls by the pressure of the hand.

In *skating*, regelation probably plays an important rôle, as pointed out by Dr. Joly. With a sharp skate, the skater's weight bears upon a very small surface of ice, which may cause it to melt even though several degrees below zero. Thus the skate *melts* rather than wears a slight groove in the ice. If the ice is very cold the skate will not "bite," *i.e.*, it will not melt a groove, unless very sharp. Friction is also probably much reduced by the film of water between the skate and the ice.

189. Glaciers.—Glaciers are great rivers of ice that flow slowly down the mountain gorges, sometimes (in the far north) reaching the sea, where they break off in huge pieces called ice-

bergs, which float away to menace ocean travel. Glaciers owe both their origin and their motion, in part, to regelation. Due to the great pressure developed by the accumulated masses of snow in the mountains or in the polar regions, part of the snow is melted and frozen together as solid ice, forming glaciers, just as the two pieces of ice mentioned above were frozen together.

As the glacier flows past a rocky cliff that projects into it, the ice above, although at a temperature far below zero, melts because of the high pressure, flows around the obstacle, and freezes again below it. The velocity of glaciers varies from a few inches a day to ten feet a day (Muir Glacier, Alaska), depending upon their size and the slope of their beds. The mid-portion of a glacier flows faster than the edge and the top faster than the bottom, evidencing a sort of tar-like viscosity.

Glaciers in the remote past have repeatedly swept over vast regions of the globe, profoundly modifying the soil and topography of those regions. The northern half of the United States shows abundant evidence of these ice invasions (see Geology). At present, glaciers exist only in high altitudes or high latitudes.

190. The Ice Cream Freezer.—Experiments show that ice, in the presence of common salt, may melt at a temperature far below 0° C. (−22° C. or −7°.4 F.). This fact makes possible the production of very low temperatures by artificial means. The most familiar example of the practical application of this principle is the ice cream freezer. The broken ice, mixed with salt, surrounds an inner vessel which contains the cream. The rotation of the inner vessel serves the two-fold purpose of agitating the cream within, and mixing the salt and ice without. The revolving vanes within aerate the cream, thus making it light and "velvety." The freezing would take place, however, without revolving either vanes or container, but the process would require more time, and the product would be inferior. As the ice melts, the water thus formed dissolves more salt, and the resulting brine melts more ice, and so on. One part (by weight) of salt to three parts of crushed ice or snow gives the best results. This is the proper proportion to form a saturated brine at that low temperature.

The theory of the production of low temperatures by freezing mixtures, such as salt and ice, is very simple. Every gram of ice that melts requires 79.25 calories of heat to melt it. If this heat is *supplied*, by a flame for example, the temperature remains at

0° C. until practically all of the ice is melted. If the melting of the ice is caused by the presence of some salt or other chemical, the requisite 79.25 *calories* of heat for *each gram* melted *must come* from the *freezing mixture* itself, and from its surroundings, mainly the *inner vessel* of the freezer, thus causing a fall of temperature. Still lower temperatures may be obtained with a mixture of calcium chloride and snow. The cheapness of common salt, and the fact that −22° C. is sufficiently cold for rapid freezing, accounts for its universal use. In fact, while being frozen, that is, while being agitated, the cream should be but a few degrees below zero to secure the maximum "lightness."

PROBLEMS

1. How much heat would be required to change 20 gm. of ice at −10° C. to water at 20° C.?

2. Now much heat would be required to change 40 gm. of water at 30° C. to steam at 140° C.? The heat of vaporization at 140° C. is about 510 cal. per gm.

3. If 40 gm. of water at 80° C. is mixed with 30 gm. of water at 20° C., what will be the temperature of the mixture? Neglect the heat capacity of the calorimeter. Suggestion: Call the required temperature t, and then solve for it.

4. Find the "water equivalent" of a brass calorimeter that weighs 150 gm.

5. Same as problem 3, except that the heat capacity of the calorimeter containing the cold water is considered. The weight of the calorimeter is 60 gm., and the specific heat of the material of which it is composed is 0.11.

6. A certain calorimeter, whose water equivalent is 20, contains 80 gm. of water at 40° C. When a mass of 200 gm. of a certain metal at 100° C. is introduced, the temperature of the water and the calorimeter rises to 55° C. Find the specific heat of the metal.

7. How many B.T.U.'s would be required to change 100 lbs. of ice at 12° F. to water at 80° F.? (Secs. 181 and 182.)

8. How many B.T.U.'s would be required to change 100 lbs. of water at 80° F. to steam at 320° F.? When the water in the boiler is heated to 320° F. the steam pressure is about 90 lbs. per sq. in., and the heat of vaporization, in the *metric system*, is about 495 cal. per gm.

9. How many pounds of soft coal would be required to change 100 lbs. of water at 70° F. to steam at 212° F.?

Assume that 10 per cent. of the energy is lost through incomplete combustion, and that 30 per cent. of the remaining heat escapes through the smokestack, or is lost by radiation, etc. See table, Sec. 181.

10. How high would the energy obtainable from burning a ton of coal raise a ton of material, (*a*) assuming 12.5 per cent. efficiency for the steam engine? (*b*) assuming 100 per cent. efficiency?

CHAPTER XV

VAPORIZATION

191. Vaporization Defined.—Vaporization is the general term applied to the process of changing from a liquid or solid to the vapor state. Vaporization takes place in three different ways, *evaporation*, *ebullition* (Sec. 192), and *sublimation*. The first two refer to the change from liquid to vapor, the last, from solid to vapor. If a solid passes directly into the vapor state without first becoming a liquid, it is said to sublime, and the process is *sublimation*. Snow sublimes—slowly disappearing when perfectly dry and far below zero. Other substances besides snow sublime; notably camphor, iodine, and arsenic.

In whatever manner the vaporization occurs, it requires heat energy to bring it about, and when the vapor condenses an equal amount of heat (the heat of vaporization, Sec. 182) is evolved. Hence a molecule must contain more energy when in the vapor state than when in the liquid state, due, according to the kinetic theory (Sec. 171), to the greater rapidity of its to-and-fro motion. The above *absorption* and *evolution* of heat which accompany vaporization and condensation, respectively, are of the utmost importance in nature (Sec. 185) and also commercially. In steam heating, the heat is evolved—about 540 calories for each gram of steam condensed—at the place where the condensation occurs, namely, in the radiator. Note the similar absorption of heat in the melting of ice (utilized in the ice-cream freezer, Sec. 190) and the evolution of heat in the freezing of water. Thus, vaporization and melting are *heat-absorbing* processes; while the reverse changes of state, condensation and freezing, are *heat-liberating* processes.

192. Evaporation and Ebullition.—The heat energy of a body is supposed to be due to its molecular motion (Sec. 160), which, as the body is heated, becomes more violent. The *evaporation* of a liquid may be readily explained in accordance with this theory. Let A (Fig. 128) be an air-tight cylinder containing some water B, and provided with an air-tight piston P. Suppose

this piston, originally in contact with the water, to be suddenly raised, thereby producing above the water a vacuum. As the water molecules near the surface of the water move rapidly to and fro some of them escape into the vacuous space above, where they travel to and fro just as do the molecules of a gas. After a considerable number of these molecules have escaped from the water, many of them in their to-and-fro motion will again strike the water and be retained. Thus we see that there is a continual passage of these molecules from the water to the vapor above, and *vice versa*. The vapor above is said to be *saturated* when, in this interchange, *equilibrium* has been reached; *i.e.*, when the rate at which the molecules are *returning* to the water is equal to the rate at which they are *escaping* from it.

The saturated water vapor above the water in A exerts a pressure due to the impact of its molecules against the walls, just as

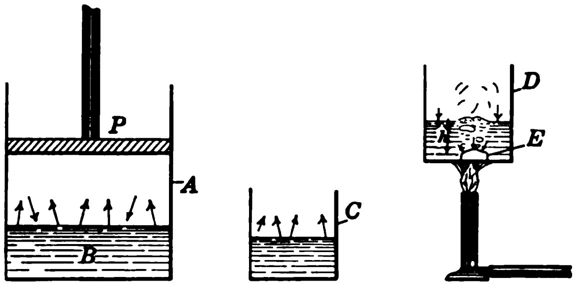

FIG. 128.

any gas exerts pressure. This vapor pressure is about 1/40 atmosphere when the water is at room temperature and becomes 1 atmosphere when the water and the cylinder are heated to the boiling point.

Ebullition.—When water is placed in an open vessel (C, Fig. 128) *evaporation* into the air takes place from the surface, as already described for vessel A. When heated to the boiling point (D, Fig. 128), bubbles of vapor form at the point of application of heat and rise to the surface, where the vapor escapes to the air. When *vaporization* takes place in this manner, *i.e.*, by the formation of bubbles within the liquid, it is called *Ebullition*, or boiling; while when it takes place simply from the surface of the liquid, it is called *Evaporation*.

As has already been stated, the pressure of saturated water

vapor at 100° C. is one atmosphere. This will be evident from the following considerations. In the formation of the steam bubble E below the surface of the water in the open dish D, it is clear that the pressure of the vapor in the bubble must be equal to the atmospheric pressure or it would collapse. Indeed it must be a trifle greater than atmospheric pressure, because the pressure upon it is one atmosphere plus the slight pressure (hdg) due to the water above it. We are now prepared to accept the general statement that *any liquid will boil in a shallow open dish when it reaches that temperature for which the pressure of its saturated vapor is one atmosphere.* This temperature, known as the boiling point at atmospheric pressure or simply the boiling point, differs widely for the various substances.

193. Boiling Point.—Unless otherwise stated, the *Boiling Point* is understood to be that temperature at which boiling occurs at *Standard Atmospheric Pressure* (760 mm. of mercury). For pure liquids, this is a perfectly definite, sharply defined temperature, so definite, indeed, that it may be used in identifying the substance. Thus if a liquid boils at 34.°9 we may be fairly sure that it is ether; at 61°, chloroform; at 290°, glycerine. The boiling points for a few substances are given in the following table.

BOILING POINTS AT ATMOSPHERIC PRESSURE

Substance	Temperature	Substance	Temperature
Helium..........	−267° C.	Alcohol (wood)..	66° C.
Hydrogen........	−253	Alcohol (ethyl)...	78.4
Nitrogen.........	−194	Water..........	100
Oxygen..........	−184	Glycerine.......	290
Carbon dioxide[1]...	− 80	Mercury........	357
Ammonia........	− 38.5	Sulphur.........	448
Ether............	34.9	Zinc............	about 930
Chloroform.......	61	Lead...........	about 1500

Solutions of solids in liquids have a higher boiling point, as well as a lower freezing point (Sec. 186) than the pure solvent. Thus a 24 per cent. brine, which we have seen freezes at −22° C., boils at about 107° C. The elevation of the boiling point is approximately proportional to the concentration for weak solutions. A 24 per cent. *sugar* solution boils at about 100.°5 C.

194. Effect of Pressure on the Boiling Point.—When a change of state is accompanied by an *increase* in volume, we readily see

[1] Carbon dioxide (CO_2) sublimes at −80° C. and atmospheric pressure. Under a pressure of 5.1 atmospheres it *melts* and also *boils* at −57° C.

that subjecting the substance to a high pressure will *oppose* the change; while if the change of state is accompanied by a *decrease* in volume, the reverse *is true*, *i.e.*, pressure will then *aid* the process. Consequently, since water expands in changing to either ice or steam, subjecting it to high pressure makes it "harder" either to freeze or boil it; *i.e.*, pressure *lowers* the freezing point, (Sec. 186) and *raises* the boiling point. The latter volume change is vastly greater than the former; accordingly the corresponding temperature change is greater. Thus, when the pressure changes from one atmosphere to two, the change of boiling point (21°) is much greater than the change of freezing point (0.°0075). When the steam gauge reads 45 lbs. per sq. in. or 3 atmospheres, the absolute steam pressure on the water in the boiler is 4 atmospheres and the temperature of the water is 144° C. When the steam gauge reads 200 lbs., a pressure sometimes used, the temperature of the boiler water is 194° C. On the other hand, to make water *boil* in the receiver of an air pump at *room temperature* (20°), the pressure must be reduced to about 1/40 atmosphere. (See table below.)

BOILING POINT OF WATER AT VARIOUS PRESSURES[1]

Temperature	Pressure in cm. of mercury	Temperature	Pressure in atmospheres	Temperature	Pressure in atmospheres
0° C.	0.46	70° C.	0.3	140° C.	3.5
10	0.92	80	0.46	150	4.7
20	1.74	90	0.70	160	6.1
30	3.15	100	1.00	170	7.8
40	5.49	110	1.40	180	10.0
50	9.20	**121°**	**2.00**	190	12.4
60	14.88	130	2.67	200	15.5

Franklin's Experiment on Boiling Point.—Benjamin Franklin discovered that if a flask partly filled with water is boiled until the air is all expelled (Fig. 129, left sketch), and is then tightly stoppered and removed from the flame (right sketch), then pouring cold water (the colder the better) upon the flask causes the water to boil, even after it has cooled to about room temperature. The explanation is simple. When the temperature of the water is 50° C. the vapor pressure in the flask is 9.2 cm. of mercury.

[1] This is also a table of the saturated vapor pressure of water at various temperatures. (See close of Sec. 192, also Sec. 196.)

(See table above.) Suppose that under these conditions cold water is poured upon the flask. This chilling of the flask condenses some of the contained vapor, thereby causing a slight drop in pressure, whereupon more water bursts into steam. Indeed, so long as the temperature is 50°, the vapor pressure will be *maintained* at 9.2 cm.; hence the colder the water which is poured on, the more rapid the condensation, and consequently the more violent the boiling. The flask should have a round bottom or the atmospheric pressure will crush it when the pressure within becomes low. Inverting the flask and placing the stopper under water, as shown, precludes the possibility of air entering the flask and destroying the vacuum.

This lowering of the boiling point as the air pressure decreases is a serious drawback in cooking at high altitudes. At an altitude

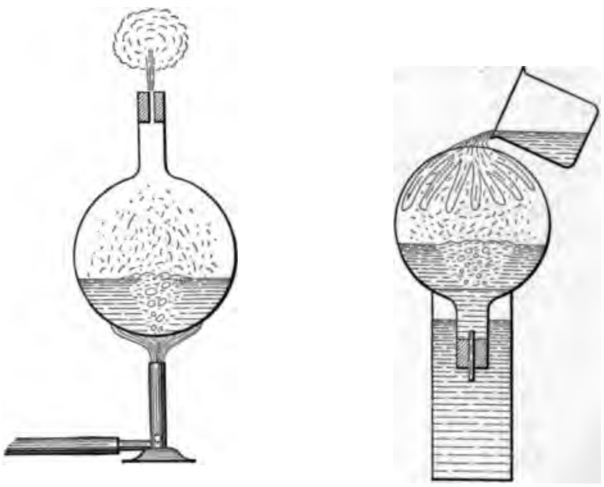

FIG. 129.

of 10,000 ft. (*e.g.*, at Leadville, Colorado), water boils at about 90° C., and at the summit of Pike's Peak (alt. 14,000 ft.), at about 85° C. At such altitudes it is very difficult to cook (by boiling) certain articles of food, (*e.g.*, beans), requiring in some cases more than a day. It will be understood that when water has reached the boiling point, further application of heat does not cause any further temperature rise, but is used in changing the boiling water to steam. In sugar manufacture, the "boiling down" is done in "vacuum pans" at reduced pressure and re-

duced temperature to avoid charring the sugar. By boiling substances in a closed vessel or boiler so that the steam is confined, thereby raising the pressure, and consequently raising the boiling point, the cooking is more quickly and more thoroughly done. This method is used in canning factories.

Superheating, Bumping.—After pure water has boiled for some time and the air which it contains has been expelled, it sometimes boils intermittently with almost explosive violence known as "bumping." A thermometer inserted in the water will show that the temperature just previous to the "bumping" is slightly above normal boiling point; in other words the water is *Superheated.* A few pieces of porous material or a little unboiled water added will stop the bumping. We have seen (Sec. 186) that water may also be *supercooled* without freezing. Dufour has shown that water in fine globules immersed in oil may remain liquid from $-20°$ to $178°$ C.

195. Geysers.—The geyser may be described as a great hot spring which, at more or less regular intervals, spouts forth a column or jet of hot water. Geysers are found in Iceland, New Zealand, and Yellowstone National Park. One of the Iceland geysers throws a column of water 10 ft. in diameter to a height of 200 ft. at intervals of about 6 hours. Grand Geyser, of the National Park, spouts to a height of 250 ft. Old Faithful, in the National Park, is noted for its regularity.

Geysers owe their action to the fact that water under great pressure must be heated considerably above 100° before it boils, and perhaps in some cases also to superheating of the lower parts of the water column just before the eruption takes place. A deep, irregular passage, or "well," filled with water, is heated at the bottom by the internal heat of the earth to a temperature far above the ordinary boiling point before the vapor pressure is sufficient to form a bubble. When this temperature is reached (unless superheating occurs) a vapor bubble forms and forces the column of water upward. At first the water simply flows away at the top. This, however, reduces the pressure on the vapor below, whereupon it rapidly expands, and the highly heated water below, now having less pressure upon it, bursts into steam with explosive violence and throws upward a column of boiling water. This water, now considerably cooled, flows back into the "well." After a few hours the water at the bottom of the well again becomes heated sufficiently above 100° to form

steam bubbles under the high pressure to which it is subjected, and the geyser again "spouts."

Bunsen, who first explained the action of the natural geyser, devised an artificial geyser. It consisted of a tin tube, say 4 ft. in length and 4 in. in diameter at the lower end, tapering to about 1 in. in diameter at the top, with a broad flaring portion above to catch the column when it spouts. If filled with water and then heated at the bottom, it spouts at fairly regular intervals. If constructed with thermometers passing through the walls of the tube, it will be found that the thermometers just previous to eruption read higher than 100°, and that the lowest one reads highest.

In the case of steam boilers under high pressure, the water may be from 50° to 80° hotter than the normal boiling point, and if the boiler gives way, thereby reducing the pressure, part of this water bursts into steam. This *additional supply of steam* no doubt contributes greatly to the violence of boiler explosions.

196. Properties of Saturated Vapor.—If, after the space above the water in A (Fig. 128) has become filled with saturated vapor, the piston P is suddenly forced down, there will then be more molecules per unit volume of the space than there were before. Consequently, the rate at which the molecules return to the water will be greater than before, and therefore greater than the rate at which they are escaping from the water. In other words, some of the vapor condenses to water. This condensation takes place *very* quickly and continues until equilibrium is restored and the vapor is still simply *saturated vapor.*

If, on the other hand, the piston P had been suddenly moved upward instead of downward, the vapor molecules in the space above the water, having somewhat more room than before, would not be so closely crowded together and hence would not return to the water in such great numbers as before. In other words, the rate of escape of molecules from the water would be greater than their rate of return; consequently the number of molecules in the space above the water would increase until equilibrium was reached, *i.e.*, until the space was again filled with *saturated vapor.*

In the case of a saturated vapor above its liquid, we may consider that there are two *opposing tendencies always at work.* As the temperature of the liquid rises, the tendency of the liquid to change to vapor increases, *i.e.*, more *liquid vaporizes.* The effect of increasing the external pressure applied to the vapor is,

on the other hand, to tend to condense it to the liquid state. At all times, and under all circumstances, the pressure *applied to* the vapor is equal to the pressure *exerted by* the vapor. Referring to Fig. 128, it may readily be seen that if the vapor pressure acting upward upon P is equal to, say 5 lbs. per sq. in. at any instant, that the downward pressure exerted by the piston upon the vapor below it, is likewise 5 lbs. per sq. in. Of course this would be equally true for any other pressure.

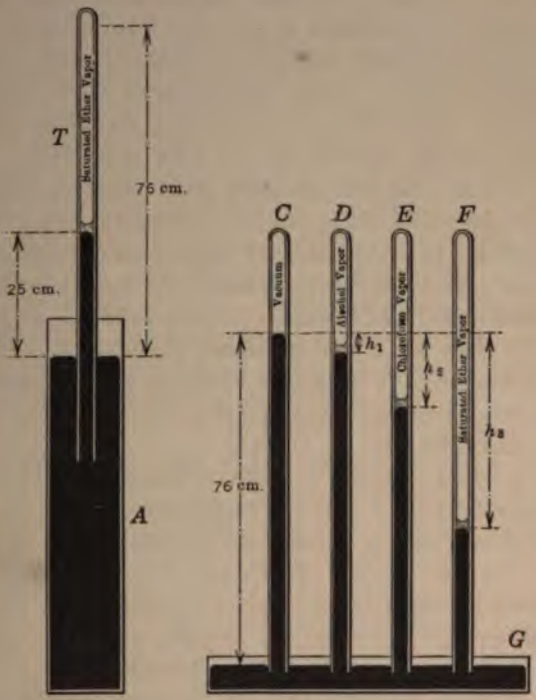

FIG. 130.

These characteristics of a saturated vapor *above its own liquid* are beautifully illustrated in the following experiment. A barometer tube T (Fig. 130, left sketch) is filled with mercury, stoppered, and carefully inverted in a mercury "well" A about 80 cm. deep. Upon removing the stopper, the mercury runs out of the tube, leaving the mercury level about 76 cm. higher in the tube than in the well, as explained in Sec. 136. Next, without admitting any air, introduce, by means of an ink filler, sufficient ether to make about 1 cm. depth in the tube. This ether rises

and quickly evaporates, until the upper part of the tube is filled with its saturated vapor, whose pressure at room temperature is about 2/3 atmosphere. Consequently, the mercury drops until it is about 25 cm. (1/3 of 76) above that in the well.

Now, as the tube is quickly moved upward more ether evaporates, maintaining the pressure of the saturated vapor constantly at 2/3 atmosphere, as evidenced by the fact that the level of the mercury still remains 25 cm. above that in the well. If the tube is suddenly forced downward, some ether vapor condenses, and the mercury still remains at the same 25-cm. level. After the tube has been raised high enough that *all* of the ether is evaporated, further raising it causes the pressure of the vapor to decrease (in accordance with Boyle's Law), as shown by the fact that the level of the mercury in the tube then rises.

Finally, if the tube and contents are heated to 34.9° C., the boiling point for ether, its saturated vapor produces a pressure of one atmosphere, and the mercury within and without the tube comes to the same level, and *remains* at the same level though the tube be again raised and lowered. If the tube is severely chilled, the mercury rises considerably higher than 25 cm. This shows that the pressure of the saturated vapor, or the pressure at which boiling occurs, rises rapidly with the temperature. (See table for *Water*, Sec. 194.)

Saturated Vapor Pressure of Different Liquids.—The pressure of the saturated vapor of liquids varies greatly for the different liquids, as shown by the experiment illustrated in Fig. 130 (right sketch). The four tubes C, D, E, and F are filled with mercury and are then inverted in the mercury trough G. The mercury then stands at a height of 76 cm. in each tube. If, now, a little alcohol is introduced into D, some chloroform into E, and some ether into F, it will be found that the mercury level lowers by the amounts h_1, h_2, and h_3, respectively. The value of h_1 is 4.4 cm., which shows that at room temperature the pressure of the saturated vapor of alcohol is equal to 4.4 cm. of mercury. Since $h_2/h_1=4$ (approx.), we see that at room temperature the pressure of the saturated vapor is about 4 times as great for chloroform as for alcohol.

197. Cooling Effect of Evaporation.—If the hand is moistened with ether, alcohol, gasoline, or any other liquid that evaporates *quickly*, a decided cooling effect is produced. Water produces a similar but less marked effect. We have seen that it

requires 536.5 calories to change a gram of boiling water to steam. When water is evaporated at *ordinary* temperatures it requires somewhat more than this, about 600 calories. If this heat is not supplied by a burner or some other external source, it must come from the *remaining water* and the containing vessel, thereby cooling them below room temperature.

There are *two factors* which determine the magnitude of the cooling effect produced by the evaporation of a liquid. One of these is the *volatility* of the liquid; the other, the value of its *heat of vaporization*. From the table (Sec. 182) we see that the heat of vaporization is about 3 times as great for water as for alcohol. Consequently, if alcohol evaporated 3 times as fast as water under like conditions, then alcohol and water would produce about equally pronounced cooling effects. Alcohol, however, evaporates much more than 3 times as fast as water, and therefore gives greater cooling effect, as observed.

If three open vessels contain alcohol, chloroform, and ether, respectively, it will be found that a thermometer placed in the one containing alcohol shows a temperature slightly lower than room temperature; while the one in chloroform reads still lower, and the one in ether the lowest of all. A thermometer placed in water would read almost exactly room temperature. The main reason for this difference is the different rates at which these liquids evaporate, although, as just stated, the value of the heat of vaporization is also a determining factor. Ether, being by far the most volatile of the three, gives the greatest cooling effect. Observe that the more volatile liquids are those having a low boiling point, and consequently a high vapor pressure at room temperature. In some minor surgical operations the requisite numbness is produced by the chilling effect of a spray of very volatile liquid. Other practical uses of the cooling effect of evaporation are discussed in Secs. 198, 199, and 200. The converse, or the heating effect due to condensation, is utilized in all heating by steam (Sec. 191), and it also plays an important rôle in influencing weather conditions.

198. The Wet-and-dry-bulb Hygrometer.—The cooling effect of evaporation is employed in the wet-and-dry-bulb hygrometer, used in determining the amount of moisture in the atmosphere. It consists of two ordinary thermometers which are just alike except that a piece of muslin is tied about the bulb of one. The muslin is in contact with a wick, the lower end of

which is in a vessel of water. By virtue of the capillary action of the wick and muslin, the bulb is kept moist. This moisture evaporating from the bulb cools it, causing this thermometer to read several degrees lower than the other one.

If the air is very dry, this evaporation will be rapid and the difference between the readings of the two thermometers will be large; whereas if the air is almost saturated with moisture, the evaporation will be slow and the two thermometer readings will differ but slightly. Consequently, if the two readings differ but little, rain or other precipitation may be expected. The method of finding the amount of water vapor in the air by means of these thermometer readings, is discussed in a subsequent chapter.

As a mass of air m comes into contact with the wet (colder) bulb it *gives* heat to the bulb, and as it absorbs moisture from the bulb it also *takes* heat from it. A few moments after the apparatus is set up, equilibrium is reached, as shown by the fact that the temperature of the wet bulb is constant. It is then known that the amounts of heat "given" and "taken" by the bulb are equal. This fact is utilized in the derivation of certain theoretical formulas for computing the amount of moisture in the air directly from the two thermometer readings. The practical method, however, is to use tables (Sec. 222) compiled from experiments.

199. Cooling Effect due to Evaporation of Liquid Carbon Dioxide.—Carbon dioxide (CO_2) is a gas at ordinary temperatures and pressures, but if cooled to a low temperature and then subjected to high pressure it changes to the liquid state. If the pressure is reduced it quickly changes back to the vapor state. We have seen that the pressure of water vapor is about 1/40 atmosphere at room temperature. Liquid carbon dioxide is so extremely volatile, that is, it has so great a tendency to change to the vapor state, that its vapor pressure at room temperature has the enormous value of 60 atmospheres. It follows then, that when an air-tight vessel is partly filled with liquid carbon dioxide at room temperature, a portion of it quickly changes to vapor until the pressure in the space above the liquid becomes 60 atmospheres. Carbon dioxide is shipped and kept in strong sealed iron tanks to be used for charging soda fountains, etc.

If such a tank is inverted (Fig. 131) and the valve is opened, a stream of liquid carbon dioxide is forced out by the 60-atmosphere pressure of the vapor within. As soon as this liquid carbon dioxide escapes to the air, where the pressure is only one atmosphere, it changes almost instantly to vapor, and takes from the

air, from the nozzle, and from the remaining liquid, its heat of vaporization, about 40 calories per gram at room temperature. This abstraction of heat chills the nozzle to such an extent that the moisture of the air rapidly condenses upon it as a frosty coating. It also chills, in fact freezes, part of the liquid jet of carbon dioxide, forming carbon dioxide "snow." This snow is so cold ($-80°$ C.) that mercury surrounded by it quickly freezes.

200. Refrigeration and Ice Manufacture by the Ammonia Process.—There are several systems or methods of ice manufacture, in *all* of which, however, the chilling effect is produced by the heat *absorption* (due to heat of vaporization) that accompanies the vaporization of a volatile liquid. The most important of these liquids are ammonia (NH_3) and carbon dioxide (CO_2). Economy demands that the vapor be condensed again to a liquid, in order to use the same liquid repeatedly.

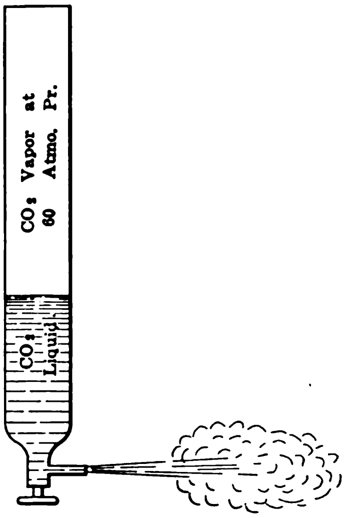

FIG. 131.

In the *Compression System*, the vapor is compressed by means of an air pump until it becomes a liquid. The heat *evolved* in this process (heat of vaporization) is disposed of usually by flowing water, and the cooled liquid (*e.g.*, ammonia) is again allowed to evaporate. Thus the cycle, consisting of evaporation accompanied by heat *absorption*, and condensation to liquid accompanied by heat *evolution*, is repeated indefinitely. Since the former occurs in pipes in the ice tank (freezing tank), we see that the heat is *literally pumped from* the *freezing tank* to the *flowing water*.

Ammonia is a substance admirably adapted to use in this way. Its heat of vaporization is fairly large (295 cal. per gm.), and it is very volatile—that is, it evaporates very quickly, its vapor pressure at room temperature being about 10 atmospheres. At $-38°.5$ C. its vapor pressure is one atmosphere; hence it would boil in an open vessel at that low temperature. The liquid commonly *called* ammonia is simply water containing ammonia gas

which it readily absorbs. Ammonia is a gas at ordinary temperatures, but when cooled and subjected to several atmospheres' pressure it changes to a liquid. If carbon dioxide is used instead of ammonia, the cost of manufacturing the ice is somewhat greater. The greater compactness of the apparatus, however, coupled with the fact that in case of accidental bursting of the pipes, carbon dioxide is much less dangerous than ammonia, has resulted in its adoption on ships.

The essentials of the *Ammonia Refrigerating apparatus* are shown diagrammatically in Fig. 132. A is the cooling tank which receives a continual supply of cold water through pipe c.; B is an air pump; C is a freezing tank filled with brine; D is a pipe filled with liquid ammonia; and E is a pipe filled with ammonia vapor.

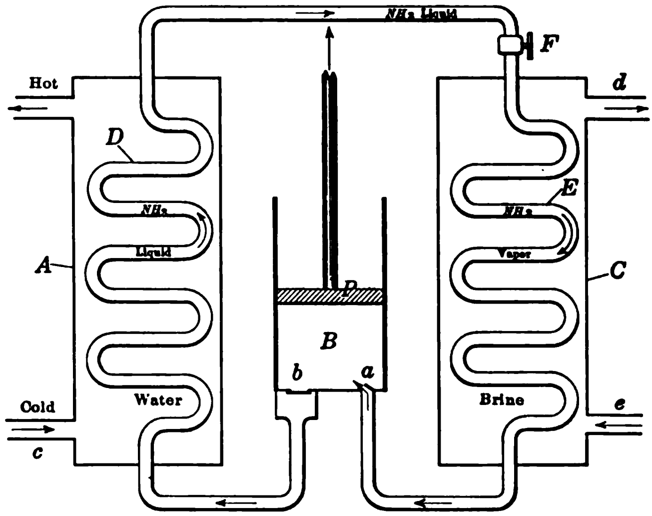

Fig. 132.

If valve F were *slightly* opened, liquid ammonia would enter E and evaporate until the pressure in E was equal to the vapor pressure of ammonia at room temperature or about 10 atmospheres. Whereupon evaporation, and therefore all cooling action, would cease. If, however, the pump is operated, ammonia gas is withdrawn from E through valve a and is then forced into pipe D through valve b under sufficient pressure to liquefy it. This constant withdrawal of ammonia gas from pipe E permits more liquid ammonia to enter through F and evaporate. The am-

monia, as it evaporates in E, *withdraws* from E and from the surrounding brine its heat of vaporization (about 300 cal. per gm.); while each gram of gas that is condensed to a liquid in D imparts to D and to its surroundings about 300 calories. Thus we see that heat is withdrawn from the very cold brine in C and imparted to the much warmer water in A. This action continues so long as the pump is operated. Brine is used in C because it may be cooled far below zero without freezing.

The Refrigerator Room.—The cold brine from C may be pumped through d into the pipes in the refrigerator room and then back through pipe e to the tank. The brine as it returns is not so cold as before, having abstracted some heat from the refrigerator room. This heat it now imparts to pipe E. Thus, through the circulation of the brine, heat is carried from the cooling room to the tank C, and we have just seen that due to the circulation of the ammonia, heat is carried from the brine tank C to the water tank A.

The pipe E, instead of passing into the brine, may pass back and forth in the refrigerator room. The stifling ammonia vapor, which rapidly fills the room, in case of the leaking or bursting of an ammonia pipe, makes this method dangerous.

In the *Can System* of ice manufacture, the cans of water to be frozen are placed in the brine in C, and left there 40 or 50 hours as required. In the *Plate System*, the pipe E passes back and forth on one face of a large metal plate, chilling it and forming a sheet of ice of any desired thickness upon the other face, which is in contact with water. For every 8 or 10 tons of ice manufactured, the engine that operates the pump uses about one ton of coal.

Observe that in "pumping" the heat, as we may say, from the cold freezing tank to the much warmer flowing water, we are causing the heat to flow "uphill," so to speak; for heat of *itself* always tends to flow from hotter to colder bodies, that is, "downhill." Observe also that it takes *external* applied *energy* of the steam engine that operates the pump to *cause* this "uphill" flow of heat.

201. Critical Temperature and Critical Pressure.—In 1869, Dr. Andrews performed at Glasgow his classical experiments on carbon dioxide. He found that when some of this gas, confined in a compression cylinder at a temperature of about 32° or 33° C., had the pressure upon it changed from say 70 atmospheres to 80 atmospheres, then the volume decreased, not by 1/8 as required

by Boyle's Law (Sec. 139), but much more than this. He also found that carbon dioxide gas cannot be changed to the liquid state by pressure, however great, if its temperature is above 31° C. This temperature (31°) is called the Critical Temperature for carbon dioxide.

If carbon dioxide gas is at its critical temperature, it requires 73 atmospheres' pressure to change it to the liquid state. This pressure is called the *Critical Pressure* for carbon dioxide. If the temperature of any gas is several degrees lower than its critical temperature, then the pressure required to change it to the liquid state is considerably less than the critical pressure. Below is given a table of critical temperatures and critical pressures for a few gases.

CRITICAL TEMPERATURES AND CRITICAL PRESSURES FOR A FEW SUBSTANCES

Substance	Critical temperature	Critical pressure in atmospheres
Hydrogen[1] (H)............	−241° C.	14
Nitrogen (N).............	−146	34
Air (O and N)............	−140	39
Oxygen (O)..............	−118	50
Ethylene (C_2H_4)...........	10	52
Carbon dioxide (CO_2)......	30.92	73
Ammonia (NH_3)...........	130	115
Water vapor (H_2O).........	364	194.6

202. Isothermals for Carbon Dioxide.—In Fig. 134, the isothermals which Andrews determined for carbon dioxide are shown. For the meaning of isothermals and the method of obtaining them, the student is referred to "Isothermals for Air" (Sec. 140).

The essential parts of the apparatus used by Andrews are shown in section in Fig. 133. A glass tube *A* about 2.5 mm. in diameter, terminating in a fine capillary tube above, was filled with carbon dioxide gas and plugged with a piston of mercury *a*. This tube was next slipped into the cap *C* of the compression chamber *D*. A similar tube *B*, filled with *air*, and likewise stoppered with mercury, was placed in the compression chamber *E*.

As *S* was screwed into the compression chamber *D*, the pressure

[1] The values −234.°5 C. and 20 atmospheres, sometimes given as the critical temperature and critical pressure, respectively, for hydrogen, are incorrect; the first, because of extrapolation error in the readings of the resistance thermometer, the second, because of manometer error in the original determination.

on the water in the two chambers, and consequently the pressure on the mercury and gas in the two tubes A and B, could be increased as desired. Of course, as the pressure was increased, the mercury rose higher and higher in tubes A and B to, say, m_1 and m_2. Knowing the original volume of air in B and also the bore of the capillary portion of tube B, the pressure in the chamber could be determined. Thus, if the volume of air in tube B above m_2 were 1/50 of the original volume, then the pressure in both chambers would be approximately 50 atmospheres. At such pressures there is a deviation from Boyle's law, which was taken into account and corrected for. Knowing the bore of A, the volume of carbon dioxide above m_1 could be found.

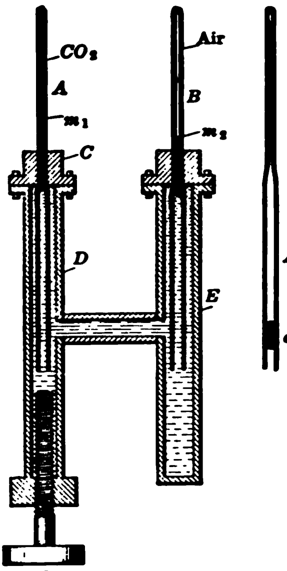

FIG. 133.

Plotting the values of the volumes so found as abscissæ, with the corresponding pressures as ordinates, when the temperature of the apparatus was 48.°1 C., the isothermal marked 48.°1 (Fig. 134) was obtained. The form of the 48.°1 isothermal shows that at this temperature the carbon dioxide vapor followed Boyle's law, at least roughly.

When, however, the experiment was repeated with the apparatus at the temperature 31.°1 C., it was found that when the pressure was somewhat above 70 atmospheres (point a on the 31.°1 isothermal) a *slight increase* in pressure caused a very *great decrease* in volume, as shown by a considerable rise in m_1. As the pressure was increased slightly above 75 atmospheres, as represented by point b on the curve, a further *slight reduction* of volume was accompanied by a comparatively *great increase* in pressure, as shown by the fact that the portion bc of the isothermal is nearly vertical. Note also that the portion ab of the isothermal is nearly horizontal.

If the experiment were again repeated at, say 30° C., then as the pressure reached about 70 atmospheres, liquid carbon dioxide would collect on m_1, and this liquid would be seen to have a sharply defined meniscus separating it from the vapor above. At 31.°1 no such meniscus a
The limiting temperature (30.°92 C.) at which the meniscus j
to appear under increasing pressure, is called the *Critical Tempera*

Let us now discuss the 21.°5 isothermal, which isothermal wa
mined by keeping the apparatus at 21.°5 v ' ·asing the p
As the *volume* was *decreased* from that i int A t

represented by point B, the *pressure increased* from about 50 atmospheres to 60. Now as S was screwed farther into the chamber, the *volume decreased* from point B to point C with *practically no increase in pressure* (note that BC is practically horizontal). During this change the saturated carbon dioxide vapor was changing to the liquid state, as shown by the fact that the liquid carbon dioxide resting on m_1 could be seen to be increasing. At C the gas had all been changed to liquid carbon dioxide, and since liquids are almost incompressible, a very *slight compression, i.e.*, a very slight rising of meniscus m_1, was accompanied by a very *great increase* of pressure, as evidenced by the nearly vertical direction of CD.

It will be observed, that while the volume is reduced from that represented by point B to that represented by point C, the carbon dioxide

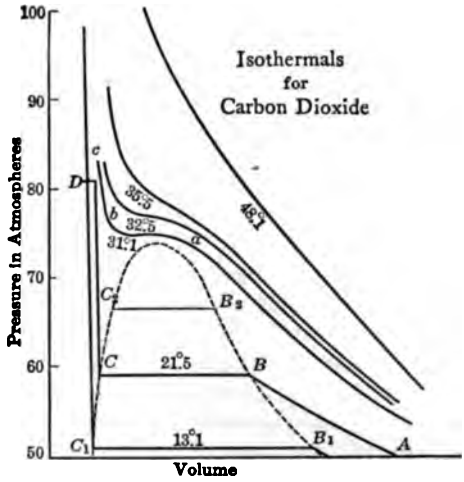

FIG. 134.

is changing to the liquid state, and therefore gaseous and liquid carbon dioxide coexist in tube A. Likewise at 13.°1 the two states, or *phases*, coexist from B_1 to C_1, while if the temperature were, say 28° C., the two phases would coexist for volumes between B_2 and C_2. Accordingly, the region within the dotted curve through B, B_1, B_2, C, C_1, C_2, etc., represents on the diagram all possible corresponding values of pressure, volume, and temperature at which the two phases may coexist. Thus, if the state of the carbon dioxide (temperature, pressure, and volume) is represented by a point anywhere to the right, or to the right and above this dotted curve, only the gaseous phase exists; to the left, only the liquid phase. We may now define the *Critical Temperature* of any substance as the *highest temperature at which the liquid and gaseous phases of that substance can coexist.*

This definition suggests the following simple method of determining critical temperatures. A thick-walled glass tube is partly (say 1/4) filled with the liquid, *e.g.* water, the space above being a vacuum, or rather, a space containing saturated water vapor. The tube is then heated until the meniscus disappears. The temperature at which the meniscus disappears is the *critical temperature* (364° C. for water), and the pressure then tending to burst the tube, is termed the *critical pressure*. It will be noted that as the water is heated, its vapor pressure becomes greater, finally producing the critical pressure (194.6 atmospheres) when heated to the critical temperature.

The Distinction between a Vapor and a Gas.—When a gas is cooled below its critical temperature it becomes a vapor. Conversely, when a vapor is heated above its critical temperature it becomes a gas. A vapor and its liquid often coexist; a gas and its liquid, never.

203. The Joule-Thomson Experiment.—In 1852, Joule and Thomson (Lord Kelvin) performed their celebrated "Porous Plug" experiment. They forced various gases under high pressure through a plug of cotton or silk into a space at atmospheric pressure. In every case, except when

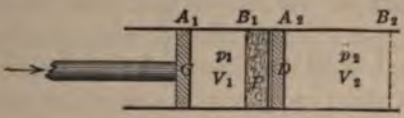

FIG. 135.

hydrogen was used, the gas was *cooler* after passing through the plug than it was before. Hydrogen, on the contrary, showed a slight *rise* in temperature. We may note, however, that at very low temperatures (below −80° C.) hydrogen also experiences a cooling effect.

The principle involved in this experiment will be explained in connection with Fig. 135. Let P be a stationary porous plug in a cylinder containing two pistons C and D. Let piston C, as it moves (slowly) from A_1 to B_1 against a high pressure p_1, force the gas of volume V_1 through the plug, and let this gas push the piston D from A_2 to B_2, and let it have the new volume V_2 and the new pressure p_2 (1 atmosphere). Now, from the proof given in Sec. 156, we see that piston C does the work p_1V_1 *upon* the gas in forcing it through the plug; while the work done *by* the gas in forcing D from A_2 to B_2 is p_2V_2. Accordingly, if $p_2V_2 = p_1V_1$, *i.e.*, if the work done *by* the gas is equal to the work done *upon* it, then the gas should (on this score at least) be neither heated nor cooled by its passage through the plug. All gases, however, deviate from Boyle's law, and for all but hydrogen the product pV at ordinary te-peratures increases as p decreases. Hence here $p_2V_2 > p_1V_1$ (> = greater than), which means that the work done *by* the gas (which te to cool it) exceeds the work done *upon* the gas (ends to heat

As a result, then, the gas is either cooled or else it abstracts heat from the piston, or both.

Cooling Effect of Internal Work.—From the known deviation from Boyle's law exhibited by air, it can be shown that the temperature of the air in passing through the plug should drop about 0.°1 C. for each atmosphere difference in pressure between p_1 and p_2. Thomson and Joule found a difference of nearly 1° C. per atmosphere. This *additional* cooling effect is attributed to the *work done against intermolecular attraction* (internal work done) *when a gas expands*. The work done by the gas in expanding is due, then, *in part* to the resulting *increase in* pV (deviating from Boyle's law), and *in part* to the *work done against intermolecular attraction* in increasing the average distance between its molecules. Both of these effects, though small, are more marked at low temperatures, and by an ingenious but simple arrangement for securing a cumulative effect, Linde has employed this principle in liquefying air and other gases (Sec. 206). In Linde's apparatus, the gas passes through a small opening in a valve instead of through a porous plug.

204. Liquefaction of Gases.—About the beginning of the present century, one after another of the so-called permanent gases were liquefied, until now there is no gas known that has not been liquefied. Indeed most of them have not only been liquefied, but also frozen.

In 1823, the great experimenter Faraday liquefied chlorine and several other gases with a very simple piece of apparatus. The chemical containing the gas to be liquefied was placed in one end of a bent tube, the other end of which was placed in a freezing mixture producing a temperature lower than the critical temperature of the gas. The end of the tube containing the chemical was next heated until the gas was given off in sufficient quantity to produce the requisite pressure to liquefy it in the cold end of the tube.

In 1877, Pictet and Cailletet independently succeeded in liquefying oxygen. Later Professor Dewar and others liquefied air, and in 1893 Dewar froze some air. A few years later (1897) he liquefied and also (1899) froze some hydrogen. Subsequently (1903) he produced liquid helium, a substance that boils at 6° on the absolute scale or at $-267°$ C. He also invented the Dewar flask (Sec. 206), in which to keep these liquids.

In liquefying air and other gases having low critical temperatures, the great difficulty encountered is in the production and maintenance of such low temperatures. To accomplish this, the

cooling effect of the evaporation of a liquid and the cooling effect produced when a gas expands (Sec. 178) have both been utilized.

There are two distinctly different methods of liquefying air, known as the "Cascade" or Series Method, due to Raoult Pictet (Sec. 205), and the "Regenerative Method," due to Linde and others (Sec. 206).

205. The Cascade Method of Liquefying Gases.—In Fig. 136 is shown a diagrammatic sketch of the apparatus of Pictet, as modified and used with great success in the latter part of the 19th Century by Dewar, Olszewski, and others. It consists of three vessels *A*, *B*, and *C*, the two air pumps *D* and *E*, and the carbon dioxide tube *F*, together with the connecting pipes as shown.

The pump *D* forces ethylene through pipe *K*, valve *G*, and pipe *M* into the vessel *B* from which vessel the ethylene (now in the vapor state) returns to the pump through pipe *N*. Pump *E* maintains a similar

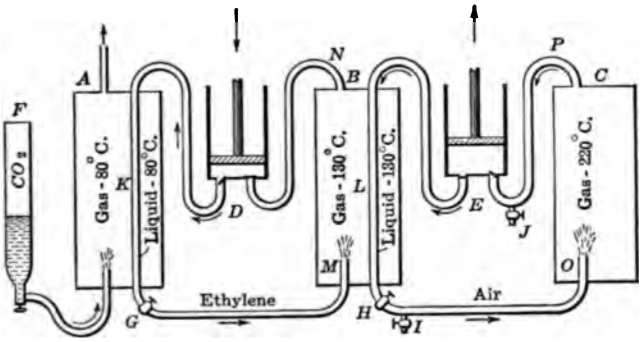

FIG 136

counterclockwise circulation of *air* through *L*, *H*, *O*, *C*, and *P*, as is indicated by the arrows.

The vaporization of the carbon dioxide in *A* produces a temperature of $-80°$ C. (Sec. 199). This cold gas, coming in contact with the spiral pipe *K* (shown straight to avoid confusion), cools it enough that the ethylene within it liquefies under the high pressure to which it is subjected. As this cold liquid ethylene vaporizes at *M*, it cools the *air* in *L* to such an extent that it in turn liquefies under the high pressure produced by pump *E*. As this liquid air passes through valve *H* and vaporizes in *C*, it produces an extremely low temperature. As pointed out in the discussion of the ammonia refrigerating apparatus, the maintenance of a *partial vacuum* into which the liquid may vaporize, as in *B* and *C*, *causes* more *rapid vaporization*, and therefore *enhances* the *chilling effect*. The liquid air may be withdrawn at *I*, and fresh air may be admitted at *J* to replenish the supply.

In liquefying air by this method, it is necessary to use ethylene, or some other intermediate liquid which produces a very low temperature when vaporized. For if L simply passed through vessel A, no pressure, however great, would liquefy the air within it, since $-80°$ C. is above the critical temperature for air. Gases have, however, been liquefied when at temperatures considerably above the critical temperatures, by subjecting them to enormous pressures and then suddenly relieving the pressure.

206. The Regenerative Method of Liquefying Gases.—The regenerative method of liquefying gases employs the principle (established by Thomson and Joule, Sec. 203) that a gas is chilled

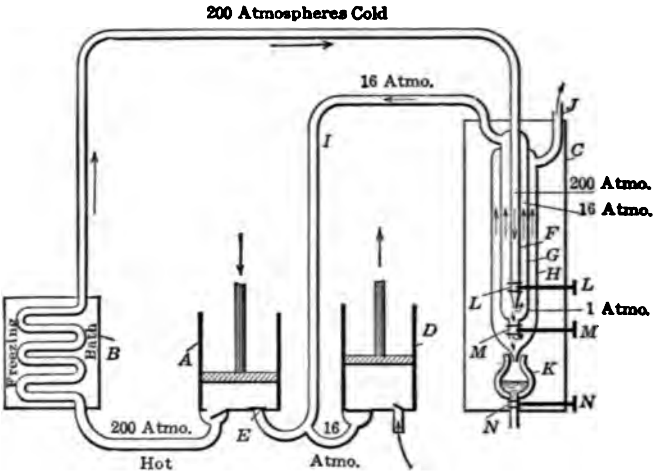

FIG. 137.

as it escapes through an orifice from a region of high pressure to a region of low pressure. This method has made possible the liquefaction of every known gas, and also the production of liquid air in large quantities and at a greatly reduced cost. From about 1890 to 1895 Dr. Linde, Mr. Tripler, and Dr. Hampson were all working along much the same line, in accordance with a suggestion made by Sir Wm. Siemens more than thirty years before; namely, that the gas, *cooled by expansion as it escapes through an orifice, shall cool the oncoming gas about to expand, and so on, thus giving a cumulative effect.* Dr. Linde, however, was the first to produce a practical machine.

The essential parts of Linde's apparatus are shown in Fig. 137. A is an air pump which takes in the gas (air, *e.g.*) through valve

E at about 16 atmospheres, and forces it under a pressure of about 200 atmospheres through the coiled pipes in the freezing bath B. From B, the air passes successively through the three concentric pipes or tubes F, G, and H in the vessel C, as indicated by the arrows. A portion of the air from G returns again through pipe I and valve E to the pump, thus completing the cycle. The cycle is repeated indefinitely as long as the pump is operated. It will be understood that the freezing bath B cools the air which has just been heated by compression. It also "freezes out" most of the moisture from the air. The pump D supplies to the pump A, under a pressure of 16 atmospheres, enough air to compensate for that which escapes through J from the outer tube H, and also for that which is liquefied and collects in the Dewar flask K.

Explanation of the Cooling Action.—The three concentric tubes F, G, and H (which it should be stated are, with respect to the rest of the apparatus, very much smaller than shown, and in practice are coiled in a spiral within C), form the vital part of the apparatus. The air, as it passes from the central tube F through valve L, has its pressure reduced from 200 atmospheres to about 16 atmospheres. This process cools it considerably. The valves are so adjusted that about 4/5 of this cooled air flows upward, as indicated by the curved arrow, through G (thereby cooling the downflowing stream in F) and then flows through I back to the pump A. The remaining 1/5 flows directly from valve L through valve M. As this air passes through valve M its pressure drops from 16 atmospheres to 1 atmosphere, producing an *additional* drop in temperature. At first all of the air that passes through valve M passes up through the outer tube H and escapes through J. We have just seen that the downflowing air in F is *cooled* by the upflowing air in G, and as this downflowing air passes through valve L it is *still further cooled* (by expansion), and therefore as it passes up through G it *still further cools* the downflowing stream in F, and so on. Thus both streams become *colder* and *colder* until so low a temperature is reached that the *additional cooling* produced by the expansion at M causes part (about 1/4) of the air that passes through M to liquefy and collect in the Dewar flask K. From K, the liquid air may be withdrawn through valve N.

Quite recently liquid air has been manufactured at the rate of about one quart per H.P.-hour expended in operating the pumps.

Properties and Effects of Liquid Air.—Liquid air is a clear, bluish liquid, of density 0.91 gm. per $cm.^3$. It boils at a temperature of

$-191°.4$ C. and its nitrogen freezes at $-210°$, its oxygen at $-227°$. It is attracted by a magnet, due to the oxygen which it contains. If liquid air is poured into water it floats at first; but, due to the fact that nitrogen (density 0.85, boiling point $-196°$ C.) vaporizes faster than oxygen (density 1.13, boiling point $-183°$), it soon sinks, boiling as it sinks, and rapidly disappears. Felt, if saturated with liquid air, burns readily.

At the temperature of liquid air, mercury, alcohol, and indeed most liquids, are quickly frozen. Iron and rubber become almost as brittle as glass; while lead becomes elastic, *i.e.*, more like steel.

The Dewar Flask.—If liquid air were placed in a closed metal vessel it would vaporize, and quickly develop an enormous pressure. Even if this pressure did not burst the container, the air would soon be warmed above its critical temperature and cease to be a liquid, so that a special form of container is required. Professor Dewar performed a great service for low-temperature research when he devised the double-walled flask (K, Fig. 137). In such a container, liquid air has been kept for hours and has been shipped to a considerable distance. The space between the walls is a nearly perfect vacuum, which prevents, in a large measure, the passage of heat into the flask. Silvering the walls reflects heat away from the flask and therefore improves it. These flasks must not be tightly stoppered even for an instant or they will explode, due to the pressure caused by the vaporization of the liquid air. The constant but slow evaporation from the liquid air keeps it cooled well below its critical temperature, in fact at about $-191°$ C., the boiling point for air at atmospheric pressure.

The Thermal Bottle.—The *Thermal Bottles* advertised as "Icy hot," etc., are simply Dewar flasks properly mounted to prevent breakage. They will keep a liquid "warm for 12 hours," or "cold for 24 hours." Observe that a liquid when called "warm" differs more from room temperature than when called "cold."

CHAPTER XVI

TRANSFER OF HEAT

207. Three Methods of Transferring Heat.—Heat may be transferred from one body to another in three ways; viz., by *Convection*, by *Conduction*, and by *Radiation*.

When air comes in contact with a hot stove it becomes heated and expands. As it expands, it becomes lighter than the surrounding air and consequently rises, carrying with it heat to other parts of the room. This is a case of transfer of heat by *convection*. Obviously, only liquids and gases can transfer heat by convection.

If one end of a metal rod is thrust into a furnace, the other end soon becomes heated by the *conduction* of heat by the metal of which the rod is composed. In general, metals are good conductors, and all other substances relatively poor conductors, especially liquids and gases.

On a cold day, the heat from a bonfire may almost blister the face, although the air in contact with the face is quite cool. In this case, the heat is transmitted to the face by *radiation*. The earth receives an immense amount of heat from the sun, although interplanetary space contains no material substance and is also very cold. This heat is transmitted by radiation. These three methods of heat transfer will be taken up in detail in subsequent sections.

208. Convection.—Heat transfer by convection is utilized in the hot-air, steam, and hot-water systems of heating. In these systems the medium of heat transfer is air, steam, and water, respectively. It will be noted in every case of heat transfer by convection, that the *heated medium moves and carries the heat with it*. Thus, in the *hot-air* system, an air jacket surrounding the furnace is provided with a fresh-air inlet near the bottom; while from the top, air pipes lead to the different rooms to be heated. As the air between the jacket and the furnace is heated it becomes lighter and rises with considerable velocity through the pipes leading to the rooms, where it mingles with the other air of the room and thereby warms it.

The convection currents produced by a hot stove, by means of which all parts of the room are warmed, are indicated by arrows in Fig. 138. As the air near the stove becomes heated, and therefore less dense, it rises, and the nearby air which comes in to take its place is in turn heated and rises. As the heated air rises and flows toward the wall, it is cooled and descends as shown.

Fig. 139 illustrates the convection currents established in a vessel of water by a piece of ice. The water near the ice, as it is cooled becomes more dense and sinks. Other water coming in from all sides is in turn cooled and sinks, as indicated by the arrows.

In *steam heating*, pipes lead from the steam boiler to the steam radiators in the rooms to be heated. Through these pipes, the

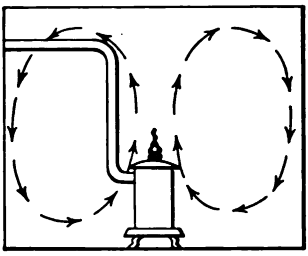

Fig. 138.

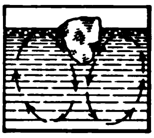

Fig. 139.

hot steam passes to the radiators, where it condenses to water. In condensing, the steam gives up its heat of vaporization and thereby heats the radiator. The water formed by the condensation of the steam runs back to the boiler.

In the *hot-water heating system*, the heated water from the boiler (B, Fig. 140) rises through pipes leading to the radiators (C, D, E, and F) where it gives up heat, thereby warming the radiators, and then descends, colder and therefore denser, through other pipes (G and H) to the boiler, where it is again heated. This cycle is repeated indefinitely. The current of water up one pipe and down another is evidently a *convection* current, established and maintained by the difference in density of the water in the two pipes. The rate of flow of the water through the radiators, and hence the heating of the rooms, may be controlled by the valves c, d, e, and f. Hot water may be obtained from the faucets I, J, K, and L. The tank M furnishes the necessary pressure, allows for the expansion of the water when heated, and provides

a safeguard against excessive pressure should steam form in the boiler.

If the boiler B were only partly filled with water, steam would pass to the radiators and there condense, and the system would become a *steam-heating system.* In this case it would be necessary to provide radiators of a type in which the condensed steam would not collect.

In heating a vessel of water by placing it upon a hot stove, the water becomes heated both by conduction and convection. The

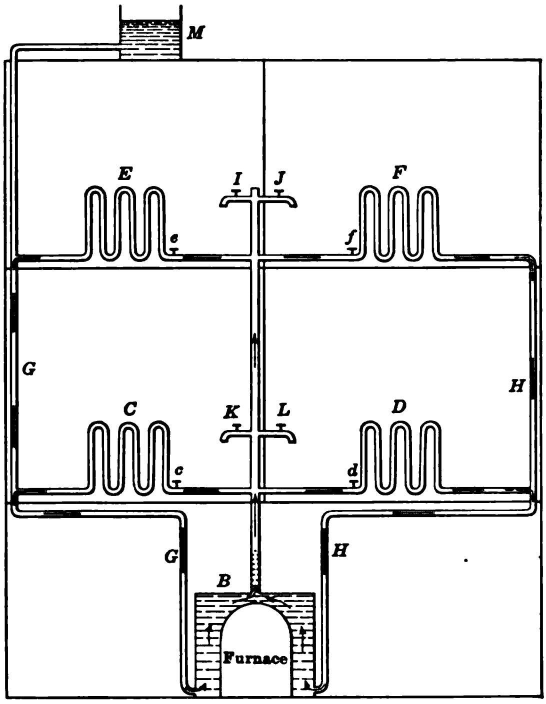

FIG. 140.

heat passes through the bottom of the vessel by conduction and heats the bottom layer of water by conduction. This heated layer is less dense than the rest of the water and rises to the surface, carrying with it a large quantity of heat. Other water, taking its place, is likewise heated and rises to the surface. In this way convection currents are established, and the entire body of water is heated.

Winds are simply convection currents produced in the air by uneven heating. The hotter air rises, and the cooler air rushing in to take its place is in turn heated and rises. This inrush of air persists so long as the temperature difference is maintained, and is called *wind* (Chapter XVII).

209. Conduction.—If one end A of a metal rod is heated, the other end B is supposed to become heated by conduction in the following manner. The violent heat vibrations of the molecules at the end A cause the molecules near them to vibrate, and in like manner these molecules, after having begun to vibrate, cause the layer of molecules *adjacent to them* on the side toward B to vibrate, and so on, until the molecules at the end B are vibrating violently; *i.e.*, until B is also *hot.*

This vibratory motion is readily and rapidly transmitted from layer to layer of the molecules of metals; therefore *metals* are said to be *good conductors.*

Brick and *wood* are poor conductors of heat, which fact makes them valuable for building material. Evidently it would require a great deal of heat to keep a house warm if its walls were composed of materials having high heat conductivity. Asbestos is a very poor conductor of heat, for which reason it is much used as a wrapping for steam pipes to prevent loss of heat, and also as a wrapping for hot air flues to protect nearby woodwork from the heat which might otherwise ignite it.

Clothing made of wool is much warmer than that made of cotton, because wool is a much poorer conductor of heat than cotton, and therefore does not conduct heat away from the body so rapidly.

Liquids, except mercury, are very poor conductors. That water is a poor conductor of heat may be demonstrated by the following experiment. A gas flame is directed downward against a shallow metal dish floating in a vessel of water. After a short time the water in contact with the dish will boil, while the water a short distance below experiences practically no change in temperature, as may be shown by thermometers inserted. It will be observed that convection currents are not established when water is heated from above. A test tube containing ice cold water, with a small piece of ice held in the bottom, may be heated near the top until the top layers of water boil without appreciably melting the ice.

Gases are very poor conductors of heat—much poorer even than

liquids. The fact that air is a poor conductor is frequently made use of in buildings by having "dead air" spaces in the walls. It is well known that if a slight air space is left between the plaster and the wall, a house is much warmer than if the plaster is applied directly to the wall. If a brick wall is wet it conducts heat much better than if dry, simply because its pores are filled with water instead of with air. From the table of *Thermal Conductivities* given below, it will be seen that water conducts heat about 25 times as well as air. Fabrics of a loose weave are warmer than those of a dense weave of the same material (except in wind protection), because of the more abundant air space. A wool-lined canvass coat protects against both wind and low temperature.

Davy's Safety Lamp.—If a flame is directed against a *cold* metal surface, it will be found that the metal cools below the combustion point the gases of which the flame is composed, so that the flame does

FIG. 141.

FIG. 141a

not actually touch the metal. This fact may be demonstrated by pasting one piece of paper on a block of metal, and a second piece on a block of wood, and thrusting both into a flame. The second piece of paper quickly ignites, the first does not. A thin paper pail quickly ignites if exposed to a flame when empty, but not when filled with water.

If a piece of wire gauze is held above a Bunsen burner or other gas jet, the flame will burn *above* the gauze *only* (Fig. 141), *if lighted above*, and *below only* (Fig. 141*a*), *if lighted below*. The flame will not "strike through" the gauze until the latter reaches red heat. Evidently, the gas (Fig. 141*a*) as it passes through the wire gauze is cooled below its ignition temperature. If a lighted match is now applied below the gauze (Fig. 141), or above it (Fig. 141*a*), the flame burns both above and below as though the gauze were absent.

The miner's *Safety Lamp*, invented by Sir Humphry Davy, has its flame completely enclosed by iron gauze. The explosive fire-damp as it passes through the gauze, burns within, but not without, and thus gives the miner warning of its presence. After a time the gauze might become heated sufficiently to ignite the gas and cause an explosion.

Boiler "Scale."—The incrustation of the tubes of tubular boilers with lime, etc., deposited from the water, is one of the

serious problems of steam engineering. The incrusted material adds to the thickness of the walls of the tubes, and is also a very poor conductor of heat in comparison with the metal of the tube. Consequently it interferes with the transmission of the heat from the heated furnace gases to the water, and thereby lowers the efficiency of the boiler. Furthermore, the metal, being in contact with the flame on the one side and the "scale" (instead of the water) on the other, becomes hotter, and therefore burns out sooner than if the scale were prevented.

210. Thermal Conductivity.—If three short rods of similar size and length, one of copper, one of iron, and one of glass, are held by one end in the hand while the other end is thrust into the gas flame, it will be found that the copper rod quickly becomes unbearably hot, the iron rod less quickly, while the glass rod does not become uncomfortably hot, however long it is held. This experiment shows that copper is a better conductor than iron, and that iron is a better conductor than glass; but it does not enable us to tell how many times better. To do this we must compare the thermal conductivities of the two metals, from which (see table) we find that copper conducts about five times as well as iron, and over 500 times as well as glass. The fact that glass is such a very poor conductor explains why the thin glass of windows is so great a protection against the cold.

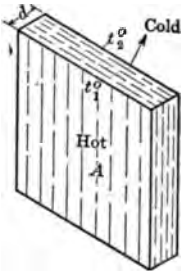

FIG. 142.

If one face of a slab of metal (Fig. 142) is kept at a higher temperature than the other face, it will be evident that the number of calories of heat Q which will pass through the slab in T seconds will vary directly as the time T, as the area A of the face, and also directly as the difference in temperature between the two faces (*i.e.*, $t_1 - t_2$, in which the temperature of the hotter face is $t_1°$ and the colder, $t_2°$). It is also evident, other things being equal, that less heat will flow through a thick slab than through a thin one. Indeed, we readily see that the quantity Q will vary inversely as the thickness (d) of the slab. Accordingly we have

$$Q \propto AT\frac{(t_1-t_2)}{d} = KAT\frac{(t_1-t_2)}{d} \qquad (86)$$

in which K is a constant, whose value depends upon the character of the material of which the slab is composed, and is called the *Thermal Conductivity* of the substance.

Since Eq. 86 is true for all values of the variables, it is true if we let A, T, (t_1-t_2), and d all be unity. This, however, would reduce the equation to $K=Q$. Hence, K is *numerically the number of calories of heat that will flow in unit time* (the second) *through a slab of unit area and unit thickness* (*i.e.*, through a cubic centimeter) *if its two opposite faces differ in temperature by unity* (1° C).

Temperature Gradient.—Observe that $\frac{t_1-t_2}{d}$ is the fall in temperature per centimeter in the direction of heat flow. This quantity is called the *Temperature Gradient.* The heat conductivity, then, is the *rate of flow of heat* (calories per sec.) through a conductor, divided by the product of the *cross-sectional area* and the *temperature gradient.* It is better in determining the heat conductivity for materials which are good conductors, such as the metals, to use a rod instead of a slab.

The rod is conveniently heated at one end by steam circulation, and cooled at the other end by water circulation. The temperature of the water as it flows past the end of the rod rises from $t_3°$ to $t_4°$. If M grams of water flows past in T seconds, then $Q=M(t_4-t_3)$, and the *rate of flow* of heat through the rod is $\frac{M(t_4-t_3)}{T}.$ Two thermometers are inserted in the rod at a distance d apart, one near the hot end, the other near the cold end. Let the former read $t_1°$ and the latter, $t_2°$. The temperature gradient is, then, $\frac{t_1-t_2}{d}$. The remaining quantity A of Eq. 86, which must be known before K can be calculated, is the cross-sectional area of the rod. If the rod is of uniform diameter and is packed in felt throughout its length to prevent loss of heat, then the rate of heat flow, and also the *temperature gradient, will be the same at all points in the rod.*

The temperature gradient may be thought of as *forcing heat* along the rod, somewhat as the *pressure gradient forces water* along a pipe. A few thermal conductivities are given in the table below.

THERMAL CONDUCTIVITIES

Substances	Thermal conductivity K	Substance	Thermal conductivity K
Silver	1.006	Marble	0.0014
Copper	0.88 to 0.96	Water	0.0014
Aluminum	0.34	Hydrogen	0.00033
Iron	0.16 to 0.20	Paraffine	0.00025
Mercury	0.016	Air	0.000056
Glass	0.0015	Flannel	0.000035

The value of the thermal conductivity varies greatly in some cases for different specimens of the same substance. Thus, for

hard steel, it is about one-half as large as for soft steel, and about one-third as large as for hard steel. Different kinds of copper give different results. The values given in the table are approximate average values.

211. Wave Motion.—The kinds of wave motion most commonly met are three in number, typified by *water waves, sound waves,* and *ether waves.* The beautiful waves which travel over a field of grain on a windy day, are quite similar to water waves in appearance, and similar to *all waves* in one respect; namely, that the medium (here the swaying heads of grain) does not move forward, but its parts, or *particles* simply *oscillate* to-and-fro *about* their respective *equilibrium positions.*

Water Waves.—There are many kinds of water waves; varying in *form* from the smooth ocean "swell" due to a distant storm, to the "choppy" storm-lashed billows of the tempest; and varying in *size* from the large ocean waves 20 ft. or more in height, to the tiny ripples that speed over a still pond before a sudden gust of wind. The *Tide* (Sec. 30) consists of two wave crests on opposite sides of the earth, which travel around the earth in about 25 hrs. Consequently, at the equator, the wave length is over 12,000 miles, and the velocity about 1000 miles per hour.

Restoring Force.—In all cases of wave motion, at least in material media, there must be a restoring force developed which acts upon the displaced particle of the medium in such a direction as to tend to bring it back to its equilibrium position. As the head of grain sways to-and-fro, the supporting stem, alternately bent this way and that, furnishes the restoring force. As the vibrating particle reaches its equilibrium position, it has kinetic energy which carries it to the position of maximum displacement in the opposite direction. Thus the swaying head of grain when the stem is erect is in equilibrium, but its velocity is then a maximum and it moves on and again bends the stem.

In the case of large water waves, the restoring force is the *gravitational pull* which acts *downward* on the "crest," and the *buoyant force* which acts *upward* on the "trough" of the wave. These waves are often called gravitational water waves. In the case of fine ripples, the restoring force is mainly due to surface tension. The velocity of long water waves increases with the wave length (distance from crest to crest), while with ripples, the reverse is true; *i.e.*, the finer the ripples are, the faster they travel.

Sound Waves.—As the prong of a tuning fork vibrates to-and-

fro, its motion in one direction condenses the air ahead of it; while its return motion rarefies the air at the same point. These condensations and rarefactions travel in all directions from the fork with a velocity of about 1100 ft. per sec., and are called *Sound Waves.* Obviously, if the tuning fork vibrated 1100 times per sec., one condensation would be one foot from the tuning fork when the next condensation started; while if the fork vibrated 110 times per sec. this distance between *Condensations,* called the *wave length* λ, would be 10 ft. In other words the relation $v = n\lambda$ is true, in which v is the velocity of sound, and n, the number of vibrations of the tuning fork per second. Sound waves are given off by a vibrating body, and are transmitted by any elastic medium, such as air, water, wood, and the metals. The velocity varies greatly with the medium, but the relation $v = n\lambda$ always holds.

Ether Waves.—Ether waves consist in vibrations of the *Ether* (Sec. 214), a medium which is supposed to pervade all space and permeate all materials. These vibrations are produced, in the case of heat or light waves, by atomic vibrations in a manner not understood. The ether waves used in wireless telegraphy are produced by special electrical apparatus which we cannot discuss here.

Ether waves are usually grouped in the following manner. Those which affect the eye (*i.e.,* produce the sensation of light) are called *light waves,* while those too long to affect the eye are called *heat waves.* Those waves which are too short to affect the eye do affect a photographic plate, and are sometimes called *actinic waves.* It should not be inferred that light waves do not produce heat or chemical (*e.g.,* photographic) effects, for they do produce both. Certain waves which are still longer than heat waves, and which are produced electrically, are called *Hertz waves.* These waves are the waves employed in wireless telegraphy. They were discovered in 1888 by the German physicist, H. R. Hertz (1857–94).

The longest ether waves that affect the eye are those of red light ($\lambda = 1/35000$ in. approx.). Next in order of wave length are orange, yellow, green, blue, and violet light. The wave length of violet light is about one-half that of red, while *ultra-violet* light of wave length less than one-third that of violet has been studied by photographic means. An occupant of a room flooded with ultra-violet light would be in total darkness, and yet with a

camera, using a short exposure, he could take a photograph of the objects in the room. The wave lengths longer than those of red light, up to about 1/500 inch, have been much studied, and are called heat waves, or *infra-red*. It is interesting to note that the shortest Hertz waves that have been produced are but little longer than the longest heat waves that have been studied. If this small "gap" were filled, then ether waves varying in length from several miles to 1/200000 in. would be known.

Since the velocity v for all ether waves is 186,000 miles per sec., the frequency of vibration n for any given wave length λ is quickly found from the relation $v = n\lambda$. Thus the frequency of vibration of violet light for which $\lambda = 1/70000$ in. is about 800,000,000,000,000. This means that the source of such light, the vibrating atom, or atomic particle (electron) sends out 800,000,000,000,000 vibrations per second!

Direction of Vibration.—A water wave in traveling south, let us say, would appear to cause the water particles to vibrate up and down. Careful examination, however, will show that there is combined with this up-and-down motion a north-and-south motion; so that any particular particle is seen to describe approximately a circular path. A sound wave traveling south causes the air particles to vibrate to and fro north and south; while an ether wave traveling south would cause the ether particles to vibrate up and down or east and west, or in some direction in a plane which is at *right angles* to the direction in which the wave is traveling. For this reason, the ether wave is said to be a *Transverse Wave* and the sound wave, a *Longitudinal Wave.* The phenomena of *polarized light* seem to prove beyond question that light is a transverse wave.

212. Interference of Wave Trains.—A succession of waves, following each other at equal intervals, constitutes a wave train. A vibrating tuning fork or violin string, or any other body which vibrates at a constant frequency, gives rise to a train of sound waves. Two such wave trains of different frequency produce interference effects, known as beats, which are familiar to all.

Interference of Sound Waves.—Let a tuning fork A of 200 vibrations per second be sounded. The train of waves from this fork, impinging upon the ear of a nearby listener, will cause the tympanum of his ear to be alternately pushed in and out 200 times per sec., thus giving rise to the perception of a musical tone of *uniform intensity*. If, now, a second fork B of, say 201 vibrations per second, is sounded, the train

of waves from it, let us say the "B train," will interfere with the "A train" and produce an alternate waxing and waning in the intensity of the sound, known as "beats." In this case there would be 1 beat per sec. For, consider an instant when a compressional wave from the A train and one from the B train both strike the tympanum together. This will cause the tympanum to vibrate through a relatively large distance, *i.e.*, it will cause it to have a vibration of large amplitude, and a loud note (maximum) will be heard. (The amplitude of a vibration is half the distance through which the vibrating body or particle, as the case may be, moves when vibrating; in other words, it is the maximum displacement of the particle from its equilibrium position.) One-half second later, a *compressional wave* from the A train and a *rarefaction* from the B train will both strike the tympanum. Evidently these two disturbances, which are said to be *out of phase* by a half period, will produce but little effect upon the tympanum, in fact none if the two wave trains have *exactly equal amplitudes*. Consequently, a minimum in the tone is heard. Still later, by 1/2 sec., the two trains reach the ear exactly in phase, and another maximum of intensity in the tone is noted, and so on. Obviously, for a few waves before and after the maximum, the two trains of waves will be nearly in phase, and a fairly loud tone will be heard. This tone dies down gradually as the waves of the two trains get more and more out of phase with each other, until the minimum is reached.

Had the tuning forks differed by 10 vibrations per second, there would have been 10 beats per second. To tune a violin string to unison with a piano, gradually increase (or decrease) the tension upon it until the beats, which come at longer and longer intervals, finally disappear entirely. If increasing the tension produces more beats per second, the string is already of too high *pitch*.

Interference of Light Waves.—By a proper arrangement, two trains of light waves of equal frequency and equal amplitude may be produced. If these two trains fall upon a photographic plate from slightly different directions, they will reinforce each other at some points of the film, and annul each other at other points. For certain portions of the plate, the two trains are constantly one-half period out of phase. Such portions are in total darkness, and therefore remain clear when the plate is "developed," producing, with the alternate "exposed" strips, a beautiful effect. We here have the strange anomaly of *light added to light producing darkness*, for either beam *alone* would have affected the entire photographic plate.

213. Reflection and Refraction of Waves.—In Fig. 143, let AB be a stone pier, and let abc, etc., be water waves traveling in the direction bO. Then $a'b'c'$, etc. (dotted lines), will be the reflected water waves, and will travel in the direction Ob', such that bO and Ob' make equal angles θ_1 and θ_2 with the normal

(NO) to the pier. This important law of reflection is stated thus: *The angle of reflection* (θ_2) *is equal to the angle of incidence* (θ_1).

If AB is a mirror and *abc*, etc., light waves, or heat waves, then the construction will show accurately the reflection of light or heat waves, as the case may be.

Proof: If the reflected wave has the same velocity as the incident wave, which is strictly true in the case of heat and light, then, while the incident light (let us say) travels from a_1 to a_2, the reflected light will travel from c_1 to c_2. The triangles $a_1c_1a_2$ and $c_2a_2c_1$ will be not only similar, but equal. Therefore $\theta_3 = \theta_4$. But $\theta_3 = \theta_1$ and $\theta_4 = \theta_2$, hence $\theta_1 = \theta_2$, which was to be proved.

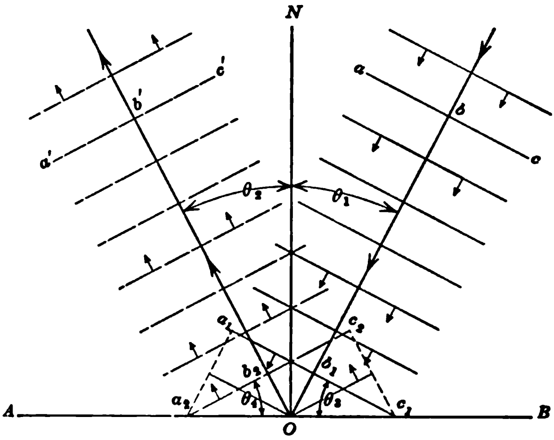

FIG. 143.

Refraction.—Let *abc* (Fig. 144) represent a light wave or a heat wave, traveling in the direction bO. Then, as the portion a reaches a', portion c will have reached c' instead of c''. The ratio cc'/cc'' is about 3/4, since light and heat radiation travel about 3/4 as fast in water as in air. The reciprocal of this ratio, *i.e.*, the velocity in air divided by the velocity in water, is called the *index of refraction* for water. The index of refraction for glass varies with the kind of glass and the length of the wave, from about 1.5 to 2. Since the ray is always normal to the wave front, the ray Ob' deviates from the direction bO by the angle α, called the *angle of deviation*. The fact that the ray bends sharply *downward* as it enters the water, accounts for the apparent sharp *upward* bending of a straight stick held in a slanting position partly beneath the surface of the water.

The fact that light and heat radiation travel more slowly in glass than in air, thus causing all rays which strike the glass obliquely to be deviated, makes possible the focusing of a bundle of rays at a point by means of a glass lens, and therefore makes possible the formation of images by lenses. Since practically all optical instruments consist essentially of a combination of lenses, we see the great importance of the refractive power of glass and other transparent substances. Indeed were it not for the fact that light travels more slowly through the crystalline lens of the eye than through air, *vision itself would be impossible.*

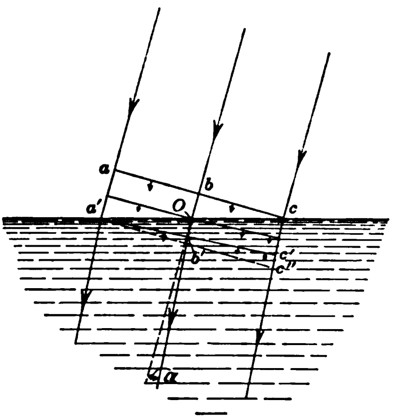

Fig. 144.

The production of the rainbow and prismatic colors in general depends upon the fact that the velocity of light in glass, water, etc., depends upon the wave length, being greatest for red and least for violet. Consequently red light is deviated the least, the violet the most.

214. Radiation.—If a glowing incandescent lamp is placed under the receiver of an air pump, it will be found that it gives off heat and heats the receiver, whether the receiver contains air or a vacuum. It is evident, then, that the air is not the medium of transfer of heat by radiation. Likewise, in the case of heat and light received from the sun, the medium of transfer cannot be air. Since the transmission of a vibratory motion from one point to another requires an intervening medium, physicists have been led to postulate the **Ether** as such a medium, and have ascribed to it such properties as seem best to explain the observed phenomena. The ether is supposed to fill all space and also to permeate all

materials. Thus we know that the heat of the sun passes readily through glass by radiation. This is effected, however, by the ether in the glass and not by the glass itself. Indeed the glass molecules prevent the ether from transmitting the radiation so well as it would if the glass were absent.

While immense quantities of heat are transferred from the sun to the earth by radiation, it is well to call attention to the fact that what we call *radiant heat* or heat radiation, is *not strictly heat*, but energy of wave motion. Radiant heat does not heat the medium through which it passes (unless it is in part absorbed), but heats any body which it strikes—a good reflector least, a lamp-black surface most. Both heat radiation and light may be reflected, and also refracted (Sec. 213). The moon and the planets, in the main, shine by reflected sunlight. We see all objects which are not self-luminous, by means of irregularly (scattering) reflected light. At South Pasadena, Cal., a 10-H.P. steam engine is run by a boiler which is heated by means of sunlight reflected from a great number of properly placed mirrors.

215. Factors in Heat Radiation.—It has been shown experimentally that the higher the temperature of a body becomes, the faster it radiates heat energy. Obviously, the amount of heat radiated in a given time will also be proportional to the amount of heated surface. It has also been found that two metal spheres, A and B, alike as to material, size and weight, but differing in finish of surface, have quite different radiating powers. Thus if A is highly polished, so as to have a mirror-like surface, while B is coated with lamp black, it will be found that B radiates heat much faster than A. This is easily tested by simply heating A and B to the same temperature and then suspending them to cool. It will be found that B cools much more rapidly than A, which shows that B *parts* with its heat more quickly, *i.e.*, *radiates* better, than A. A lamp-black surface is about the best radiating surface, while a polished mirror surface is about the poorest. The radiating powers of other substances lie between those of these two. From the above discussion, we see that the high polish of the nickel trimmings of stoves decreases their efficiency somewhat.

Prevost's Theory of Heat Exchanges.—According to this theory, a body radiates heat to surrounding bodies whether it is *warmer* than they or *colder*. In the former case it radiates *more* heat to the surrounding bodies *than it receives* from them, and its temperature *falls;* while in the latter case it radiates *less* heat *than it*

receives, and its temperature *rises*. The fall in temperature experienced by a body when placed near ice, a result which would at first seem to indicate that cold can be radiated and that it is not therefore merely the absence of heat, is easily explained by this exchange theory. The body radiates heat no faster to the ice than it would to a warmer body, but it receives less in return, and therefore becomes colder.

Laws of Cooling.—Newton considered that the amount of heat H, radiated from a body of temperature t, to its surroundings of temperature t', was proportional to the difference in temperature; *i.e.*,

$$H = K(t - t')$$

in which K is a constant, depending upon the size and character of the surface. This law is very nearly true for slight differences in temperature only. Thus a body loses heat almost exactly twice as fast when 2° warmer than its surroundings as it does when 1° warmer. Experiment, however, shows that if this temperature difference is, say, 20°, the amount of heat radiated is more than 20 times as great as when it is 1°.

The quite different law, expressed by the equation

$$H = K(T^4 - T'^4)$$

is due to Stefan, and is known as Stefan's Law. In this equation T and T' are the temperatures of the body and its surroundings, respectively, on the *absolute scale*. Stefan's law, applied to radiation by *black* bodies, accords with experimental results.

216. Radiation and Absorption.—It has been found experimentally that surfaces which radiate heat rapidly when hot, absorb heat rapidly when cold. Thus if the two metal spheres mentioned in Sec. 215 were placed in the sunshine, B would be warmed very much more quickly than A. Evidently the same amount of solar heat radiation would strike each, but A reflects more and consequently absorbs less than B, which has smaller reflecting power. There is a close proportionality between radiation and absorption. For example, if B, when hot, loses heat by radiation *twice as fast* as A does when *equally* hot, then if both are equally cold and are placed in the sunshine, B will absorb heat practically *twice as fast* as A. That is, good absorbers of heat (when cold) are good radiators of heat (when hot). If two thermometers, one of which has its bulb smoked until black,

are placed side by side in the sunshine, the one with the blackened bulb will indicate a higher temperature than the other.

217. Measurement of Heat Radiation.—By means of the thermopile (Sec. 174), and other sensitive devices, such as the bolometer, many measurements of intensity of heat radiation have been made. When white light, *e.g.*, sunlight, passes through a prism, the different colors of light take slightly different directions, and a "spectrum" of the colors, red, orange, yellow, green, blue, and violet, is produced.

By exposing the bolometer successively in the violet, blue, green, yellow, orange and the red, and then moving it still farther, into the invisible or infra-red part of the spectrum, it is found that the radiant energy increases with the wave length, and reaches a maximum in the infra-red. In other words, the wave length of the sun's radiation which contains the most energy is slightly greater than that of the extreme red. It has been found by experiment, using various sources of known temperature for producing the light, that the wave length of maximum energy is shorter, the hotter the source. From these considerations the temperature of the sun is estimated to be about 6000° C. In such experiments, a rock-salt prism must be used, since glass absorbs infra-red radiations to a great extent.

218. Transmission of Heat Radiation Through Glass, Etc.—Just as light passes readily through glass and other transparent substances, so heat radiation passes readily through certain substances. In general, substances *transparent to light* are also *transparent to heat radiation*, but there are some exceptions to this rule.

A thin pane of glass gives very little protection from the sun's heat, but if held between the face and a hot stove it is a great protection. It may be remarked that in the former case, the glass is not noticeably warmed, while in the latter case it is warmed. It is apparent, then, that the glass transmits solar radiation better than it does the radiation from the hot stove. This selective transmission of radiation is really due to "selective absorption." The glass absorbs a greater percentage of the radiation in the latter case than in the former, which accounts not only for the fact that it *transmits less* heat in the case of the radiation from the stove, but also for the fact that it is *heated more*.

In this connection, we may state that it has been shown by experiment that a body, say a piece of iron, when heated to a white heat, gives off simultaneously heat waves varying greatly in

length. As it is heated more and more, it gives off more and more energy of *all wave lengths;* but the energy of the *shorter* wave lengths *increases most rapidly.* Accordingly, the wave length of maximum energy becomes shorter the hotter the source, as stated in Sec. 217.

Just before the iron reaches "red heat" the heat waves are all too long to be visible. As it becomes hotter, somewhat shorter waves, corresponding to red light, are given off, and we say that the iron is "red hot." If heated to a still higher temperature, so as to give off a great deal of light, in fact light of all different wave lengths, we say that it is "white hot." A hot stove, then, gives off, in the main, very *long* heat waves; while the sun, which is intensely heated, gives off a great deal of its heat energy in the *short* wave lengths.

The above-mentioned fact, that glass affords protection from the heat radiation from a stove, and no appreciable protection in the case of solar radiation, is explained by saying that glass *transmits* short heat waves much better than long heat waves, *i.e.*, glass is more transparent to short than to long heat waves. More strictly, it might be said that glass does not prevent the transmission of short heat waves by the ether permeating it, to so great an extent as it does the long heat waves.

The "Hotbed."—The rise in temperature of the soil in a *Hotbed*, when the glass cover is on, above what it would be if the glass were removed, is in part due to this behavior of glass in the transmission of heat radiation. The greater part of the solar heat that strikes the glass, being of short wave length, passes readily through the glass to the soil, which is thereby warmed. As the soil is warmed, it radiates heat energy, but in the form of long heat waves which do not readily pass through the glass, and hence the heat is largely retained. The fact that the glass prevents a continual stream of cold air from flowing over the soil beneath it, and still permits the sun to shine upon the soil, accounts in large part for its effectiveness.

"Smudging" of Orchards.—Very soon after sunset, blades of grass and other objects, through loss of heat by radiation, usually become cool enough to precipitate part of the moisture of the air upon them in the form of *Dew* (Secs. 220, 221). It is well known that heavy dews form when the sky is clear. If the sky is overcast, even by fleecy clouds, a portion of the radiated heat is reflected by the clouds back to the earth, and the cooling of

objects, and consequently the formation of dew upon them, is less marked.

Many fruit growers have placed in the orchard, a thermostat, so adjusted that an alarm is sounded when the freezing point is approached. As soon as the alarm is sounded, the "smudge" fires (coal, coal oil, etc.) are started. These fires produce a thin veil of smoke, which hovers over the orchard and protects it from frost, somewhat as a cloud would. In addition to the protection afforded by the smoke, the considerable amount of heat developed by the fires is also important. If the wind blows, such protection is much less effective. Frosts, however, usually occur during still, clear nights.

219. The General Case of Heat Radiation Striking a Body.—Heat radiation, *e.g.*, solar radiation, when it strikes a body, is in general divided into three parts: the part (a) which is *reflected;* the part (b) which is *absorbed* and therefore tends to heat the body; and the part (c) which is *transmitted*, or passes through the body. The sum of these, *i.e.*, $a+b+c$, is of course equal to the original energy that strikes the body. In some cases, the part reflected is large, *e.g.*, if the body has polished surfaces. In other cases, the part absorbed is large, *e.g.*, for lamp black or, in general, for dull surfaces, and also for certain partially transparent substances. The part transmitted is large for quartz and rock salt; much smaller for glass, water and ice, and absent for metals unless they are in the form of exceedingly thin foil.

PROBLEMS

1. If a piece of plate glass 80 cm. in length, 50 cm. in width, and 1.2 cm. in thickness, is kept 20° C. hotter on one side than on the other, how many calories of heat pass through it every minute by conduction alone?

2. A copper vessel, the bottom of which is 0.2 cm. thick, has an area of 400 cm.^{2}, and contains 3 kilograms of water. What will be the temperature rise of the water in it in 1 minute, if the lower side of the bottom is kept 3° C. warmer than the upper side of the bottom?

3. Assuming the sun to be directly over head, what power (in H.P.) does it radiate in the form of heat upon an acre of land at noon. See Sec. 161.

4. A wall 10 in. thick is made of a material, the thermal conductivity of which is 0.00112. The wall is made "twice as warm" by rebuilding it with an additional thickness of "dead air" space. Find the thickness of the air space. (In practice, convection currents diminish considerably the effectiveness of so-called "dead air" spaces.)

5. How many pounds of steam at 140° C. (heat of vap. 509 cal. per gm.) will a boiler furnish per hour if it has 1000 sq. ft. of heating surface of iron (thermal conductivity 0.16) 0.25 in. in thickness, which is kept 5° hotter next the flame than next the water? Note that the heat of vaporization and the conductivity are given in C.G.S. units.

CHAPTER XVII

METEOROLOGY

220. General Discussion.—Meteorology is that science which treats, in the main, of the variations in heat and moisture of the atmosphere, and the production of storms by these variations. Although the earth's atmosphere extends to a height of a great many miles, the weather is determined almost entirely by the condition of the lower, denser strata, extending to a height of but a few miles.

Clouds.—Clouds have been divided into eight or ten important classes, according to their appearance or altitude. Their altitude varies from 1/2 mile to 8 or 10 miles, and their appearance varies from the dense, gray, structureless rain cloud, called *Nimbus*, to the interesting and beautiful "wool-pack" cloud, known as *Cumulus*, which resembles the smoke and "steam" rolling up from a locomotive. All clouds are composed either of minute droplets of water or tiny crystals of snow, floating in the air. *Fog* is merely a cloud at the surface of the earth. Thus, what is a cloud to the people in the valley, is a fog to the party on the mountain side enveloped by the cloud. The droplets in a fog are easily seen. The upper clouds may travel in a direction quite different from that of the surface wind, and at velocities as high as 200 miles per hour.

221. Moisture in the Atmosphere.—The constant evaporation from the ocean, from inland bodies of water, and from the ground, provides the air with moisture, the amount of which varies greatly from time to time. Although the water vapor seldom forms as much as 2 per cent. of the weight of the air, nevertheless, *water vapor* is the most *important factor* in determining the character of the weather. When air contains all of the moisture it will take up, it is said to be *saturated*. If saturated air is heated, it is capable of taking up more moisture; while if it is cooled, it precipitates a portion of its moisture as fog, cloud, dew, or rain. If still further cooled, it loses still more of its water vapor. Indeed the statement that the air is saturated with

water vapor does not indicate how much water vapor it contains, unless the temperature of the air is also given.

When unsaturated air is cooled more and more, it finally reaches a temperature at which precipitation of its moisture occurs. This temperature is called the *Dew Point.* If air is nearly saturated, very little cooling brings it to the dew point. After the dew point is reached, the air cools more slowly, because every gram of water vapor precipitated, gives up nearly 600 calories of heat (its heat of vaporization) to the air. Thus, if on a clear chilly evening in the fall, a test for the amount of water vapor in the air shows the dew point to be several degrees below zero, then frost may be expected before morning; while if the dew point is well above zero, there is little probability of frost. This might be taken as a partial guide as to whether or not to protect delicate plants. The fact should be emphasized, that if the moisture in the air is visible, it is in the form of droplets, since *water vapor, like steam, is invisible.*

222. Hygrometry and Hygrometers.—Hygrometry deals with the determination of the amount of moisture in the atmosphere. The devices used in this determination are called hygrometers. Only two of these, the chemical hygrometer and the wet-and-dry-bulb hygrometer, will be discussed.

The Chemical Hygrometer consists of a glass tube containing fused calcium chloride ($CaCl_2$), or some other chemical having great affinity for water. Through this tube (previously weighed) a known volume of air is passed. This air, during its passage, gives up its moisture to the chemical and escapes as perfectly dry air. The tube is again weighed, and the gain in weight gives the amount of water vapor in this known volume of air.

The Wet-and-dry-bulb Hygrometer.—From the two temperature readings of the wet-and-dry-bulb thermometer (Sec. 198), in connection with a table such as given below, the dew point may be found. Having found the dew point, the amount of moisture per cubic yard is readily found from the second table.

The manner of using these tables will be best illustrated by an example. Suppose that when a test is made, the dry-bulb thermometer reads 60° F., and the wet-bulb thermometer 52° F., or 8° lower. Running down the vertical column (first table) for which $t-t'$ is 8° until opposite the dry-bulb reading 60°, we find the dew point 45°.6 F. This shows that if the temperature of the air falls to 45°.6 F., precipitation will commence. Opposite to

dew point 45° (the nearest point to 45.6) in the second table, we find 0.299 and 0.0133; which shows that every cubic yard of air contained approximately 0.0133 lbs. of water vapor on the day of this test, and that the water vapor pressure was 0.299 inches of mercury.

DEW POINTS FROM WET-AND-DRY-BULB HYGROMETER READINGS
Dry bulb temperature t. Wet bulb temperature t'. Difference $t-t'$.

$t°-t'°$	=2°	4°	6°	8°	10°	12°	14° F.
$t°$							
40° F.	36.2	30.8	25.6	20.8	16.0	11.2	6.4
45	41.4	35.8	31.2	27.0	22.1	17.4	12.8
50	45.8	41.6	37.4	33.0	29.0	24.8	20.6
55	51.0	47.0	43.0	39.0	35.2	31.0	27.0
60	56.4	52.8	49.2	45.6	42.3	38.4	34.8
65	61.6	59.2	55.0	51.4	48.0	44.6	41.0
70	67.0	64.0	61.0	58.0	55.3	52.0	49.0
75	72.0	69.0	66.0	63.0	60.6	57.0	54.0
80	77.0	74.0	71.0	68.0	65.0	62.0	59.0

DEW POINTS AND THE CORRESPONDING PRESSURES AND DENSITIES OF WATER VAPOR

Dew point	Pressure in inches of mercury	Density in lbs. per cu. yd.
20° F.	0.102	0.0051
25	0.130	0.0066
30	0.161	0.0076
35	0.200	0.0091
40	0.252	0.0111
45	0.299	0.0133
50	0.358	0.0159
55	0.433	0.0196
60	0.512	0.0223
65	0.617	0.0255
70	0.732	0.0306

223. Winds, Trade Winds.—Winds originate in the uneven heating of the earth's atmosphere at different points. This heating is in part due to the direct action of the sun, and in part to the heat of vaporization given off when a portion of the moisture in the air changes to the liquid state. When air is heated it expands, and therefore becomes lighter and rises with considerable velocity. The current of colder air, rushing in to take its place, is called *Wind.* This effect is easily noticed with a large bonfire on a *still* day. The violent upward rush of the heated air above the fire carries cinders to a great height. The

cool air rushing in to take its place produces a "wind" that *blows toward the fire from all directions.*

The Trade Winds.—An effect similar to that produced by the bonfire, as above described, is constantly being produced on a grand scale in the tropical regions. The constant high temperature of the equatorial regions heats the air highly and causes it to rise. The air north and south of this region, rushing in to take the place of this rising air, constitutes the *Trade Winds.*

On account of the rotation of the earth, trade winds do not blow directly toward the equator but shift to the westward. Thus in the West Indies, the trade winds are N. E. winds, *i.e.*, they blow S. W. The *trade* winds *south* of the equator are S. E. winds, *i.e.*, they blow N. W.

The westward deviation of the trade winds, both north and south of the equator, may be accounted for as follows. Objects near the equator describe each day, due to the rotation of the earth, paths which are the full circumference of the earth; while objects some distance either north or south of the equator describe shorter paths in the same time, and therefore have less velocity. Consequently, as a body of air moves toward the equator, it comes to points of higher eastward velocity, and therefore "falls behind," so to speak; that is, it drifts somewhat to the westward.

Between the trade winds of the two hemispheres lies the equatorial "zone of calms." This zone, which varies from 200 to 500 miles in width, has caused sailing vessels much trouble with its prolonged calms, violent thunder storms, and sudden squalls.

Since a rising column of air is cooled by expansion, it precipitates its moisture; whereas a descending column, warmed as it is by compression, is always capable of absorbing more moisture and is, therefore, relatively dry. In the zone of calms, the air from the two trade winds which meet in this region must rise. As it rises, it is cooled and precipitates its moisture in torrents of rain. Wherever the prevailing wind blows from the sea across a mountain range near the coast, the rain will be excessive on the mountain slope toward the sea, where the air must rise to pass over the mountain. As the air descends upon the opposite side of the range, it is very dry and produces a region of scant rain and, in many cases, a desert. The rainfall on portions of the southern slope of the Himalaya Mountains is about

30 ft. per year; while to the north of the range lie large arid or semi-arid districts.

224. Land and Sea Breezes.—Near the seashore, especially in warm countries, the breeze usually blows toward the shore from about noon until shortly before sunset, and toward the sea from about midnight until shortly before sunrise. The former is called a *Sea Breeze;* the latter, a *Land Breeze.*

These breezes are due to the fact that the temperature of the land changes quickly, while the temperature of the ocean is nearly constant (Sec. 185). Consequently, by noon, the air above the land has become considerably heated, and is therefore less dense than the air over the ocean. This heated air, therefore, rises, and the air from the ocean, rushing in to take its place, is called the sea breeze. The rising column of air becomes cooler as it rises, and flows out to sea. Thus, air flows from sea to land near the earth, and from land to sea in the higher regions of the atmosphere. Toward midnight the land and the air above it have become chilled. This chilled, and therefore dense air flows out to sea, as a land breeze; while the air from the ocean flows toward the land in the higher region. It will be observed, then, that the convection circulation at night is just the reverse of the day circulation.

It is observed that the sea breeze first originates some distance out at sea and blows toward the land. A feasible explanation is this: As the air over the land first becomes heated it expands and swells up like a large blister. The air above, lifted by the "blister," flows away out to sea in the higher regions of the atmosphere, thereby causing an excess pressure upon the air there. The air then flows away from this region of excess pressure toward the land, where the deficit in pressure exists.

225. Cyclones.—Strictly, the term cyclone applies to the periodical rotary storms, about 1000 miles or so in diameter, which occur in various parts of the earth. Every few days they pass across the central portion of the United States, in a direction somewhat north of east. Their courses may be followed from day to day by means of the U. S. weather maps. The barometric pressure is usually about one-half inch of mercury less at the center ("storm center") of a cyclone, than at the margin. This region of low pressure, called a "low area," is due, at least in part, to the condensation of water vapor that occurs in cloud formation, and the consequent

heating of the air by the heat of vaporization thereby evolved. This heated air rises, and the surrounding air, rushing in, produces wind. Due to the rotation of the earth, these winds, instead of blowing straight toward the storm center, are, in general, deflected to the *right* of the storm center. Occasionally, due so some local disturbance, the wind may blow in a direction nearly opposite to that which would be expected from the above rule; but, in general, the surrounding air moves toward the storm center in a spiral path. The rotation is counterclockwise (viewed from above) in the Northern hemisphere, and clockwise in the Southern. Any body in motion (*e.g.*, a rifle ball) in the *Northern hemisphere* tends to deviate *to the right* from its path, and in the *Southern hemisphere to the left.*[1] This fact accounts for the rotatory motion of these storms, as explained below.

Cause of the Rotary Motion of Cyclones.—Let Fig. 145 represent a top view of a level table A upon which rests a heavy ball B loosely surrounded by a very light frame C to which is attached a string DD_1. Evidently, if the table is at rest and the lower end of the string D_1, which passes through a hole in the center of the table, is pulled, ball B will roll in a straight path to the center. If, however, the string is pulled while the table, and consequently the ball, are rapidly revolving in the direction indicated by the arrow a, then the ball will follow a *left-handed* spiral path as indicated by the broken line. For, as the ball moves nearer to the center, it reaches portions of the table of smaller and smaller radius, and consequently portions having less tangential velocity than its own. Therefore, the ball rolls "ahead," *i.e.*, to the *right*, of the straight line D as shown. If the table and the ball were rotated in the opposite direction (clockwise), similar reasoning would show that the ball would then travel toward the center in a *right-handed spiral path.*

It will next be shown that any area of the globe, having a diameter of, say, a few hundred miles, may be *considered* to be a flat surface,

[1] Although this tendency of a moving body to drift to the right in the Northern hemisphere and to the left in the Southern hemisphere is of such great importance in determining the motion of storms, its effect on projectiles is very slight indeed. Thus, in latitude 40°, due to this cause, an army rifle projectile veers to the right (no matter in what direction it is fired) by only about 3 in. on a 1000-yd. range. Due to the same cause, a heavy locomotive, when at full speed on a level track, bears only about 50 lbs. more on the right rail than on the left.

rotating about a vertical axis at its center, and that consequently, air which tends to move toward the center of the area, as it does in cyclones, will trace a spiral path similar to that traced by the ball (Fig. 145). That an area about the north pole has such rotation, with the pole as axis, is evident. Since the earth rotates from west to east, this rotation viewed from above, is counterclockwise the same as shown in Fig. 145. The rotational velocity, say ω_1, is of course one revolution per day.

Such an area at the equator would revolve once a day about a *horizontal* (N. and S.) axis, but would obviously have no rotation about a

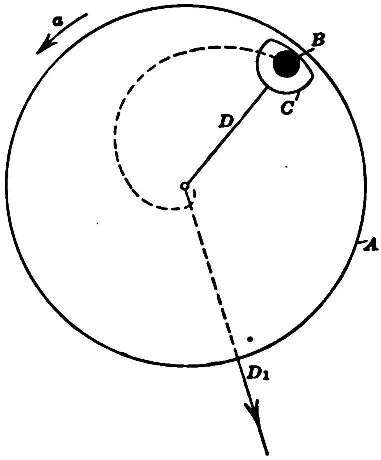

FIG. 145.

vertical axis. This fact accounts for the absence of cyclones near the equator.

It can be shown that such an area, in latitude θ, has an angular velocity ω about the vertical axis, given by the equation

$$\omega = \omega_1 \sin \theta$$

The rotation of the area is counterclockwise in the northern hemisphere (see rotation at the north pole above) and clockwise in the southern. Consequently, the air moves (*i.e.*, the wind blows) toward the center of a cyclone in a left-handed spiral path in the former case, and in a right-handed spiral path in the latter case, as explained above for the ball.

Hurricanes and Typhoons.—The hurricanes of the West Indies, and the Typhoons of China, might be called the "cyclones" of the tropical and sub-tropical regions. They are more violent and of smaller diameter than cyclones, their diameter rarely exceeding 400 miles, though they sometimes *gradually change* to cyclones and travel long distances through the temperate zones.

226. Tornadoes.—Tornadoes resemble hurricanes, but are much smaller, and usually more violent. Because of the terrific violence, narrow path, brief duration, and still more brief warning given, tornadoes have not been very satisfactorily studied, and much difference of opinion exists with regard to them. The visible part of a tornado consists of a depending, funnel-shaped cloud, tapering to a column which frequently extends to the ground. Due to the centrifugal force caused by the rapid rotation of the column, the air pressure within it is considerably reduced. Consequently as moist air enters the column it is cooled by expansion and its moisture condenses, forming the cloud which makes the column visible. At sea, tornadoes are called *Water Spouts.* The column is not water, however, but cloud and spray.

Origin.—Tornadoes usually develop to the southeast of the center of a cyclone. Sometimes several may rage simultaneously at different points in the same cyclone. Occasionally conditions of the atmosphere arise which are especially favorable to the formation of tornadoes. These conditions are a warm layer of air saturated with moisture next to the earth, with a layer of much cooler air above it. As, due to local disturbance, some of this heated moist air rises to the cooler regions, it precipitates part of its moisture, thus freeing a considerable amount of heat. This heat prevents the rising air from cooling so rapidly as it otherwise would, and consequently helps to maintain its tendency to rise. As this air rises, it is followed by other satusated air, which in turn receives heat by condensation of its water vapor. Thus the action, when once started, continues with great violence. The air rushing in from the surrounding country to take the place of the ascending air current acquires a rotary motion, just as already explained in connection with cyclones As the tornado advances it is constantly furnished with a new supply of hot, damp air and it will continue just so long as this supply is furnished, *i.e.*, until it passes over the section of country in which these favorable conditions exist. Tornadoes travel across the United States in a direction which is usually about east. A tornado may be likened to a forest fire, in that the one requires a continuous supply of moist air, the other, a continuous supply of fuel.

Tornadoes sometimes do not reach to the earth, which indicates that the favorable stratum of air upon which they "feed" is, at

least sometimes, at a considerable altitude. Some think it is always at a high altitude. This, the writer doubts. The moist stratum is probably very deep.

Extent.—The destructive paths of tornadoes vary in width from 100 ft. to 1/2 mile, and in length from less than a mile to 200 miles. A tornado which wrecks weak buildings over a path 1/8 mile in width may leave the ground practically bare for a width of 100 ft. or so.

Velocity.—The velocity of tornadoes varies from 10 to 100 miles per hour. It is estimated, however, that the wind near the center sometimes attains a velocity of 200 or 300 miles per hour, or even greater.

Judging by the effects produced, the velocity must be very great. An iron bed rail has been driven through a tree by a tornado. A thin-bladed shovel has been driven several inches into a tree. Such a shovel would not withstand driving into a tree with a sledge hammer. Splintered boards are frequently driven deep into the ground, and, by way of contrast, mention may be made of a ladder which was laid down, at a considerable distance from the path, so gently as to scarcely leave a mark on the ground. Shingles and thin boards have been found in great numbers 6 or 7 miles from the path, and probably 10 or 15 miles from where they began their flight.

The rapid rotational velocity at the center, tends to produce a vacuum, as already mentioned. It is conjectured that the pressure at the center of the tornado may be as much as 3 or 4 lbs. per square inch less than normal. If this be true, then, as the tornado reaches a building filled with air at nearly normal pressure, there will be an excess pressure within the building of say 3 lb. per square inch, or over 400 lbs. per square foot, tending to make the building explode. The position of the wreckage sometimes indicates that this is just what has taken place.

In spite of the great violence of tornadoes, few people are killed by them, because of their infrequency and limited extent. If a man were to live a few hundred thousand years he might reasonably expect to be caught in the path of a tornado, and if immune from death except by tornadoes he could not reasonably expect to live more than a few million years.

CHAPTER XVIII

STEAM ENGINES AND GAS ENGINES

227. Work Obtained from Heat—Thermodynamics.—Thermodynamics deals with the subject of the transformation of heat into mechanical energy, and *vice versa*, and the relations that obtain in such transformation under different conditions. No attempt will be made to give more than a brief general treatment of this important subject.

What is known as the *First Law of Thermodynamics* may be stated as follows: *Heat* may be transformed into *mechanical energy*, and likewise, *mechanical energy* may be transformed into heat, and in all cases, the ratio of the work done, to the heat so transformed, is constant. Conversely, the ratio of the work supplied, to the heat developed (in case mechanical energy is changed to heat energy by friction, etc.), gives the *same constant*. This constant, in the British system, is 778. Thus, if one B.T.U. of heat is converted into mechanical energy, it will do 778 ft.-lbs. of work; conversely, if 778 ft.-lbs. of work is converted into heat, it produces one B.T.U. For example, if 778 ft.-lbs. of energy is used in stirring 1 lb. of water, it will warm the water 1° F. The similar relation in the metric system is expressed by the statement that 1 calorie equals 4.187×10^7 ergs.

Illustrations of the First Law of Thermodynamics.—By means of the steam engine and the gas engine, heat is converted into mechanical energy. In bringing a train to rest, its kinetic (mechanical) energy is converted into heat by the brakes, where a shower of sparks may be seen. In inflating a bicycle tire, work is done in compressing the air, and this heated air makes the tube leading from the pump to the tire quite warm. In the fire syringe, a snug-fitting piston, below which some tinder is fastened, is quickly forced into a cylinder containing air. As the air is compressed it is heated sufficiently to ignite the tinder. In gas engines, preignition may occur during the compression stroke, due in part to the heat developed by the work of compression.

The Second Law of Thermodynamics.—The second law of thermodynamics is expressed by the statement that heat will not flow of itself (*i.e.*, without external work), from a colder to a warmer body. In the operation of the ammonia refrigerating apparatus, heat is taken continuously from the very cold brine and given to the very much warmer cooling tank; but the work required to cause this "uphill" flow of heat is done by the steam engine which operates the air pump.

Lord Kelvin's statement of the second law amounts to this: Work cannot be obtained by using up the heat in the coldest bodies present. Carnot (Sec. 236) showed that when heat passes from a hotter to a colder body (through an engine) the maximum fraction of the heat which may be converted into work is $\frac{T_1 - T_2}{T_1}$, in which T_1 and T_2 are, respectively, the temperatures of the two bodies on the *absolute* scale.

228. Efficiency.—While it is possible to convert mechanical energy, or work, entirely into heat, thereby obtaining 100 per cent. efficiency, it is impossible in the reverse process to transform more than a small percentage of heat energy into mechanical energy. It is, indeed, a very good steam engine that changes into work 1/5 of the heat energy of the steam furnished it by the boiler. Considering the large amount of heat that radiates from the furnace, and also the heat that escapes through the smoke stack, there is a further reduction in the efficiency. The *total* efficiency of a steam engine is the product of three efficiencies; that of the *furnace*, that of the *boiler*, and that of the *engine*.

The furnace wastes about 1/10 of the coal due to incomplete combustion, through escape of unburnt gases up the smoke stack, and unburnt coal into the ash pit. The *furnace efficiency* is, therefore, about 9/10 or 90 per cent. About 4/10 of the heat developed by the furnace escapes into the boiler room or up the smoke stack; so that the *boiler efficiency* is about 6/10 or 60 per cent. A good "condensing" engine converts into work about 1/5 of the heat energy furnished it by the boiler, in other words, its efficiency is about 20 per cent. The total efficiency E of the steam engine, which may be defined by the equation

$$E = \frac{\text{work done}}{\text{energy of fuel burned}}$$

has, then, the value $1/5 \times 6/10 \times 9/10$, or about 11 per cent.

Calculation of Efficiency.—The efficiency of the steam engine varies greatly with the care of the furnace, and the type and size of the boiler and engine. Few engines have a total efficiency above 12 per cent., and many of the smaller ones have as low as 4 or 5 per cent. efficiency, or even lower. Coal which has a heat of combustion of 14,000 B.T.U. per lb., contains $14{,}000 \times 778$ ft.-lbs. of energy per pound. One H.P.-hr. is 3600×550 ft.-lbs. Accordingly $\frac{3600 \times 550}{14000 \times 778}$ lb., or approximately 1/5 lb. of coal would do 1 H.P.-hr. of work if the efficiency of the engine were 100 per cent. If an engine requires 4 lbs. of coal per H.P.-hr., its efficiency is approximately $1/4 \times 1/5$, or 5 per cent. In order to make an *accurate* determination of the efficiency, the heat of combustion would have to be known for the particular grade of coal used.

Limiting, or Thermodynamic Efficiency.—Carnot (Sec. 236) showed that the efficiency of an ideal engine, which, of course, cannot be surpassed, is determined by the *two extreme temperatures of the working fluid* (steam or gas). If heat (say in steam) is supplied to the engine at 127° C. or 400° A., and the engine delivers it to the condenser at 27° C., or 300° A., then the maximum theoretical efficiency is $\frac{400-300}{400}$, or 25 per cent. Obviously, then, a gain in efficiency is obtained by using steam at a very high pressure, and consequently at a high temperature. The high efficiency of the gas engine is due partly to the great temperature difference employed, and partly to the fact that the "furnace" is in the cylinder itself, thereby reducing heat "losses."

Some gas engines (Sec. 237) have more than 30 per cent. efficiency. Gas engines are usually more troublesome than steam engines and also less reliable in their operation; nevertheless, because of their greater economy of fuel, they are coming into very general use.

The lightness of gas engines recommends them for use on automobiles, motorcycles, and flying machines. Engines weighing about 2.5 lbs. per H.P. have been made for use on aeroplanes. Indeed, the lightness of the gas engine has made possible the development of the aeroplane.

229. The Steam Engine.—A modern steam engine, fully

equipped with all of its essential attachments, is a very complicated mechanism.

In order to bring out more clearly the fundamental principles involved in the action of the steam engine, it seems best to omit important details found in the modern engine, since these details are confusing to the beginner, and therefore serve to obscure the underlying principles. In accordance with this idea, an exceedingly primitive engine is shown in Fig. 146. In Fig. 147, an engine is shown which is essentially modern, although certain details of construction are purposely omitted or modified, especially in the indicator mechanism, and in the valve mechanism.

In Fig. 146, A is a pipe which carries steam from the boiler to the cylinder B, through either valve a or valve b, depending upon which is open. P is the piston, and C is the piston rod, which

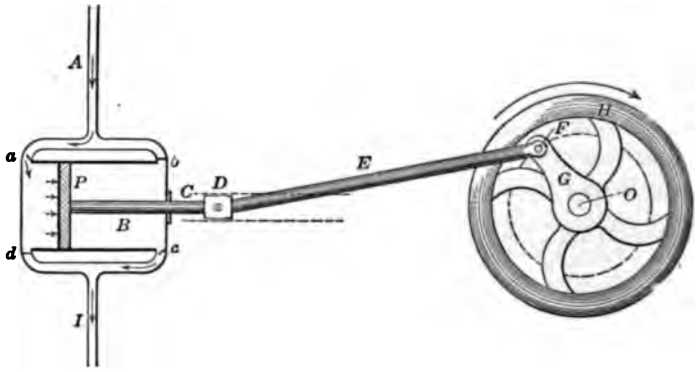

FIG. 146.

passes through the end of the cylinder (through steam-tight packing in a "stuffing box") to the crosshead D. As the piston is forced back and forth by the steam, as will be explained below, the crosshead moves to and fro in "guides," indicated by the broken lines. The crosshead, by means of the connecting rod E attached to the crank pin F, causes the crank G to revolve as indicated. The crank G revolves the crank shaft O, to which is usually attached a very heavy flywheel H in order to "steady" the motion.

If valves a and c are open, and b and d closed, the steam passes from the boiler into the cylinder, and forces P to the right. The exhaust steam to the right of P (remaining from a former stroke) is driven out through c to the air. When P reaches the

right end of the cylinder, valves a and c are closed, and b and d are opened, thus permitting steam to enter at b and force P to the left end again; whereupon the entire operation is repeated. These valves are automatically opened and closed at just the right instant by a mechanism connected with the crank shaft (Sec. 233). In practice, valve a would be closed when P had traveled to the right about 1/3 the length of the cylinder (Sec. 231).

Speed Regulation.—A Centrifugal Governor, driven by the engine, controls the steam supply, and hence the speed, by opening wider the throttle valve (valve not shown) in A if the speed is too low, and by partially closing it when the speed is too high, as explained in Sec. 63. It may be mentioned that some governors control the speed by regulating the cut-off (Sec. 231); that is, by admitting steam to the cylinder during a small fraction of the stroke, in case the speed becomes too high.

Compound and Triple Expansion Engines.—In the *Compound Engine,* the exhaust steam from cylinder B passes through pipe I to a second cylinder, where it drives the piston to and fro, just as the steam from pipe A drives the piston shown in the figure. If the exhaust steam from this second cylinder operates a third cylinder we have a *Triple Expansion Engine*—so-called because the steam expands three times. Obviously, because of this expansion, the second cylinder must be larger than the first, and the third larger than the second. By using steam at very high pressure (about 200 lbs. per sq. in.), and expanding it successively in there different cylinders, a much higher efficiency is obtained than with a single-cylinder engine. It will be evident, that the more the steam condenses on the walls of the cylinder, the more rapidly its pressure drops with expansion. It may be mentioned that the greater efficiency of the triple expansion engine is due principally to a reduction of this condensation.

Superheating.—Another method of reducing condensation is to superheat the steam. If the steam is conducted from the boiler to the engine through coiled pipes surrounded by moderately hot flame, it may thereby have its temperature raised as much as 200° F., and is then said to be superheated 200°. Superheated steam does not so readily condense upon expansion in the engine as does ordinary steam, and consequently gives a higher efficiency.

Increasing the Efficiency.—The efficiency of the steam engine

has been increased, step by step, by means of various improvements, prominent among which are, the expansive use of steam in the cylinder (Sec. 231), the expansion from cylinder to cylinder as in triple expansion engines, and the condensation of the exhaust steam ahead of the piston (Sec. 230) to eliminate "back pressure." To these may be added the use of higher steam pressure, and also the use of superheated steam.

230. Condensing Engines.—It will be observed, that in the above noncondensing engine, the steam from the boiler has to force the piston against atmospheric pressure (15 lbs. per sq. in). By leading the exhaust pipe I to a "condenser," which condenses most of the steam, this "back pressure" is largely eliminated. The *Condenser* consists of an air-tight metal enclosure, kept cool either by a water jet playing inside, or by cold water circulating on the outside. The former is called the *Jet Condenser* and the latter, the *Surface Condenser*. A pipe from an air pump leads to the condenser, and by means of this pipe, the air pump removes the water and air, maintaining in the condenser a fairly good vacuum. Assuming that the boiler pressure is, say, 60 lbs. per sq. in. (*i.e.*, 60 lbs. per sq. in. above atmospheric pressure), and that the condenser maintains in the cylinder, "ahead" of the piston, a partial vacuum of 2 lbs. per sq. in. pressure; it will be evident that the available working pressure will be increased to 73 lbs. per sq. in. ($15-2=13$, and $60+13=73$), and that therefore the efficiency will be increased in about the same ratio.

231. Expansive Use of Steam, Cut-off Point.—If, when the piston shown in Fig. 146 has moved to the right 1/4 the length of the cylinder, *i.e.*, when it is at 1/4 stroke, valve a is closed, then only 1/4 as much steam will be used as would have been used had valve a remained open to the end of the stroke. But the work done by the piston during the stroke will be more than 1/4 as much in the first case as in the second, hence steam is economized. If a is closed at 1/4 stroke, the *Cut-off Point* is said to be at 1/4 stroke.

The work done per stroke, if the valve a remains open during the full stroke, is FL (or Fd, since Work$=Fd$), in which F is the force exerted on the piston (its area A times the steam pressure p), and L is the length of the stroke. Consequently, the work done per stroke is pAL. If the cut-off is set at 1/4 stroke, then the full pressure is applied for the first quarter stroke only, and therefore the work done by the steam in this quarter stroke

is $pAL/4$. During the remaining 3/4 stroke, the enclosed steam expands to four times as great volume, and because of the cooling effect of expansion, it has its pressure reduced at the *end* of the stroke to *less* than $p/4$, the value which Boyles' Law would indicate. Assuming the *average pressure* during the last 3/4 stroke to be even as low as $p/3$, we have for the work of this 3/4 stroke

$$\frac{p}{3}\times\frac{3}{4}L\times A=\frac{1}{4}pAL$$

We see, then, that by using the *expansive* power of the steam during 3/4 of the stroke, we obtain the work $\frac{1}{4}$ pAL, which, added to $\frac{1}{4}$ pAL obtained from the first 1/4 stroke, gives $\frac{1}{2}$ pAL for the total work. But the work obtained per stroke by keeping the valve a open during the full stroke was pAL. Hence the total work per stroke, using the cut-off, is 1/2 as great as without, and the steam consumption is only 1/4 as great; therefore, the *Efficiency* is doubled, in this instance, by the use of the cut-off.

232. Power.—Since power is the rate of doing work, or, in the units usually employed, the amount of work done per second (Sec. 81), it will be at once evident that the product of the work per stroke, or PAL (Sec. 231), and the number of strokes (to the right) per second, or N, will give the power developed by the steam, which enters at the *left* end of the cylinder. That is, power $=PALN$, in which P is the *average difference in pressure upon the two sides of the piston during the entire stroke.* The average pressure is easily found from the indicator card (Sec. 234). As an aid to the memory, the symbols may be rearranged so as to spell the word $PLAN$.

If P is expressed in pounds per square inch, and A in square inches, then the *average* force PA exerted by the piston will be in pounds. If L is the length of the cylinder in feet, then PAL, the work done per stroke, will be expressed in foot-pounds. Finally, since N, the number of revolutions per second, is also the number of strokes to the *right* per second, the power, $PLAN$, developed by the *left end* of the cylinder, is given in foot-pounds per second. Dividing this by 550 gives the result in horse power; *i.e.*,

$$\text{H.P. (one end)}=\frac{PLAN}{550} \qquad (87)$$

If N represents the speed of the engine in revolutions per minute (R.P.M.), then, since 1 H.P. = 33000 ft.-lbs. per min., we have

$$\text{H.P. (one end)} = \frac{PLAN}{33000} \tag{87a}$$

If the cut-off point for the stroke to the left does not occur at exactly the same fraction of the stroke as it does for the stroke to the right, then the average pressure pushing the piston to the left will not be the same as that pushing it to the right, and the power developed by the right end of the cylinder will differ from that developed by the left end. This difference usually amounts to but a few per cent. of the total power.

233. The Slide Valve Mechanism.—The slide valve V (Fig. 147) is operated by what virtually amounts to a crank of length OO', called, however, an *Eccentric*. The eccentric consists of a circular disc J whose center is at O', attached to the crank shaft whose center is at O. Over J passes the strap K connected with the eccentric rod L. As the crank shaft revolves clockwise, O', which is virtually the right end of rod L, moves in the small dotted circle as indicated. This circular motion causes L to move to and fro, thus imparting to the slide valve V a to-and-fro motion. By adjusting the eccentric until the angle between OO' and the crank G has the proper value, the valve opens and closes the ports at the proper instants.

With the valve in the position shown, the steam from the boiler, entering the steam chest S through pipe A, passes through steam port a into the cylinder. The exhaust steam, from the preceding stroke, escapes through steam port b and exhaust port c into the exhaust pipe, which conducts the steam in the direction away from the reader to the condenser (not shown). An instant later, port a will be closed (cut-off point), and the steam then in the left end of the cylinder will expand (expansion period, Sec. 231) and push the piston to the right. As the piston approaches the right end, the valve V will close port b and at the same time open port a into the exhaust port c. This releases the steam in the left end of the cylinder, and is called the *release point*. Since b is closed before the piston reaches the right end of its stroke, there still remains some exhaust steam in the right end of the cylinder. This steam acts as a "cushion" and reduces the jarring. During the last part of the stroke, then, the piston is compressing exhaust steam. This is called the *compression*

period. About the time the piston reaches the right end of its stroke, valve *V* has moved far enough to the left to open port *b* to the steam chest, thus admitting "live" steam to the right end of the cylinder, and the return stroke, similar in all respects to the one we have just described, occurs.

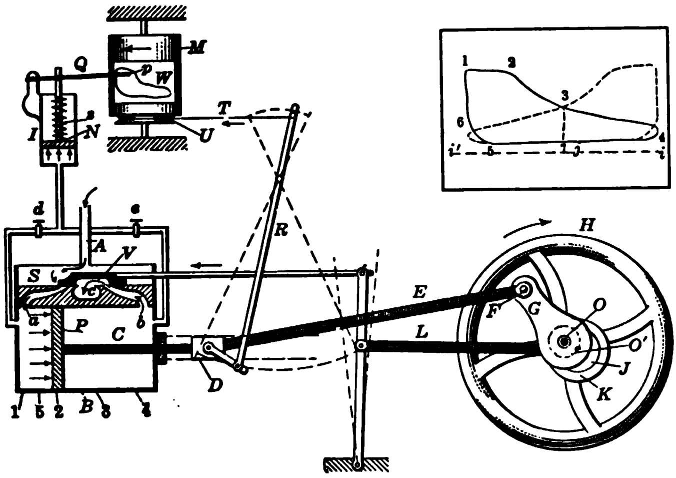

FIG. 147.

234. The Indicator.—The essentials of the indicator are shown in Fig. 147 (left upper corner). *I* is a small vertical cylinder containing a piston *N*, and is connected by pipes with the ends of the engine cylinder, as shown. If valve *e* is closed and valve *d* is open, it will be evident that, as the pressure in the left end of the cylinder rises and falls, the piston *N*, which is held down by the spring *s*, will rise and fall, and cause the pencil *p* at the end of the lever *Q* to rise and fall.

M is a drum, to which is fastened a "card" *W*. This drum is free to rotate about a vertical axis when the cord *T*, passing over pulley *U*, is pulled to the right. As the pull on *T* is released, a spring (not shown) causes the drum to rotate in the reverse direction.

It will therefore be seen that the to-and-fro (horizontal) motion of the crosshead *D*, by means of lever *R* and string *T*, causes the drum to rotate to and fro, and consequently move the card to and fro under the pencil *p*. If the pencil were stationary it would trace a straight horizontal line on the card.

Thus we see that the change of pressure in the cylinder causes the pencil to move up and down, while the motion of the drum causes the card to move horizontally under the pencil. In practice, both of these motions take place simultaneously, and the pencil traces over and over the curve shown. It will be seen that the motion of the card under the pencil exactly reproduces, on a reduced scale, the motion of the piston and crosshead. That is to say, when the piston P has moved to the right, say 1/4 the length of the cylinder, or is at "quarter stroke," the pencil p is 1/4 way across the indicator card, and so on.

The indicator card is shown, drawn to a larger scale, in the upper, right corner of Fig. 147. At the instants that the piston, in moving to the right, passes points 1, 2, 3, 4, the pencil p traces respectively, the corresponding points 1, 2, 3, 4, on the indicator card. As the piston moves back to the left from 4 to 5, pencil p traces from 4 to 5 on the curve. The indicator card shows that full steam pressure acts on P during its motion from 1 to 2; that at 2 the inlet valve at the left closes (*i.e.*, cut-off occurs, see slide valve, Sec. 233); and that the pressure of the enclosed steam, as it expands and pushes the piston through the remainder of the stroke, decreases, as indicated by the points 2, 3, and 4.

As the piston, on the return stroke, reaches the point marked 5, port a is closed and the *compression period* (Sec. 233) begins. This is shown on the indicator diagram by the rounded corner at 5. At the point marked 6, steam is again admitted through port a, and the pencil p rises to point 1 on the diagram. The different periods shown on the indicator diagram are, then, *admission* of steam from 1 to 2, *expansion* from 2 to 4, *exhaust* from 4 to 5, and *compression* from 5 to 6.

If the back pressure of the exhaust steam were entirely eliminated by the condenser, the pencil on the return stroke would trace a lower line than 4–5, say, ii'. The distance j is then a measure of the back pressure, which would be about 2 or 3 lbs. per sq. in. when using a condenser, and about 15 lbs. per sq. in. without a condenser.

To obtain the indicator diagram for the other end of the cylinder (shown in broken lines in the figure), valve d is closed and valve e is opened. This curve *should be* (frequently it is not) a duplicate of the curve just discussed, in the same sense that the right had is a duplicate of the left.

Use of the Indicator Card.—The indicator card enables the operator to tell whether the engine is working properly; *e.g.*, whether the admission or the cut-off are premature or delayed, requiring valve adjustment; or whether or not the "back pressure" is excessive due to fault of the condenser, and so on.

Another use of the indicator card is in determining the average working pressure which drives the piston. By subjecting the indicator piston to known changes of pressure as read by a steam gauge, we may easily determine how many pounds pressure per square inch corresponds to an inch rise of the pencil p. Having thus *calibrated* the indicator, suppose we find that an increase of 40 lbs. per sq. in. causes p to rise 1 in. Let the vertical dotted line through 3 across the indicator curve be 1.5 in. in length. We then know that at 1/2 stroke the available working pressure on the piston, or the difference between the pressure on the left and the exhaust pressure on the right side of the piston, is 60 lbs. per sq. in. Further, suppose that when we divide the total area of the curve by its horizontal length we obtain 2 in. for its *average* height. We then know that the *average* working pressure P for the entire stroke is 80 lbs. per sq. in. This average value of p, thus found, is the P of Eq. 87, which gives the horse power (H.P.) of the engine.

Since the average height of the indicator diagram gives the average working pressure on the piston, and since its length is proportional to the length of the stroke of the piston, we see that its area is proportional to, and is therefore a measure of, the work done per stroke, and hence a measure of the *power*. Accordingly, any adjustment of valves or other change which *increases* this *area* without altering the speed, produces a proportional *increase* in *power*. If, further, the same amount of steam is used as before, then there is a proportional *increase* in *efficiency*.

235. The Steam Turbine.—In recent years, some large and very efficient steam turbines have been installed. Because of their freedom from jarring, which is so great in the reciprocating steam engines, and also because of their high speed, they are being used more and more for steamship power.

In the steam turbine, a stream of steam impinges against slanting vanes and makes them move just as air makes windmill vanes move (Sec. 149). It differs from the windmill, however, in that the stream of steam must be confined, just as water is in the water turbine. Note that the windmill might be called an

air turbine. The steam turbine differs from the windmill also in that each portion of steam must pass successively several movable vanes alternating with fixed vanes, as indicated in Fig. 148. The *rotor vanes*, attached to the rotating part called the *rotor*, are indicated by heavy curved lines. The *stator vanes* are stationary and are attached to the tubular shell which surrounds the rotor and confines the steam. The stator vanes are indicated in the sketch by the light curved lines. It will be understood that the reader is looking toward the axis of the rotor, which is indicated by the horizontal line.

As the steam passes to the right, the fixed vanes deflect it somewhat downward, and the movable vanes, somewhat up-

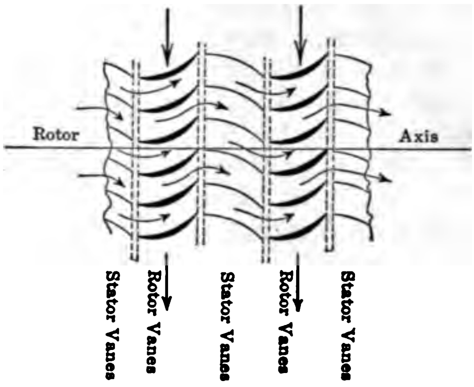

Fig. 148.

ward, as indicated by the *light* arrows. The reaction to this upward thrust exerted upon the steam by the movable vanes causes these vanes to move downward (as explained in connection with Fig. 100, Sec. 149, and as indicated by the heavy arrows) with an enormous velocity, and with considerable force.

To allow for the expansion of the steam, the above-mentioned tubular shell increases in diameter to the right, and the rotor vanes increase in length to the right. The stator vanes are also longer at the right.

If the steam, as it passes to the right from the turbine, enters a condenser, the effective steam pressure and likewise the efficiency, will be increased just as is the case with the reciprocating steam engine.

236. Carnot's Cycle.—Nearly a century ago, the French physicist, Sadi Carnot, who may be said to have founded the science of thermodynamics, showed by a line of reasoning in which he used a so-called "ideal engine" (Fig. 149), that by taking some heat H_1, from one body and giving a smaller amount H_2, to a colder body, an amount of heat $H_1 - H_2$ may be converted into work, and that the percentage of the heat that may be so converted depends only upon the temperatures of the two bodies.

In Fig. 149 (Sketch I–II), let a cylinder with non-conducting walls, a non-conducting piston, and a perfect conducting base in contact with

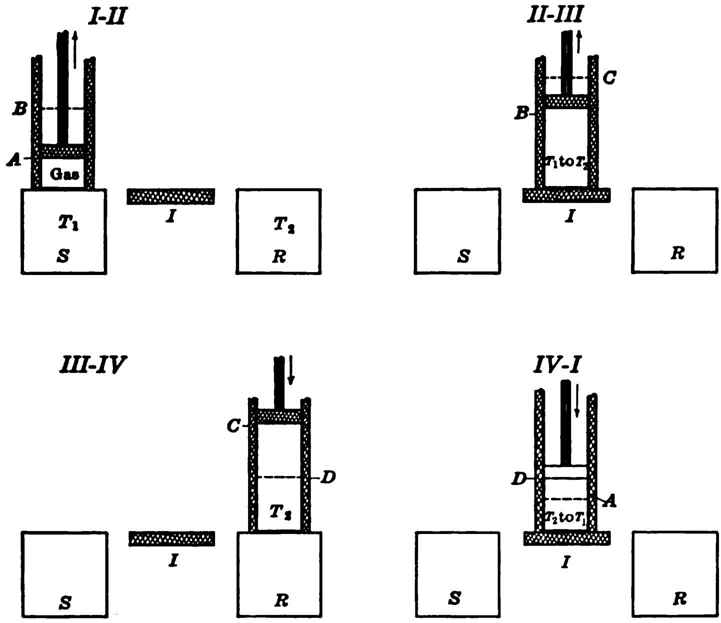

FIG. 149.

the perfect conducting "source" S, contain some gas at a temperature T_1. (Parts that are perfect non-conductors of heat are shown crosshatched.) Let the gas be a perfect gas, *i.e.*, one which obeys Boyle's law and Charles' law. Let I be a perfectly non-conducting slab, R, the perfectly conducting "refrigerator," and let S be kept constantly at the temperature T_1, and R, at the temperature T_2 on the *absolute* scale.

We shall now put the gas through four different stages, I, II, III, and IV. In Fig. 149, we shall indicate the four processes of changing from stages I to II, II to III, III to IV, and from IV back to I, by the four sketches marked respectively, I–II, II–III, III–IV, and IV–I. The pis-

ton, in the four stages, assumes successively the positions A, B, C, and D, and the corresponding pressures and volumes of the gas are indicated, respectively, by the points A, B, C, and D on the pressure-volume diagram (Fig. 150).

Process 1: As the gas is permitted to change from stage I to II (sketch marked I–II) by pushing the piston from A to B, it does work on the piston (force $\times$ distance or pressure $\times$ volume, Sec. 203), and therefore would cool itself were it not in contact with the perfect conductor S. This contact maintains its temperature at T_1° A., *i.e.*, the gas *takes* an amount of *heat*, say H_1, *from source* S, and its expansion is

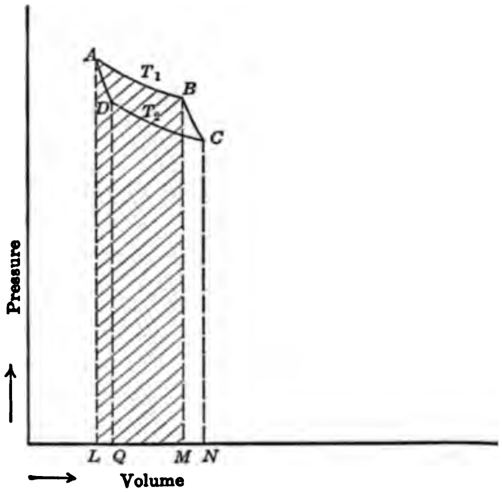

FIG. 150.

represented in Fig. 150 by the portion AB of an *isothermal*. Since work is the product of the average pressure and the change in volume LM (Sec. 203), we see that the work done by the gas is proportional to, and is represented by, the shaded area $ABML$.

Process 2: The cylinder is next placed on the non-conducting slab I, and the gas is permitted to expand and push the piston from B to C. In this process (sketch II–III), since the gas is now completely surrounded by non-conductors of heat, the work of expansion is done at the expense of the *heat of the gas itself*, and its temperature is thereby lowered. Consequently, as the volume increases, the pressure decreases more rapidly than for the previous *isothermal* expansion. In case the energy (heat) of expansion must come from the gas itself, as in this instance, the expansion is *Adiabatic*. AB is an *isothermal line* and BC is an *Adiabatic line*. The gas is now at stage III, and the work done by the gas

in expanding from B to C is represented by the area $BCNM$ which lies below the curve BC (compare Process 1).

Process 3: The cylinder is next placed upon the cold body or "refrigerator" R (sketch III–IV), and the gas is compressed from C to D. Since R is a perfect conductor, this will be an *isothermal compression*, and, as the volume is slowly reduced, the pressure will gradually increase as represented by the *isothermal* CD. The gas is now at stage IV, and is represented by point D on the diagram. The work done *upon* the gas in this process is, by previous reasoning, represented by the area $CDQN$. The work of compressing the gas develops heat in it, but this *heat*, say H_2, is *immediately given to the refrigerator.*

Process 4: Finally, the cylinder is again placed upon the non-conducting slab I, and the piston is forced from D back to the original position A. Since the gas is now surrounded by a perfect non-conductor, the heat of compression raises its temperature to T_1. As the volume is gradually decreased, the pressure increases more rapidly than before, because of the accompanying temperature rise, which accounts for the fact that DA is steeper than CD. In this case, of course, we have an *Adiabatic Compression* and the line DA is an *adiabatic line.* The work done upon the gas in this process is represented by the area $DALQ$.

Efficiency of Carnot's Cycle.—From the preceding discussion, we see that the work done by the gas during the two expansions (Processes 1 and 2) is represented by the area below ABC; while the work done upon the gas during the two compressions (Processes 3 and 4) is represented by the area below ADC. Consequently, the net work obtained from the gas is represented by the area $ABCD$.

It has just been shown (Process 1) that the gas as it expands from A to B, does work represented by the area $ABML$, and since its temperature remains constant, it must take from the source S, an amount of heat energy equal to this work. Let us call this heat H_1. Similar reasoning shows that when compressed from C to D, the gas gives to the refrigerator an amount of heat H_2 represented by the area $CDQN$. During the other two processes (adiabatic processes) the gas can neither acquire nor impart heat. Accordingly, for this cycle, the efficiency is given by the equation

$$E=\frac{\text{work done}}{\text{heat received}}=\frac{ABCD}{ABML}=\frac{ABML-CDQN}{ABML}\text{(approx.)}=\frac{H_1-H_2}{H_1} \tag{88}$$

Now, the heat contained by a gas, or any other substance, is

proportional to the temperature of the substance (assuming that the body has a constant specific heat). Consequently,

$$E=\frac{H_1-H_2}{H_1}=\frac{T_1-T_2}{T_1}$$

This equation shows (as mentioned in Sec. 228) that if the absolute temperature T_1 of the "live" steam as it enters the cylinder from the boiler is 400° A. and the temperature T_2 of the condenser is 300° A., then the maximum theoretical efficiency of the engine is $\frac{400-300}{400}$ or 25 per cent. For a rigorous, and more extended treatment of this topic consult advanced works.

237. The Gas Engine—Fuel, Carburetor, Ignition, and Governor.—In the gas engine, the pressure which forces the piston along the cylinder is exerted by a hot gas, instead of by steam as in the case of the steam engine. The gas engine also differs from the steam engine in that the fuel, commonly an explosive mixture of gasoline vapor with air, is burned (*i.e.*, explosion occurs) within the cylinder itself. For this reason, no furnace or boiler is required, which makes it much better than the steam engine for a portable source of power. Gas engines may be made very light in proportion to the power which they will develop. The weight per H.P. varies from several hundred pounds for stationary engines, to 10 lbs. for automobiles. As has already been mentioned, the lightness of the gas engine (as low as 2.5 lbs. per H.P. for aeroplanes) has made aeroplane flight possible.

The fact that a gas engine may be started in an instant (*i.e.*, usually), and that the instant it is stopped the consumption of fuel ceases, makes it especially adapted for power for automobiles, or for any work requiring intermittent power. The fact that the power can be instantly varied as required is also a point in its favor.

Fuel.—*Gasoline* is the most widely used fuel for gas engines. It is readily vaporized, and this vapor, mixed with the proper amount of air as it is drawn into the cylinder, is very explosive and is therefore readily ignited. Complete combustion is easily obtained with gasoline; so that it does not foul the cylinder as some fuels do. *Kerosene* is much less volatile than gasoline, but may be used after the cylinder has first become heated by the use of gasoline. *Alcohol* may also be used. *Crude Petroleum* is used in some engines. *Illuminating Gas*, mixed with air, may

be used as a fuel. Natural gas, where available, forms an ideal fuel, and is used in some large power plants. The use of "*Producer*" *Gas* requires considerable auxiliary apparatus, but because of its cheapness, it is profitably used by stationary engines.

Briefly, producer gas is formed by heating coal while restricting the air supply, so that the carbon burns to carbon monoxide (CO) which is a combustible gas, instead of to carbon dioxide (CO_2), which is incombustible. If some steam (H_2O) is admitted with the air, the steam is decomposed into oxygen (O) which combines with the carbon and forms more carbon monoxide (CO). The remaining hydrogen constituent (H) of the steam is an excellent fuel gas. All of these gases pass from the coal through various cooling and purifying chambers, either directly into the gas engine, or into a gas tank to be used as required.

The Carburetor.—The carburetor is a device for mixing the vapor of the gasoline, or other liquid fuel, with the air which passes into the cylinder, thus forming the "charge." The explosion of this charge develops the pressure which drives the piston. As the air being drawn into the engine rushes past a small nozzle connected with the gasoline supply (see *C*, Fig. 153, left sketch), the gasoline is "drawn" out of the nozzle (see atomizer, Sec. 156) in the form of a fine spray, which quickly changes to vapor, and is thereby thoroughly mixed with the air to form the "charge." This thorough mixing is essential to complete combustion. If kerosene is used, the air must be previously heated in order to vaporize the spray. It is well to preheat the air in any case.

Ignition.—The charge is usually ignited electrically, either by what is called the "jump spark" from an induction coil, or by the "make-and-break" method. An induction coil consists of a bundle of iron wires, upon which is wound a layer or two of insulated copper wire, called the primary coil. One end of this primary coil is connected by a wire directly to one terminal of a battery, while the other end is connected to the opposite terminal of the battery through a vibrator or other device, which opens and closes the electrical circuit a great many times per second. On top of the primary coil, and, as a rule, carefully insulated from it, are wound a great many turns of fine wire, called the secondary coil. When the current in the primary circuit is broken, a spark will pass between the terminals of the second-

ary, provided they are not too far apart. The "spark distance" of the secondary varies from a small fraction of an inch to several feet, depending upon the size and kind of induction coil. For ignition purposes, only a short spark is required. By means of a suitable mechanism, this spark is made to take place between two points in the "spark plug" (*B*, Fig. 153) within the cylinder at the instant the explosion should occur.

In the "make-and-break" method of ignition, neither the secondary nor the vibrator is needed. One terminal of the primary coil, which, with its iron wire "core," is called a "spark coil," is connected directly to the *firing pin* which passes through a hole into the cylinder. The other terminal of the primary is connected to one pole of a battery. From the other pole of the battery a wire leads to a metal contact piece which passes into the cylinder from which it is insulated, at a point near the firing pin. By means of a cam, this firing pin is made to alternately touch and then move away from the metal contact piece within the cylinder. Consequently, by proper adjustment of the cam, the circuit is broken by the firing pin and the gas is ignited at the instant the explosion is desired. If the spark occurs when the piston is past dead center it is said to be *retarded*, if before, *advanced*. Engines running at very high speed require the spark to be advanced, or the flame will not have time to reach all of the gas until rather late in the stroke. The indicator card will tell whether or not advancing the spark increases the power in a given instance. If the spark is advanced too far "back-firing" results, with its attendant jarring and reduction of power.

The electric current may be produced by a "magneto." The magneto generates current only when the engine is running; so that a battery must be used when starting the engine, after which, by turning a switch, the magneto is thrown into the circuit and the battery is thrown out.

Cooling.—To prevent the cylinder from becoming too hot, a "water jacket" is provided. The cylinder walls are made double, and the space between them is filled with water. This water, as it is heated, passes to the "radiator" and then returns to the water jacket again. The water circulation is maintained either by a pump, or by convection. The radiator is so constructed, that it has a large radiating surface. A fan is frequently used to cause air to circulate through the radiator more rapidly. In some automobiles air cooling is used entirely, the cylinder

being deeply ribbed so as to have a large surface over which the air is forced in a rapid stream.

The Governor.—Commonly some form of the *Centrifugal Governor* (Sec. 63) is used to control the speed. In the "hit-or-miss" method no "charge" is admitted when the speed is too high. This causes fluctuations in the speed which are readily noticeable. In other methods of speed control, either the quantity of "richness" (proportion of gas or gasoline vapor to air) of the charge is varied to suit the load. If the load is light, the governor reduces the gas or gasoline supply; or else it closes the intake valve earlier in the stroke, thereby reducing the quantity of the charge.

238. Multiple-cylinder Engines.—With two-cycle engines (Sec. 240), an explosion occurs every other stroke; while in the four-cycle engine (Sec. 239) explosions occur only once in four strokes (*i.e.*, in two revolutions). It will be seen that the applied torque is quite intermittent as compared with that of the steam engine. If an engine has six cylinders, with their connecting rods attached to six different cranks on the same crank-shaft, then, by having the cranks set at the proper angle apart and by properly timing the six different explosions, a nearly uniform torque is developed. The six-cylinder engine is characterized by very smooth running. The four-cylinder engines, and even the two-cylinder engines, produce a much more uniform torque than the single-cylinder engines.

239. The Four-cycle Engine.—In the so-called *four-cycle* engine, a complete cycle consists of *four strokes*, or two revolutions. The four strokes are, suction or *charging*, *compression*, *working*, and *exhaust*. The stroke, at the beginning of which the explosion occurs, is the working stroke. With this engine, every fourth stroke is a working stroke; whereas, in the steam engine, every stroke is a working stroke. The operation of this engine will be understood from a discussion of Fig. 151. In the upper sketch, marked *I* (Fig. 151), valve *a* is open and valve *b* is closed, so that as the piston moves to the right the "suction" draws in the charge from the carburetor. This is the charging stroke. On the return stroke of the piston (Sketch II), both valves are closed and the charge is highly compressed.

As the piston reaches the end of its stroke, the gas then occupying the clearance space, or "combustion chamber," is ignited by means of either the "firing pin" *c* or a "spark plug," depending

upon which method of ignition is used. Ignition may occur either at, before, or after "dead center." (See Ignitio, nSec. 237.) The "explosion," or the *burning* of the gasoline vapor, produces a very high temperature and therefore, according to

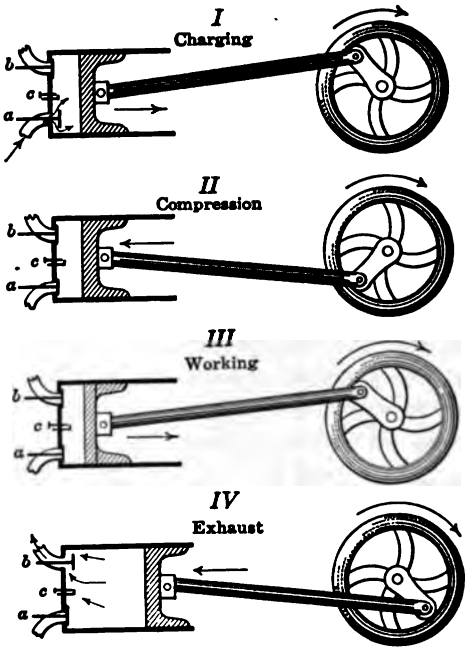

FIG. 151.

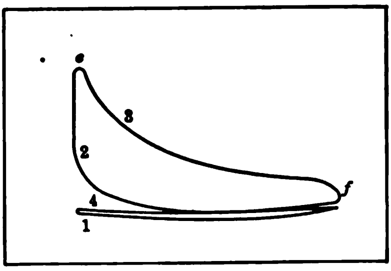

FIG. 152.

the law of Charles, a very high pressure. This high pressure pushes the piston to the right. This stroke is called the working stroke (Sketch III). As the piston again returns to the left, valve *b* is open, and the burned gases escape. This is the exhaust

stroke. The exhaust is very noisy unless the exhaust gases are passed through a muffler.

In many engines there is no piston rod, the connecting rod being attached directly to the piston as shown. The valves are operated automatically by cams, or other devices connected with the crank shaft so that by proper adjustment, exact timing may be obtained.

Indicator Card.—An indicator mechanism may be connected with the cylinder just as with the steam-engine cylinder (Sec. 234). In Fig. 152, is shown the indicator card from a four-cycle engine. The line marked 1 shows the pressure corresponding to the charging stroke (Sketch I, Fig. 151). Line 2 shows the pressure during the compression stroke (Sketch II, Fig. 151). At point

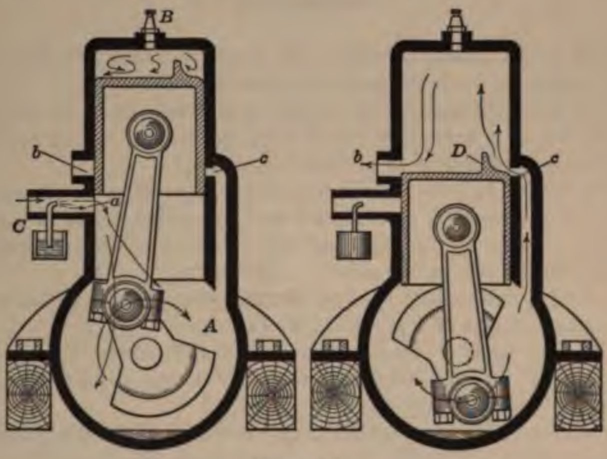

FIG. 153.

e, the explosion has occurred, and the pressure has reached a maximum. Line 3 represents the pressure during the working stroke (Sketch III), showing how it varies from the maximum down to *f*. Line 4 shows the pressure during the exhaust stroke (Sketch IV, Fig. 151).

240. The Two-cycle Engine.—The operation of the two-cycle engine will be understood from a discussion of Fig. 153. As the piston moves upward, compressing a previous charge, it produces suction at port *a* (left sketch), and draws in the charging gas from the carburetor *C* into the crank case *A*, which is air-tight in this type of engine. As the piston reaches the top of its

stroke, the charge is ignited by the spark plug *B*, and explosion occurs. As the piston now descends it is driven, with great force, by the high pressure of the heated gases. This is the working stroke. As soon as the piston passes below the exhaust port *b* (right sketch) the exhaust gas escapes, in part. An instant later, the piston is below port *c*, and part of the gas in the crank case, which gas is now slightly compressed by the descent of the piston, rushes through port *c*. As this charge enters port *c*, it strikes the baffling plate *D*, which deflects it upward, thus forcing most of the remaining exhaust gas out through port *b*. As the piston again rises, it compresses this charge preparatory to ignition, and the cycle is completed.

PROBLEMS

1. If all of the energy developed by a mass of iron in falling 778 ft. is used in heating it, what will be its temperature rise?

2. If the complete combustion of 1 lb. of a certain grade of coal develops 13,000 B.T.U.'s of heat, how much work (in ft.-lbs.) would it perform if is used in a heat engine of 10 per cent. efficiency?

3. How many H.P.-hours of potential energy does a pound of coal (13,500 B.T.U.'s per lb.) possess, and how many H.P.-hours of work can a good steam engine (say of 12.5 per cent. efficiency) obtain from it? Note that one horse-power for one second is 550 ft.-lbs.

4. How long would a ton of coal, like that mentioned in Problem 2, run a 10-H.P. steam engine of 6 per cent. total efficiency?

5. How high would the heat energy (14,000 B.T.U.'s per lb.) from a given mass of coal lift an equal mass of material, if it were possible to convert all of the heat of the coal into mechanical energy?

6. Find the H.P. of a noncondensing steam engine supplied during full stroke with steam at 80 lbs. per sq. in. pressure (80 lbs. is the available working pressure), when making 120 R.P.M. (4 strokes per sec.); the length of stroke being 2 feet and the cross section of the piston being 150 sq. in.

7. How many pounds of water at 70° F. will be changed to steam at 212° F. for each pound of coal (Prob. 2) burned in a furnace of 90 per cent. efficiency, heating a boiler of 70 per cent. efficiency.

8. An engine whose speed is 150 R.P.M., has a piston 15 in. in diameter which makes a 2-ft. stroke. The indicator diagram is 4 in. long and has an area of 9 sq. in. The indicator spring is a "50-lb. spring," *i.e.*, a rise of 1 in. by the indicator pencil indicates a change in pressure of 50 lbs. per sq. in. What is the power of the engine?

9. Find the H. P. of the engine (Prob. 6) with cut-off set at one-quarter stroke, the average pressure during the remaining 3/4 stroke being 30 lbs. per sq. in.

10. Find the H.P. of the engine (Prob. 6) with cut-off at half stroke, the pressure during the last half of the stroke being 30 lbs. per sq. in.

11. How many B.T.U.'s will a 1/2-oz. bullet develop as it strikes the target with a velocity of 1800 ft. per sec.? If this heat were all absorbed by the bullet (lead) what would be its temperature rise?

12. What is the limiting theoretical efficiency (thermodynamic efficiency) of a steam engine whose boiler is at 180° C., and whose condenser is at 50° C?

INDEX

The numbers refer to pages.